Comparing Fractions with Benchmarks Game

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GRADE LEVEL Fourth

IN THE STANDARDS 4.NF.A.2

WHAT WE LIKE ABOUT THIS TASK

Mathematically:
- Addresses standards: 4.NF.A.2 and MP3.
- Allows students to reason about the size of fractions by using benchmarks.
- Requires students to construct a viable argument and use examples to justify their reasoning (MP3).

In the classroom:
- Provides resources to allow students to compare fractions with or without a visual representation of the fractions the mathematics explicit.
- Prompts students to share their developing thinking and understanding.
- Captures student attention by using an engaging context.
- Allows for student collaboration and discussion.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:
- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.

MAKING THE SHIFTS[1]

Focus	Belongs to the Major Work[2] of fourth grade	
Coherence	Builds on work students did in grade 3 with comparing fractions and equivalence, sets students up for understanding the reasonableness of their answers with fraction operations in grade 5	
Rigor[3]	Conceptual Understanding: primary in this task.	
	Procedural Skill and Fluency: not targeted in this task.	
	Application: not targeted in this task.	

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Four.
[3]Tasks will often target only one aspect of rigor.

The steps in this routine are adapted from the [Principles for the Design of Mathematics Curricula: Promoting Language and Content Development](#).

Engage students in the Compare and Connect Mathematical Language Routine. This will support students as they identify, compare, and contrast differing mathematical approaches and representations.

Before engaging in this task (game), use the Compare and Connect strategy: "Which One Doesn't Belong?" as a warm-up.

Provide students with the following fractions:

- $\frac{3}{3}$
- $\frac{7}{6}$
- $\frac{2}{6}$
- $1\frac{1}{8}$

Ask students to determine a set of three that excludes one of the fractions. Students should be supported in attending to mathematical language while describing the attributes that allow three of the fractions to be in a set. Here are a few examples:

- $\frac{3}{3}$ doesn't belong because it is equal to one.
- $\frac{2}{6}$ doesn't belong because it is the only fraction smaller than one.
- $1\frac{1}{8}$ doesn't belong because it includes a unit fraction.
- $\frac{7}{6}$ doesn't belong because it's not a mixed number and it's greater than one.
- See additional examples in the IM commentary.

Remind the students of the language they used in this warm up. Listen for this language while students are playing the game.

LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Unit fraction
- Numerator
- Denominator
- Equal
- Greater than
- Less than
- Benchmark

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work.

This task was created as part of the Adapting Materials Project. The goal of this project was to create a replicable process for teachers intending to adapt their materials, and to help create an environment of trust, where teachers felt empowered with the knowledge, confidence, and authority to change their own instructional materials in a way that better reflects the standards. To learn more about the work of these districts, read the "Collaborative Learning and Updating Materials" article from Aligned or access the complete case study.

For more information on the specific expectations for students working with fractions in grade 4 read pages 6–10 in the progression document, *Number and Operations – Fractions*, available at www.achievethecore.org/progressions.

4.NF Comparing Fractions Using Benchmarks Game

Task

This activity is designed for pairs of students. It uses a set of cards and a fraction mat which are supplied as an attached resource, after the commentary . The goal is to place each fraction in the appropriate category on the mat. Instructions for the activity are as follows:

a. Students follow these steps with the fraction cards:
    i. Each student in the pair select a card.

    ii. Student 1 places his/her card within the appropriate category on the mat and gives an explanation for the placement.

    iii. Student 2 either agrees or disagrees with the placement.

    iv. If the partners disagree, they discuss until reaching a consensus.

    v. If the partners agree, Student 2 places his/her card and repeat steps 1 through 4 with this card.

b. After 5 rounds, each pair record observations about what methods they used to categorize the fractions.

IM Commentary

The goal of this task is to determine appropriate benchmarks for fractions with a focus on providing explanations that demonstrate deep conceptual understanding. The only benchmark fractions addressed in this task are $\frac{1}{2}$ and 1. Some important ideas to look

and listen for as the students work are

- A fraction is bigger than one if the numerator is larger than the denominator (e.g. $\frac{7}{6}$) and a fraction is less than one if the numerator is smaller than the denominator (e.g. $\frac{2}{5}$). A fraction is equal to one if the numerator is equal to the denominator (e.g. $\frac{3}{3}$).
- A fraction is bigger than $\frac{1}{2}$ if the denominator is less than two times the numerator. A fraction is equal to $\frac{1}{2}$ if the denominator is equal to two times the numerator. A fraction is less than $\frac{1}{2}$ if the denominator is more than twice the numerator.
- Students can draw pictures of fractions to compare them with $\frac{1}{2}$ and 1.
- Students can plot fractions on the number line to compare them with $\frac{1}{2}$ and 1 (3.NF.A.2).

Two different sets of cards are provided as attachments, one with a picture of the fractions being compared and one without. The pictures allow students to make a visual comparison of the fractions which is important. However, the teacher may wish for students to use a different method of comparison or to provide these pictures as they explain their decision. Some of the pictures such as those of $\frac{1}{3}$ and $\frac{2}{6}$, are not designed to visually demonstrate equivalence so the cards with pictures will still help develop non visual understanding of fractions.

Differentiated fraction mats are provided as attachments to allow students to work with broader benchmark ranges. These mats can be used in progression based on student understandings. The first mat asks students to identify fractions greater than or less than 1. The second mat asks students to identify fractions equivalent to 1/2 and 1. The third mat asks students to use the prior understandings of the previous mats to identify fractions that fall within the benchmarks of 1/2 and 1.
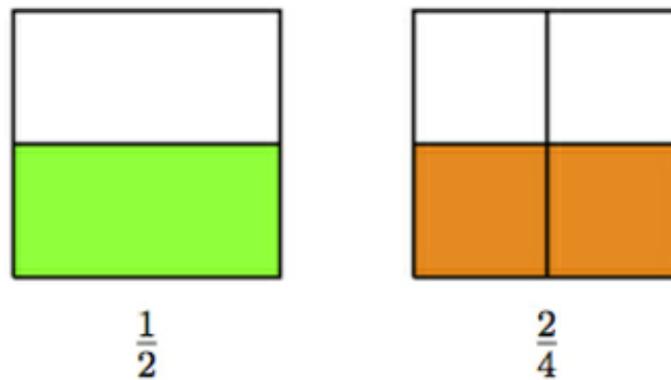
There are four attachments below:

- Fraction Cards No Picture (these have a fraction with no picture)
- Fraction Cards Picture (these have a fraction with picture: if students use these the teacher should watch to make sure that students name the whole for their fraction. The intention is that the large squares are the whole but a student could think differently and then correctly classify the fraction in a different way)
- Benchmark fraction mats (these mats have all possible categories for the fractions from less than $\frac{1}{2}$ through greater than 1)
- Differentiated fraction mats (described in the previous paragraph)

Solution

a. In order to place a fraction in the proper category students must compare the fraction to $\frac{1}{2}$ and 1. There are many strategies to do this and the student does not need to follow the particular order of reasoning described here:

- Fractions whose value is equal to 1. These are perhaps the simplest fractions to recognize: for example, $\frac{8}{8}$ is equal to 1 whole because it takes 8 equal pieces of 1/8 combine to create a whole.
- Fractions equivalent to 1/2. For example 2/4 is equivalent to 1/2 because the numerator is half of the denominator. One way to show that these fractions represent the same quantity is with a picture:

$$\frac{1}{2} \qquad \frac{2}{4}$$

- Fractions whose value is less than $\frac{1}{2}$. For example, $\frac{3}{8}$ is less than $\frac{1}{2}$ because $\frac{4}{8}$ is equivalent to $\frac{1}{2}$ and $\frac{3}{8}$ is $\frac{1}{8}$ less than $\frac{1}{2}$, therefore less than $\frac{1}{2}$.
- Fractions whose value is greater than $\frac{1}{2}$, but less than 1. For example, $\frac{5}{8}$ is greater than $\frac{1}{2}$ because $\frac{4}{8}$ is equivalent to $\frac{1}{2}$ and $\frac{5}{8}$ has an additional fourth. $\frac{5}{8}$ is less than 1 because $\frac{8}{8}$ is equivalent to 1 and $\frac{5}{8}$ needs three more $\frac{1}{8}$'s to make a whole. Alternatively, students might draw a number line and show that $\frac{5}{8}$ lies to the right of $\frac{1}{2}$ (which is equal to $\frac{4}{8}$) and to the left of 1 (which is equalto $\frac{8}{8}$):

- Fractions greater than 1. For example, $\frac{6}{4}$ is greater than 1 because $\frac{4}{4}$ is a whole and $\frac{6}{4}$ has two additional fourths in addition to the whole.

b. There are many important lessons to be learned from comparing fractions to benchmark fractions, including:

- The denominator tells me how many pieces to cut my whole into. When the whole is cut into more pieces, the pieces are smaller (this is why $\frac{1}{3}$ is less than $\frac{1}{2}$).
- The numerator tells me how many equal sized pieces I have. So $\frac{6}{10}$ is more than $\frac{5}{10}$ because I have one extra piece.
- Fractions are built from the unit fractions so it is important to understand and be able to represent the unit fractions.
- If using the fraction cards with pictures, equal sized wholes are important when comparing fractions.
- In a picture of equivalent fractions, the whole is broken into different sized pieces, but the same total amount is shaded.
- The relationship between the numerator and denominator can help me think about the value of the fraction.
- Fractions equivalent to $\frac{1}{2}$ have a numerator that is half of the denominator.
- Fractions equivalent to 1 whole have the same numerator and denominator.
- When the numerator is a bigger number than the denominator, the fraction is greater than one whole.
- When doing mathematics, patterns emerge. These patterns support students in making conjectures, supporting their reasoning, and proving mathematical claims.