Class: Algebra II

## Topic: Linear Programming

## Content Standards:

A-CED. 3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A-REI. 6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

## Prior to this Lesson:

Students demonstrated their ability to solve systems of equalities using graphical and symbolic methods: graphing, substitution, elimination. They also represented the solution to a system of inequalities by graphing the overlapping region of half planes. Yesterday, students completed a small-scale linear programming problem (by hand) in order to minimize cost. In today's lesson, we will use a large-scale linear programming problem (with technology) in order to maximize profit.

## Unit Goals:

Students will develop the techniques necessary to solve systems of equalities and graph systems of inequalities. They will extend these skills to real-world scenarios, applicable through production problems (minimize cost and/or maximize profit) when given various constraints.

Purpose of this Lesson: Students will identify the profit function and production constraints for a real-world production problem. They will then use technology (TI-84 inequality application) to create a graphical model for this system of inequalities. Students will identify and use the vertices of the feasible region to maximize the profit of production.

## Performance Objectives:

Each student will model the profit and constraints of a production problem algebraically, graph the feasible region, and test each vertex of the feasible region for maximum profit. Each student will describe linear programming and its potential use.

STUDENT
ACHIEVEMENT
PARTNERS

## Introduction:

Students respond to the following prompt:
Describe linear programming and provide one example of its potential use. (on sticky note provided)
20-second Pair-Share with a partner.
Provide background information on the history of linear programming (WWII) and the extension to production.

## Instructional Method: "Maximizing Your Efforts" Linear Programming Problem (See here for more information.)

- Identify the variables.
- Write the Profit and Constraint Functions. Discuss.
- Pass out TI-84 graphing calculators. "Inequality App" previously downloaded.
- Using document camera, demonstrate the steps required to model this production problem graphically.
- Graph the constraint function inequalities and shade their overlap.
- Locate the feasible region.
- Identify the vertices of the feasible region.
- Substitute the coordinates of the feasible region's vertices into the profit function to maximize profit.


## Guided Practice:

- Model the TI-84 graphing process using the document camera. (Students are familiar will graphing equations, but this is their first exposure to graphing inequalities.)
- Encourage students to work ahead when comfortable doing so.
- Scaffold learning with graphic organizer.


## Closure:

Return to introductory prompt in whole-class discussion.
Use guiding questions to push discussion deeper, beyond a summary of the algorithm.

## Extension:

Students are now familiar with two techniques for solving linear programming problems: computation by hand or use of TI-84 graphing technology. Student teams could each create another maximization/minimization problem, swap with another team, and solve using linear programming.
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## Problem

Games and More Co. produces both video and DVD game players. Each video player, Gamer Gallery, requires 1.5 hours for assembly and 0.25 hours for testing. Each DVD player, Major Player, requires 1 hour for assembly and 0.5 hours for testing

Each month, the Games and More manufacturing plant has 45,000 available hours for product assembly and 20,000 available hours for product testing.

Games and More Co. earns \$60 profit from each Gamer Gallery and \$75 profit from each Major Player that it sells.

How many of each type of player should Games and More Co. produce in order to obtain the greatest monthly profit? You can use linear programming to solve this manufacturing problem.

Let $x$ be the number of Gamer Gallery players, and let $y$ be the number of Major Players.

- Write an expression that represents the profit earned from selling the two different players. This is called a profit function.
$P(x, y)=$
- Determine the profit that would be earned if 50 Gamer Gallery players and 100 Major Players were sold.
- Write a system of inequalities that represents the constraints for the problem.

Assembly time constraint:

Testing time constraint:

Non-negative number of Major Player and Gamer Gallery players:

## Maximizing Your Efforts

- Solve the assembly and testing time constraints for $y$. Copy the non-negative constraints below to list all constraints.

Enter the inequalities from page 1.9 as functions on page 1.11. Use the . key to delete the equals sign and replace it with an inequality sign from the symbol palette.
Use Menu > Graph Entry/Edit > Equation $>$ Line $>\mathbf{x}=\mathbf{c}$ to enter the line $x=0$.

- Find the vertices of the feasible region. Use the Intersection tool (Menu > Points and Lines > Intersection Point(s)) to make a point at each vertex. Then use the Coordinates and Equations tool to find the coordinates of the vertices.
- Substitute each of the coordinates into the profit function from page 1.5.
$P(x, y)=$
$P(x, y)=$
$P(x, y)=$
$P(x, y)=$
- Based on these results, how many of each type of player results in the maximum monthly profit for Games and More Co.? What is the maximum possible monthly profit?
Gamer Gallery Players:

Major Players:

Maximum Monthly Profit:

