Quadratic Equations

A-REI.B.4 & A-CED.A.1 Conceptual Understanding, Procedural Skill and Fluency, and Application Mini-Assessment by Student Achievement Partners

OVERVIEW

This mini-assessment is designed to illustrate standards A-REI.B.4 and A-CED.A.1, which set an expectation for students to create, use, and solve quadratic equations. This mini-assessment is designed for teachers to use in the classroom, for self-learning, or in professional development settings to:

- Evaluate students' understanding of A-REI.B.4 and A-CED.A.1 in order to prepare to teach this material or to check for student ability to demonstrate understanding and apply these concepts;
- Gain knowledge about assessing quadratic equations; and
- Use in professional development as an illustration of CCSS-aligned assessment problems.

MAKING THE SHIFTS

This mini-assessment attends to focus as it addresses quadratic equations, which are widely applicable pre-requisites for college and careers.[1] This mini-assessment highlights coherence across grades as these questions build on similar work with linear equations. The mini-assessment targets all three aspects of rigor – conceptual understanding, procedural skill/fluency and application.

Problem	Primary Aspect(s) of Rigor
1	Conceptual Understanding
2	Procedural Skill/Fluency
3	Procedural Skill/Fluency
4	Procedural Skill/Fluency and Conceptual
5	Application

A CLOSER LOOK

A-REI.B.4. Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

The first problem in the mini-assessment consists of a series of quick conceptual questions to assess understanding of how quadratic equations work. The second problem contains a series of equations for students to solve using any process they choose, while Problem #3 explicitly requires completing the square (as is expected in the standard). The next series of problems targets conceptual understanding and procedural skill as students must first define (implicitly or explicitly) and solve for variables, and then answer the question. The last problem highlights application and addresses the modeling domain (pp. 72 and 73 of the CCSSM).

If students struggle with the longer application problem, teachers may differentiate it by giving the problem as group work, providing leading questions to struggling students, and/or leading a whole class discussion about ways to start the problem. As students gain familiarity with these items, these scaffolds can be removed so that students can engage with the mathematical practices.

A-CED.A.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

[1] For more on the widely applicable prerequisites for a range of college majors, postsecondary programs, and careers, see http://achievethecore.org/prequisites.

For a direct link, go to: http://achievethecore.org/page/976/quadratic-equations-mini-assessment-detail-pg

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO GRADE-LEVEL CONTENT

The conceptual questions in #1 can be answered efficiently if students see structure (MP.7 – Look for and make use of structure). Students must reason quantitatively and abstractly in problem #4 to define variables and interpret the solutions (MP.2 – Reason abstractly and quantitatively). Problem #5 requires MP.1 (Make sense of problems and persevere in solving them). Students will need to evaluate different entry points into the problem and continue to evaluate their progress as they persevere toward a solution. Students should look at the situation and begin to create variables and/or diagrams that help represent the givens and relationships. Students will have to engage in these processes at multiple times as they get to a certain step of the problem solving process and need to re-evaluate their progress to the ultimate goal.

Name:____Date:____________________

1. The table below contains single equations in a single variable. Check the appropriate box to show whether there is no real solutions, exactly 1 real solution, or exactly 2 real solutions. (1 point for each equation)

		No Real Solutions	Exactly 1 Real Solution	Exactly 2 Real Solutions
a.	$(a+5)^2 = 25$			
b.	$(n-5)^2 = 25$			
c.	$(z+5)^2 = -25$			
d.	$(x-5)^2 = 0$			
e.	$16 - (l+5)^2 = 25$			
f.	$(f+1)^2 = (f+2)^2$			
g.	$5b^2 = 5b^2 + 1$			

2. Solve these equations. (1 point for each equation)

a. $13g = \frac{1}{2}g^2 + 12\frac{1}{2}$

b. $7h^2 + 6 + 2h = h^2 + 4h + 26 + 5h$

c. $(1.6J - 0.2)^2 = 1$

d. $42K + 112 = 7K^2$

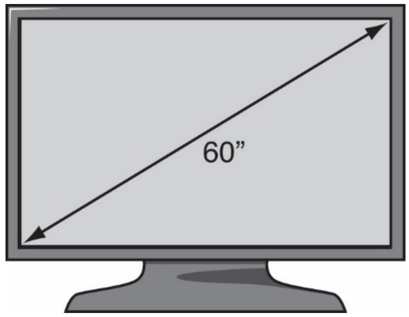
3. Solve this equation by completing the square. (1 point)

$$4L^2 + 16L = 65$$

4. The product of two consecutive positive integers is 380.

What are the two integers? (1 point)

5. TV screens are measured on the diagonal. The diagram below shows a 60” TV screen.



For this TV, the ratio of height to width[2] is 0.618.

a. What is the area of this TV screen? (2 points)

b. For any TV screen with a height-to-width ratio of 0.618, write a function $A(d)$ that gives the screen area A, in square inches, when the length of the diagonal is d (measured in inches). (1 point)

c. The cost of making any size TV screen is $0.0373A(d) + 0.5d + 10$. What is the largest screen size that can be built for $75? (2 points)

[2] Said another way, the ratio of width to height ratio of the TV is 1.618—the approximate value of a special number in mathematics called the Golden Ratio. Rectangles based on the Golden Ratio have been studied since the time of the ancient Greeks. If you are interested in learning more about the Golden Ratio, please visit: https://www.khanacademy.org/math/geometry/intro_euclid/v/the-golden-ratio.

1.

		No Real Solutions	Exactly 1 Real Solution	Exactly 2 Real Solutions
a.	$(a+5)^2 = 25$			✓
b.	$(n-5)^2 = 25$			✓
c.	$(z+5)^2 = -25$	✓		
d.	$(x-5)^2 = 0$		✓	
e.	$16-(l+5)^2 = 25$	✓		
f.	$(f+1)^2 = (f+2)^2$		✓	
g.	$5b^2 = 5b^2 + 1$	✓		

2a. $g = 1$ or 25

2b. $h = -\frac{4}{3}$ or $\frac{5}{2}$

2c. $J = -0.5$ or 0.75

2d. $K = -2$ or 8

3. $L = -6.5$ or 2.5

4. 19 and 20 [Students could solve using an equation like: $(x)(x+1) = 380$]

5. Sample Answer:

a. I can name the width of the TV to be w inches. Then, the height of the TV is 0.618*w*. So,

$$w^2 + (0.618w)^2 = 60^2$$
$$w^2 + 0.381924w^2 = 60^2$$
$$1.381924w^2 = 60^2$$
$$w^2 = \frac{60^2}{1.381924}$$
$$w = \frac{60}{\sqrt{1.381924}}$$
$$w \approx \frac{60}{1.1756} \approx 51 \text{ inches}$$

So, since the width of the TV is 51 inches, the height of the TV will be 0.618 * 51 = 31.5 inches. This will yield an area of 51 * 31.5 = **1606.5 square inches.**

b. I know from part a that the relationship between the width and the diagonal is given by the equation, $w = \frac{d}{\sqrt{1.381924}}$. I can use this to write a function giving the screen area for any length diagonal:

$$A(d) = \frac{d}{\sqrt{1.381924}} \times 0.618 \times \frac{d}{\sqrt{1.381924}}$$
$$A(d) = \frac{0.618}{1.381924} d^2$$
$$\boldsymbol{A(d) = 0.45d^2}$$

c. Since the cost of making any size TV screen is $0.0373A(d) + 0.5d + 10$, the largest screen that can be made for $75 can be solved using the following equation:

$$75 = 0.0373A(d) + 0.5d + 10$$
$$75 = 0.0373(0.45d^2) + 0.5d + 10$$
$$75 = 0.016785d^2 + 0.5d + 10$$
$$0.016785d^2 + 0.5d - 65 = 0$$
$$d = \frac{-0.5 \pm \sqrt{0.5^2 - 4(0.016785)(-65)}}{2(0.016785)}$$
$$d = \frac{-0.5 \pm \sqrt{4.6141}}{0.03357}$$
$$d = \frac{-0.5 \pm 2.15}{0.03357}$$
$$d = 49.15 \; inches$$

So, the largest TV screen that can be made for $75 is about **49 inches on the diagonal**.

Sample Rubric (5 total points):

a. 1 point for correctly determining one length.
1 point for correctly determining the area of the screen.

b. 1 point for correctly determining the function in terms of the diagonal length.

c. 1 point for correctly substituting the function from b and simplifying.
1 point for correctly solving using the quadratic equation or by graphing.