Two-Step Problems Using the Four Operations

3.OA.D.8 Application Mini-Assessment by Student Achievement Partners

OVERVIEW

This mini-assessment is designed to illustrate the standard 3.OA.D.8, which sets an expectation for students to solve two-step word problems using the four operations. This mini-assessment is designed for teachers to use either in the classroom, for self-learning, or in professional development settings to:

- **Evaluate** students' understanding of 3.OA.D.8 prior to teaching this material or to check students' abilities to demonstrate understanding of and to apply these concepts;
- **Gain knowledge about** assessing applied problem solving at the depth expected at grade 3;
- **Illustrate CCR-aligned** assessment problems;
- **Illustrate best practices** for writing tasks that allow access for all learners; and
- **Support mathematical language acquisition** by offering specific guidance.

MAKING THE SHIFTS

This mini-assessment attends to **focus** as it addresses problems with all four operations, including assessing the reasonableness of answers, which is at the heart of the grade 3 standards and a key component of the Major Work of the Grade.[1] It addresses **coherence** across grades as it builds on problem solving with addition and subtraction (2.OA.A.1) and prepares students for multi-step problem solving (4.OA.A.3). Standard 3.OA.D.8 and this mini-assessment target *application*, one of the three elements of **rigor**, through word problems.

3.OA.D.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

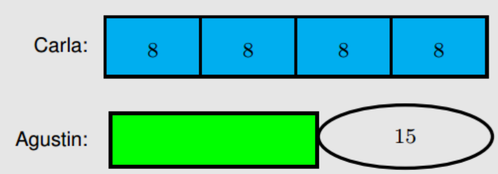
A CLOSER LOOK

Standard 3.OA.D.8 encompasses a significant amount of work for grade 3, including a variety of problem types and all four operations. Each of the problems on this mini-assessment uses the addition and subtraction situations and the multiplication and division situations (see pages 9 and 10). Because the problem-solving demands are high and the operations are paired together, this mini-assessment focuses on multiplication situations using equal groups, not arrays or measurement quantities.

Visual representations are a part of standard 3.OA.D.8, but they are included as a means to an end, not an end in themselves. Consequently, no questions explicitly ask students to use visual representations to show how they solved the problems. However, looking at students' solution strategies may be helpful for teachers to plan instruction.

A two-step problem with diagram showing problem situation and equations showing the two parts

Carla has 4 packages of silly bands. Each package has 8 silly bands in it. Agustin is supposed to get 15 fewer silly bands than Carla. How many silly bands should Agustin get?



C = number of Carla's silly bands
A = number of Agustin's silly bands

$$C = 4 \times 8 = 32$$

$$A + 15 = C$$
$$A + 15 = 32$$
$$A = 17$$

Students may be able to solve this problem without writing such equations.

[1] For more on the Major Work of the Grade, see achievethecore.org/focus.

If you feel additional scaffolding is needed, you may tell students to "Draw a picture if it helps." One example of a representation students may use is shown to the right.[2]

It is likely to take students around 25–30 minutes to answer the 7 questions on this mini-assessment.

SUPPORT FOR ENGLISH LANGUAGE LEARNERS

This lesson was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction and assessment. Go here to learn more about the research behind these supports. Features that support access in this mini-assessment include:

- Tasks that allow for multi-modal representations, which can deepen understanding of the mathematics and make it easier for students, especially ELLs, to give mathematical explanations.
- Tasks that avoid unnecessarily complex language to allow students, especially ELLs, to access and demonstrate what they know about the mathematics of the assessment.

Prior to this mini-assessment, ensure students have had ample opportunities in instruction to read, write, speak, listen for, and understand the mathematical concepts that are represented by the following terms and concepts:

- total
- have left
- how many

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work (for example, through engagement in mathematical routines).

Additionally, ELLs may need support with the following words in order to fully understand each word problem:

- birdhouse
- postcards
- gum/pack of gum

[2] This example originally appeared in the progression document, K, Counting and Cardinality; K–5 Operations and Algebraic Thinking (pg. 28).

Name:____Date:____________________

1. There were 56 birdhouses at school. Today, 4 classes made more birdhouses. Each class made 8 birdhouses. How many total birdhouses are there now?

2. Mr. Dent had 32 markers in his classroom. He buys new boxes of markers that have 9 markers in each box. Now, he has 86 markers. How many new boxes did he buy?

3. Jayson had 274 postcards in his collection. He wanted to give Sam some of his postcards. Jayson gave Sam 8 postcards from each set below:

- Arts
- Sports
- Schools
- Parks
- Beaches
- Sunsets

How many postcards does Jayson have left?

4. Adeline buys 8 packs of Fun Gum. Each pack has 7 pieces of gum. Marisol buys Juicy Gum. Each Juicy Gum pack has 9 pieces of gum.

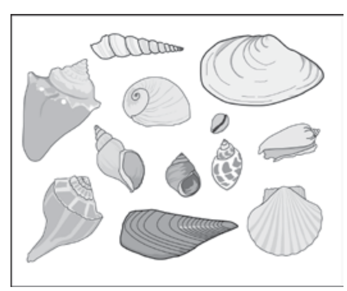
Adeline has 11 more pieces of gum than Marisol.

How many packs of gum did Marisol buy?

5. Students in 3 art classes cut 728 inches of ribbon into 8-inch long pieces. Two of the classes together cut 656 inches of ribbon. How many 8-inch long pieces of ribbon did the other class cut?

6. Last summer, Jon's family found 152 shells at the beach. This summer they were at the beach for 7 days. Each day they found 9 shells. How many fewer shells did they find this year than last year?

Shells



7. Sheldon is baking 2-inch cookies. He has 3 trays that are the same size. On one tray, he makes 5 rows with 4 cookies in each row. He cannot fit any more cookies on the tray. He fills the second tray completely and only part of the third tray.

How many cookies could Sheldon have made?

Explain your answer using numbers, words, and/or pictures

Note: The annotations below reference the structures of the addition/subtraction step and the multiplication/division step as described on pages 88 and 89 of the Common Core State Standards for Mathematics, and included for reference on pages 9 and 10.

1. There were 56 birdhouses at school. Today, 4 classes made more birdhouses. Each class made 8 birdhouses. How many total birdhouses are there now?

Students may solve by:

4 × 8 = ___
32 = ___

56 + 32 = ___
88 = ___

88 total birdhouses

> The structure of the addition/subtraction step is Add To with Result Unknown. The structure of the multiplication/division step is Equal Groups with Product Unknown. These are two of the simplest structures with all numbers within 100.

2. Mr. Dent had 32 markers in his classroom. He buys new boxes of markers that have 9 markers in each box. Now, he has 86 markers. How many new boxes did he buy?

Students may solve by:

32 + ___ = 86
___ = 54

___ x 9 = 54
___ = 6

6 new boxes of markers

> The structure of the addition/subtraction step is Add To with Change Unknown. The structure of the multiplication/division step is Equal Groups with Number of Groups Unknown. These are more complex structures with all numbers within 100.

3. Jayson had 274 postcards in his collection. He wanted to give Sam some of his postcards. Jayson gave Sam 8 postcards from each set below:
   - Arts
   - Sports
   - Schools
   - Parks
   - Beaches
   - Sunsets

   How many postcards did Jayson have left?

> The structure of the addition/subtraction step is Take From with Change Unknown, which is complex. The structure of the multiplication/division step is Equal Groups with Product Unknown, which is simpler.

Students may solve by:

6 × 8 = ___
48 = ___

274 – 48 = ___
226 = ___

226 postcards

4. Adeline buys 8 packs of Fun Gum. Each pack has 7 pieces of gum. Marisol buys Juicy gum. Each Juicy Gum pack has 9 pieces of gum.

   Adeline has 11 more pieces of gum than Marisol.

   How many packs of gum did Marisol buy?

   Students may solve by:

   Adeline – 11 = Marisol
   8 × 7 – 11 = Marisol
   56 – 11 = Marisol
   45 = Marisol

   45 ÷ 9 = ___
   5 = ___

   5 packs of gum

The structure of the addition/subtraction step is Compare with Smaller Unknown (language with "more"). The structure of the multiplication/division step is Equal Groups with Number of Groups Unknown. These are very complex structures for grade 3 when combined, so the magnitude of numbers is kept small in this question.

5. Students in 3 art classes cut 728 inches of ribbon into 8-inch long pieces. Two of the classes together cut 656 inches of ribbon. How many 8-inch long pieces of ribbon did the other class cut?

   Students may solve by:

   728 – 656 = ___
   72 = ___

   72 ÷ 8 = 9

   9 lengths of ribbon

The structure of the addition/subtraction step is Put Together/Take Apart with Addend Unknown. The structure of the multiplication/division step is Equal Groups with Number of Groups Unknown. Both steps are complex and the numbers reflect the full intent of 3.NBT and 3.OA.

6. Last summer, Jon's family found 152 shells at the beach. This summer they were at the beach for 7 days. Each day they found 9 shells. How many fewer shells did they find this year than last year?

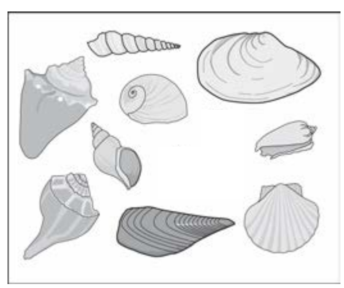
   Students may solve by:

   7 × 9 = ___
   63 = ___

   152 – 63 = ___
   89 = ___

   89 fewer shells this year

Jon's Sea Shells



The structure of the addition/subtraction step is Compare with Difference Unknown, a complex structure. The structure of the multiplication/division step is Equal Groups with Product Unknown, a simpler structure.

7. Sheldon is baking 2-inch cookies. He has 3 trays that are the same size. On one tray, he makes 5 rows with 4 cookies in each row. He cannot fit any more cookies on the tray. He fills the second tray completely and only part of the third tray.

How many cookies could Sheldon have made?

Students may solve by:
$5 \times 4 = 20$
$5 \text{ x } 4 = 20$
$20 + 20 = 40$
$40 + \_\_ < 60$

Any whole number answer between 41-59

Explain your answer using numbers, words, and/or pictures:

Assessing the reasonableness of an answer is an important part of mathematical proficiency, and stated explicitly as a goal in grade 3 as students are solving two-step word problems. This problem addresses this part of the standard by asking students to think about providing a reasonable answer within a broader range. Sometimes two students could use correct and different reasoning to arrive at their conclusions which could lead to a full class discussion. This also allows for students to engage with MP1 as they think about reasonableness and unreasonableness throughout the problem solving process.

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A+B=\square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A+\square=C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square+B=C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C-B=\square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C-\square=A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square-B=A$

	Total Unknown	Both Addends Unknown[1]	Addend Unknown[2]
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A+B=\square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C=\square+\square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A+\square=C$ $C-A=\square$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A+\square=C$ $C-A=\square$	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A+B=\square$	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C-B=\square$ $\square+B=C$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

[1] This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as." Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

[2] Either addend can be unknown; both variations should be included.

Table 3: Multiplication and division situations

	$A \times B = \square$	$A \times \square = C$ and $C \div A = \square$	$\square \times B = C$ and $C \div B = \square$
Equal Groups of Objects	Unknown Product There are A bags with B plums in each bag. How many plums are there in all?	Group Size Unknown If C plums are shared equally into A bags, then how many plums will be in each bag?	Number of Groups Unknown If C plums are to be packed B to a bag, then how many bags are needed?
Arrays of Objects		*Equal groups language*	
	Unknown Product There are A rows of apples with B apples in each row. How many apples are there?	Unknown Factor If C apples are arranged into A equal rows, how many apples will be in each row?	Unknown Factor If C apples are arranged into equal rows of B apples, how many rows will there be?
		Row and column language	
	Unknown Product The apples in the grocery window are in A rows and B columns. How many apples are there?	Unknown Factor If C apples are arranged into an array with A rows, how many columns of apples are there?	Unknown Factor If C apples are arranged into an array with B columns, how many rows are there?
Compare		$A > 1$	
	Larger Unknown A blue hat costs \$$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs \$$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. How many times as much does the red hat cost as the blue hat?
		$A < 1$	
	Smaller Unknown A blue hat costs \$$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs \$$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. What fraction of the cost of the blue hat is the cost of the red hat?

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

Multiplicative Compare problems appear first in Grade 4, with whole-number values for A, B, and C, and with the "times as much" language in the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., "A red hat costs A times as much as the blue hat" results in the same comparison as "A blue hat costs $1/A$ times as much as the red hat," but has a different subject.