Looking For and Making Use of Structure – Quadratic Equations 1

Sample task from achievethecore.org
Task by CME Project, EDC, annotation by Student Achievement Partners

GRADE LEVEL High School

IN THE STANDARDS A-REI.B.4, MP.7

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Rewards the practice of looking for and making use of structure (MP.7)
- Solvable by casting the equation into standard form, although considerable procedural fluency is required for this method. In that case, the task rewards procedural skill as well as perseverance (MP.1)

In the classroom:

- Allows individuals, groups, or the class as a whole to suggest ideas for how to proceed
- Allows the class to compare multiple methods of solving
- Does not require extensive setup on the part of the teacher

MAKING THE SHIFTS[1]

Focus	A.REI.B.4 belongs to the Widely Applicable Prerequisites for College and Careers[2]
Coherence	Chunking (seeing parts of an expression as a single object) is a key algebraic skill, useful in factoring, completing the square, and other mindful algebraic calculations, and not limited to quadratics alone
Rigor[3]	Conceptual Understanding: primary in this task
	Procedural Skill and Fluency: primary in this task
	Application: not targeted in this task

[1]For more information read Shifts for Mathematics.
[2]For more information, see Widely Applicable Prerequisites.
[3]Tasks will often target only one aspect of rigor.

COMMENTARY AND SOLUTION

The practice of looking for and making use of structure (MP.7) often amounts to deferring evaluation. In this case, instead of immediately expanding $(3x-2)^2$, the student might pause, hold both hands in his or her lap, and examine the structure of the equation. Does any idea come to mind when we see $3x-2$ and $6x-4$ next to one another? Solution 1 below develops this line of thought.

Alternatively, Solution 2 shows the route a student might take if he or she is procedurally fluent and has a habit of perseverance (MP.1). There is nothing wrong with this "straight ahead" approach; in fact, it has the virtue of being general. One will note, however, how much longer and more technically difficult that approach is, compared to the approach that exploits the structure of the equation.

Solution 1

$6x - 4$ is twice $3x - 2$, so the equation is

$$(3x-2)^2 = 2(3x-2).$$

If we put $Q = 3x - 2$, then this is $Q^2 = 2Q$ which can be put into standard form as $Q^2 = 2Q = 0$ and factored as $Q(Q - 2) = 0$. This implies $Q = 0$ or $Q = 2$, i.e., $3x - 2 = 0$ or $3x - 2 = 2$. Hence, $x = \frac{2}{3}$ or $x = \frac{4}{3}$. Both values satisfy the equation.

Solution 2

Expanding, we obtain

$$9x^2 - 12x + 4 = 6x - 4$$
$$9x^2 - 18x + 8 = 0.$$

The quadratic formula now gives $x = \frac{(18 \pm \sqrt{324-288})}{18} = \frac{(18 \pm 6)}{18} = 1 \pm \frac{2}{3} = \frac{2}{3}$ or $\frac{4}{3}$.
Alternatively, one may factor as $(3x-4)(3x-2) = 0$.

ADDITIONAL THOUGHTS

Quadratic equations come in a variety of forms, such as

$$t^2 = 49$$

$$3a^2 = 4$$

$$7 = x^2$$

$$r^2 = 0$$

$$\frac{1}{2}y^2 = \frac{1}{5}$$

$$y^2 - 8y + 15 = 0$$

$$2x^2 - 16x + 30 = 0$$

$$2p = p^2 + 1$$

$$t^2 = 4t$$

$$7x^2 + 5x - 3 = 0$$

$$\frac{3}{4}c(c-1) = c$$

$$(3x-2)^2 = 6x - 4$$

If a textbook fails to cover a variety of forms – presenting instead problem after problem in standard forms such as $ax^2 + bx + c = 0$ – then students will not see the true variety of quadratic equations. They may learn stock routines instead of general methods that work for all problems. They may develop "buggy" techniques. And they may not develop the habit of looking for and making use of structure.

For more about the ways in which students should be reasoning with equations, read pages 13 and 14 of the progressions document, *High School, Algebra*, available at www.achievethecore.org/progressions.

For a direct link, go to: http://www.achievethecore.org/page/847/quadratic-equations-1

Solve the equation

$$(3x - 2)^2 = 6x - 4$$