

BAKING BREAD

Kelly was baking bread but could only find her $\frac{1}{8}$ -cup measuring cup. She needs $\frac{1}{4}$ cup sugar, $\frac{3}{4}$ cup whole wheat flour, and $\frac{1}{2}$ cup all-purpose flour. How many $\frac{1}{8}$ cups will she need for each ingredient?

SHOW YOUR WORK:

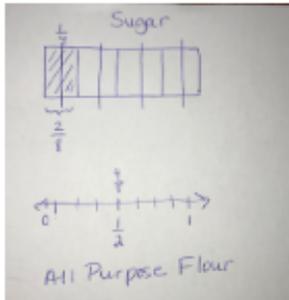
ANSWER:

TEACHER'S GUIDE BAKING BREAD

<p>Kelly was baking bread but could only find her $\frac{1}{8}$-cup measuring cup. She needs $\frac{1}{4}$ cup sugar, $\frac{3}{4}$ cup whole wheat flour, and $\frac{1}{2}$ cup all-purpose flour. How many $\frac{1}{8}$ cups will she need for each ingredient?</p>	<p>TARGETED STANDARDS: 4.NF.A.1</p>	<p>SOLUTION: Sugar $2\frac{1}{8}$ Whole Wheat Flour $6\frac{1}{8}$ All-purpose Flour $4\frac{1}{8}$</p>
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CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Students' previous work with equivalent fractions connects to their understanding of equivalent ratios and rates. In this problem, students use equivalent fractions to find numerators when a new denominator is required. Students will use this same reasoning when solving ratio and rate problems using tables and double number lines.

<p>POSSIBLE SOLUTION: Students may use equivalent fractions to change each measurement into eights, then decompose the fractions to determine the number of $\frac{1}{8}$ in each measurement.</p> <p><i>Sugar</i> $\frac{1}{4} = \frac{2}{8} = \frac{1}{8} + \frac{1}{8}$</p> <p><i>Whole Wheat Flour</i> $\frac{3}{4} = \frac{6}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$</p> <p><i>All Purpose Flour</i> $\frac{1}{2} = \frac{4}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$</p>	<p>POSSIBLE SOLUTION: Students may use equivalent fractions to change each measurement into eights, then decompose the fractions to determine the number of $\frac{1}{8}$ in each measurement.</p> 	<p>POSSIBLE STUDENT MISCONCEPTIONS: Students who do not have a solid understanding of equivalent fractions may try to apply false rules to make equivalents, such as $\frac{1}{2} = \frac{6}{8}$. In this example, the student has added the same number to the numerator and denominator to arrive at an incorrect solution. Students may misinterpret the question and combine all measurements before changing into 8ths, producing an incorrect answer of 12.</p>
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SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Use concrete manipulatives such as fraction tiles or strips to help students build $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{8}$ using pieces.
- Use guiding questions such as how many $\frac{1}{8}$ s are equal to $\frac{1}{4}$?
- Draw pictures to show equivalencies

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

- Kelly has only one measuring cup, but can still measure each ingredient accurately. What is the size of the measuring cup? (not $\frac{1}{8}$). How would your answer change if the recipe were doubled or tripled?

LANGUAGE SUPPORT:

Read the problem aloud to help students comprehend what is being asked. Students will need to understand that the ingredients are the flours and sugar.

TEACHER SUPPORT:

In this particular problem, an incorrect answer of 12 does not indicate a lack of understanding of equivalent fractions, but rather a lack of understanding of the question being asked. Ask students to explain their reasoning to uncover if they provide incorrect answers.

RUNNING WITH THE REYNOLDS

Mr. and Mrs. Reynolds went for a run. Mr. Reynolds ran for $\frac{6}{10}$ mile. Mrs. Reynolds ran for $\frac{2}{5}$ mile. Who ran farther? Explain how you know. Use the benchmarks 0 , $\frac{1}{2}$, and 1 to explain your answer.

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE RUNNING WITH THE REYNOLDS

Mr. and Mrs. Reynolds went for a run. Mr. Reynolds ran for $\frac{6}{10}$ mile. Mrs. Reynolds ran for $\frac{2}{5}$ mile. Who ran farther? Explain how you know. Use the benchmarks 0, $\frac{1}{2}$ and 1 to explain your answer.

TARGETED STANDARDS:
4.NF.A.2

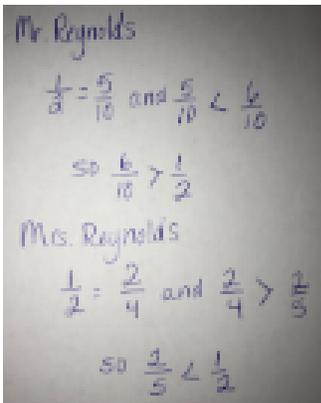
SOLUTION:
Mr. Reynolds ran farther. He ran more than $\frac{1}{2}$ of a mile, and Mrs. Reynolds ran less than $\frac{1}{2}$ of a mile.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Students' previous work with comparing fractions connects to their understanding of equivalent ratios and rates. Students' work with comparing fractions builds flexibility with numbers and allows them to reason about part-whole relationships, which they will use to understand ratio and rate relationships. In this problem, students must consider both the numerator and denominator when determining which fraction is larger. Similar reasoning is used to identify equivalent rates and ratios.

POSSIBLE SOLUTION:

Mr. Reynolds ran $\frac{6}{10}$ of a mile. $\frac{5}{10}$ is one half of a mile, so I know that he ran more than half of a mile. Mrs. Reynolds ran $\frac{2}{5}$ of a mile. $\frac{2}{4}$ is a half of a mile, and $\frac{2}{5}$ is less than $\frac{2}{4}$, so she ran less than half of a mile. (Students also may recognize that 2.5 is half of 5 and that 2 is less than 2.5, so $\frac{2}{5}$ is less than half). If Mr. Reynolds ran more than half of a mile, and Mrs. Reynolds ran less than half of a mile, then he must have run further. The same reasoning, using numerical representations might look like this:



POSSIBLE SOLUTION:

Students may use common denominators to compare the fractions. $\frac{2}{5} = \frac{4}{10}$ and $\frac{6}{10}$ is greater than $\frac{4}{10}$, so Mr. Reynolds ran farther. Using benchmarks, I can tell that $\frac{6}{10}$ is closer to $\frac{10}{10}$ or 1 whole than $\frac{4}{10}$, so Mr. Reynolds ran farther. (Or Mrs. Reynolds is closer to 0, so she ran less).

POSSIBLE STUDENT MISCONCEPTIONS:

Students may state that Mr. Reynolds ran farther, because he ran more parts of the mile (6 is greater than 2), arriving at a correct answer with incorrect reasoning. Students may also conclude that Mrs. Reynolds ran farther because $\frac{2}{5}$ is only $\frac{3}{5}$ away from one whole ($\frac{5}{5}$), while $\frac{6}{10}$ is $\frac{4}{10}$ away from one whole ($\frac{10}{10}$), so she must be closer to one mile. Both of these students lack the understanding that the denominator of a fraction tells the size of the pieces and are relying only on the number of pieces (numerator) to arrive at a conclusion.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

Use concrete and visual models to represent the fractions in the problem. Fraction tiles, tape diagrams, and especially number lines are useful representations for comparing fractions. Students may benefit from additional work with 3.NF.A.1 (fractions as numbers) and 3.NF.A.3.D (comparing fractions with like numerators and denominators). Remind students that the numerator tells the number of pieces, while the denominator tells the size of the pieces.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Another day, Mr. Reynolds ran $\frac{7}{8}$ of a mile and Mrs. Reynolds ran $\frac{9}{10}$. Who ran farther? How do you know? Use benchmark fractions to explain.

TASK SOURCE:

<https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-b-lesson-11>

WHO RAN THE FARTHEST?

Jamal ran $\frac{2}{3}$ miles. Ming ran $\frac{2}{4}$ miles. Laina ran $\frac{7}{12}$ miles. Who ran the farthest? What do you think is the easiest way to determine the answer to this question? Explain your thinking.

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE

WHO RAN THE FARTHEST?

Jamal ran $\frac{2}{3}$ mile. Ming ran $\frac{2}{4}$ mile. Laina ran $\frac{7}{12}$ mile. Who ran the farthest? What do you think is the easiest way to determine the answer to this question? Talk with a partner about your ideas.

TARGETED STANDARDS:
4.NF.A.2

SOLUTION:
Jamal ran the farthest.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Student's previous work with comparing fractions connects to their understanding of equivalent ratios and rates. Student's work with comparing fractions builds flexibility with numbers and allows them to reason about part-whole relationships, which they will use to understand ratio and rate relationships. In this problem, students must consider both the numerator and denominator when determining which fraction is larger. Similar reasoning is used to identify equivalent rates and ratios.

POSSIBLE SOLUTION:

Ming ran $\frac{2}{4}$ mile and $\frac{2}{4} = \frac{1}{2}$. Since both $\frac{2}{3}$ and $\frac{7}{12}$ are more than one half, Ming ran the shortest distance. Since $\frac{2}{3} = \frac{8}{12}$, and $\frac{8}{12}$ is larger than $\frac{7}{12}$, Jamal must have run the farthest.

POSSIBLE SOLUTION:

Ming $\frac{2}{4} = \frac{6}{12}$
Jamal $\frac{2}{3} = \frac{8}{12}$
Laina = $\frac{7}{12}$
Since $\frac{8}{12}$ has the largest numerator, Jamal must have run the farthest.

POSSIBLE STUDENT MISCONCEPTIONS:

Students who have limited fraction comparison strategies may conclude that Laina has run the farthest because her fraction has the largest numbers. This student is not seeing the fraction as a number itself. Look for errors in student thinking with equivalent fractions.

Common misconceptions include changing denominators but not numerators, and using addition instead of multiplication to make equivalents.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Use concrete and visual models to represent the fractions in the problem. Fraction tiles, tape diagrams, and especially number lines are useful representations for comparing fractions.
- Students may benefit from additional work with 3.NF.A.1 (fractions as numbers) and 3.NF.A.3.D (comparing fractions with like numerators and denominators). Remind students that the numerator tells the number of pieces, while the denominator tells the size of the pieces.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Offer the following task for students to apply the same strategies with less familiar fractions.

<http://tasks.illustrativemathematics.org/content-standards/4/NF/A/2/tasks/812>

ADDITIONAL TEACHER SUPPORT:

Additionally, students could use a common numerator strategy to compare $\frac{2}{4}$ and $\frac{2}{3}$ as a first step in solving this problem. Students may also benefit from the use of a number line to solve this problem. Situations involving distance naturally lend themselves to the visual model a number line provides.

TASK SOURCE:

<https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-c-lesson-15>

PLOTTING POINTS PART 1

A. Plot the following points on the number line without measuring:

i. $\frac{1}{3}$

ii. $\frac{3}{6}$

iii. $\frac{7}{12}$



B. Use the Number Line in Part A to compare the fractions by writing $>$, $<$, or $=$ on the lines.

i. $\frac{7}{12}$ _____ $\frac{1}{2}$

ii. $\frac{7}{12}$ _____ $\frac{5}{6}$

PLOTTING POINTS PART 2

A. Plot the following points on the number line without measuring:

i. $\frac{11}{12}$

ii. $\frac{1}{4}$

iii. $\frac{3}{8}$



B. Select two fractions from Part A, and use the given number line to compare them by writing $>$, $<$, or $=$.



C. Explain how you plotted the points in Part A.

TEACHER'S GUIDE

PLOTTING POINTS PARTS 1 & 2

See the full task in the student version.

TARGETED STANDARDS:
4.NF.A.2

SOLUTION:
[Click here to access the solution.](#)

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Student's previous work with comparing fractions connects to their understanding of equivalent ratios and rates. Student's work with comparing fractions builds flexibility with numbers and allows them to reason about part-whole relationships, which they will use to understand ratio and rate relationships. In this problem, students use fraction comparison reasoning and equivalent fractions to order fractions on a number line. They will use similar reasoning when comparing rates and ratios.

POSSIBLE SOLUTION:

Students may have different strategies for placing fractions on the number line.

1. Make equivalent fractions with denominators of 12 (or 24), divide the line into 12 parts, and locate each fraction.
2. Make equivalent fractions with denominators of 24, divide the line into 24 parts, and locate each fraction.

POSSIBLE SOLUTION:

1. Divide the number line into 3 pieces and mark $\frac{1}{3}$. Divide each third into 2 pieces to mark $\frac{5}{6}$. Divide each sixth into 2 pieces and locate $\frac{7}{12}$.
2. Mark $\frac{1}{2}$, then divide each half into 2 pieces to mark $\frac{1}{4}$. Divide each $\frac{1}{4}$ into 2 pieces to mark $\frac{3}{8}$. Each $\frac{1}{4}$ can then be divided into 3 pieces to show $\frac{11}{12}$.

POSSIBLE STUDENT MISCONCEPTIONS:

Some students have trouble dividing a number line to show fractions. For example: using 4 tick marks to show fourths, instead of using 3. These students may not fully understand fractions as numbers.

Look for errors in student thinking with equivalent fractions. Common misconceptions include changing denominators but not numerators, and using addition instead of multiplication to make equivalents.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Consider using three identical number lines for each item. Students can divide each number line and show one fraction, then compare locations among the three number lines.
- Have pre-divided number lines available for students who are having difficulty with this part.
- Use fraction tiles and relate these concrete materials to a fraction's location on the number line

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Which fraction is closer to 1, $\frac{8}{9}$ or $\frac{11}{10}$? How do you know? Use a number line to explain your thinking.

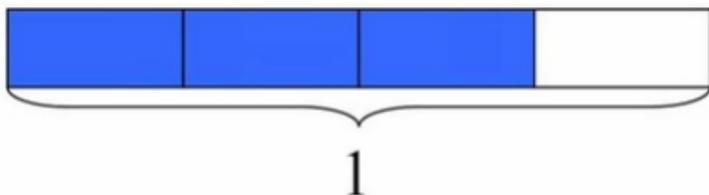
TASK SOURCE:

<https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-c-lesson-12>

SHADES OF A RECTANGLE

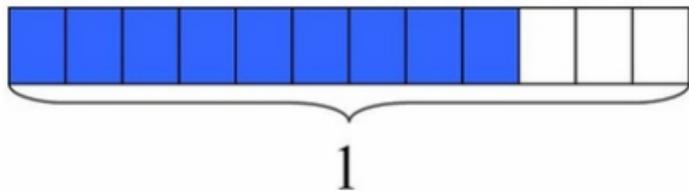
A. The rectangle below has length 1. What fraction does the shaded part represent?

ANSWER:



B. The rectangle below has the same length as the rectangle above. What fraction does the shaded part represent?

ANSWER:



C. Use the pictures to explain why the two fractions represented above are equivalent.

TEACHER'S GUIDE

SHADES OF A RECTANGLE

See the full task in the student version.

TARGETED STANDARDS:
4.NF.A.1

SOLUTION:

A. $\frac{3}{4}$ B. $\frac{9}{12}$ C. See possible solutions below.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Students' previous work with equivalent fractions connects to their understanding of equivalent ratios and rates. In this task, students use visual representations of equivalent fractions to explain how the two shaded regions are equivalent.

POSSIBLE SOLUTION:

Both pictures show the same whole and the same amount colored. Part a has 3 pieces shaded out of 4. The pieces are larger but there are fewer of them. In part b, 9 out of the 12 pieces are shaded. The same whole is cut into more pieces, so more are colored. Each $\frac{1}{4}$ of part a is equal to $\frac{3}{12}$ of part b.

POSSIBLE STUDENT MISCONCEPTIONS:

Students will easily see that the same amount of each whole is colored, but may struggle to articulate why even though the fraction names are different, they refer to the same size piece.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

Use concrete manipulatives such as fraction tiles or circles to show the equivalence of these two fractions.

Present similar problems where students first draw picture to represent a fraction (such as $\frac{1}{3}$), and then divided the whole into 2 parts, creating $\frac{2}{6}$.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Draw a third picture that shows an equivalent fraction to $\frac{3}{4}$ and $\frac{9}{12}$. How did you decide what fraction to make? Give two examples of fractions that are close to, but not equivalent to $\frac{3}{4}$ and use pictures to explain why they are not equivalent.

TASK SOURCE:

<http://tasks.illustrativemathematics.org/content-standards/4/NF/A/1/tasks/743>

BUILDING A DECK

The length of a rectangular deck is 4 times its width. If the deck's perimeter is 30 feet, what is the deck's area?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE BUILDING A DECK

The length of a rectangular deck is 4 times its width. If the deck's perimeter is 30 feet, what is the deck's area?

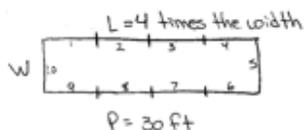
TARGETED STANDARDS:
4.OA.A.2

SOLUTION:
36 square feet

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Students solve problems involving multiplicative comparisons in grade 4 in preparation for their work with ratios and proportions (6.RP.A.1). By working with problems related to 4.OA.A.2, students understand that multiplicative relationships (such as those encountered with ratios and proportions) act differently from additive relationships.

POSSIBLE SOLUTION:



$P = 2 \times (l + w)$
 Width = 1 unit
 length = 4 units > 5 units
 $P = 10$ units
 $10 \times 3 = 30$ feet
 $a = 3$ feet per unit
 $w = 3$ ft
 $w = 3$ ft
 $L = 12$ ft
 $A = 12$ ft \times 3 ft
 $A = 36$ square feet

POSSIBLE SOLUTION:

Students may create a table to try possible combinations to determine width

length	length x4	perimeter
1	4	10 (too small)
4	16	40 (too big)
3	12	30

Area = length \times width
 $A = 3 \times 12 = 36$ square ft.

POSSIBLE STUDENT MISCONCEPTIONS:

Students often confuse area and perimeter. Ensure that students understand that perimeter tells the distance around the rectangle, while area tells about how many square feet would cover the rectangle.

Look for students who find the width, but then do not find the area. An answer of 3 or 12 may indicate that students are proficient with standard 4.OA.A.2, but did not finish the problem.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

Use tiles to have students build the rectangle and check lengths and widths. Students can then draw a picture to represent the tiles, connecting the concrete to pictorial representations of this problem

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Pose questions such as:

- If the length of the rectangle was cut in half, what would happen to the width to keep the perimeter the same. Show this relationship using a drawing.
- Suppose you want double the perimeter of this rectangle. Discuss how the length and the width change. Is the length still 4 times the width?
- What is the area of a rectangle whose perimeter is 20 and the width is $1\frac{1}{2}$ times its length?

LANGUAGE SUPPORTS:

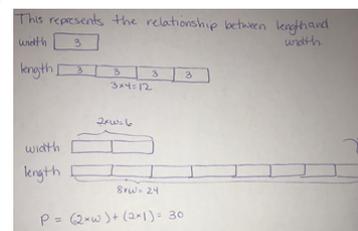
Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Area
- Perimeter
- Length
- Width
- Rectangle

Read the problem aloud as needed, and provide a visual of a rectangle showing the dimensions labeled.

ADDITIONAL TEACHER SUPPORT:

Bar models lend themselves well to solving problems involving multiplicative comparison and can be helpful in writing equations to represent the situation. Since this model is also useful for solving ratio problems, students can easily see the connection between the two. For example, a bar model for this problem may look like this:



TASK SOURCE:

<https://www.engageny.org/resource/grade-4-mathematics-module-3-topic-lesson-3>

HOW MUCH MONEY DID THEY RAISE?

A. Helen raised \$12 for the food bank last year and she raised 6 times as much money this year. How much money did she raise this year?

B. Sandra raised \$15 for the PTA and Nita raised \$45. How many times as much money did Nita raise as compared to Sandra?

C. Luis raised \$45 for the animal shelter, which was 3 times as much money as Anthony raised. How much money did Anthony raise?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE

HOW MUCH MONEY DID THEY RAISE?

- A. Helen raised \$12 for the food bank last year and she raised 6 times as much money this year. How much money did she raise this year?
- B. Sandra raised \$15 for the PTA and Nita raised \$45. How many times as much money did Nita raise as compared to Sandra?
- C. Luis raised \$45 for the animal shelter, which was 3 times as much money as Anthony raised. How much money did Anthony raise?

TARGETED STANDARDS:
4.OA.A.2

SOLUTION:
A. \$72
B. Nita raised 3 times as much money as Sandra.
C. Anthony raised \$15.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Students solve problems involving multiplicative comparisons in grade 4 in preparation for their work with ratios and proportional relationships (6.RP.A.1). This task asks students to reason multiplicatively in three different but related situations. They explore the relationship between multiplication and division when deciding how to solve each situation where the total is known and one factor is missing. The use of bar models and equations to model these situations allows students to connect to these models when used to model situations involving ratios and proportions.

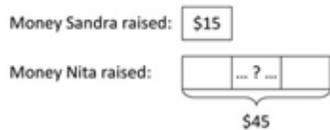
POSSIBLE SOLUTION:

Using Tape diagrams:

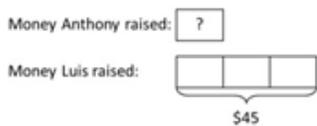
A. $12 \times 6 = 72$



B. $15 \times 3 = 45$



C. $45 \div 3 = 15$



POSSIBLE SOLUTION:

Writing and solving equations

A. Helen raised 6 x \$12, so $6 \times 12 = 72$

B. $? \times 15 = 45$, or $45 \div 15 = ?$, $? = 3$

C. $3 \times ? = 45$, or $45 \div 3 = ?$, $? = 15$

POSSIBLE STUDENT MISCONCEPTIONS:

Students may recognize these situations as multiplication situations and then simply multiply the given numbers in parts b and c.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

The numbers in this tasks are small enough to use concrete manipulatives such as base ten blocks or counters to represent the three tasks. Use of concrete representations will provide a visual context or multiplication and division. Have students draw pictures to represent each situation, and link their concrete work with a pictorial representation and equation.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

In grade 5, students explore multiplication as scaling with fractions. (5.NF.B.5) Students who benefit from an extension may find these types of problems interesting. An example task is linked here:

http://s3.amazonaws.com/illustrativemathematics/attachments/000/008/575/original/public_task_164.pdf?1462387630

LANGUAGE SUPPORTS:

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the term "Times as much."

TASK SOURCE:

<http://tasks.illustrativemathematics.org/content-standards/4/OA/A/2/tasks/263>

SNAKES AT THE ZOO

There are two snakes at the zoo, Jewel and Clyde. Jewel was six feet and Clyde was eight feet. A year later Jewel was eight feet and Clyde was 10 feet. Which one grew more?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE SNAKES AT THE ZOO

There are two snakes at the zoo, Jewel and Clyde. Jewel was six feet and Clyde was eight feet. A year later Jewel was eight feet and Clyde was 10 feet. Which one grew more?

TARGETED STANDARDS:
4.OA.A.2

SOLUTION:
There are two solutions to this problem, depending on how students interpret it.

Solution 1: Both snakes grew 2 feet, so they grew the same amount.

Solution 2: Jewel grew more, since $\frac{2}{6}$ is greater than $\frac{2}{8}$.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

This multiplicative comparison task from grade 4 (4.OA.A.2) asks students to consider how much each snake has grown, relative to itself, highlighting a proportional relationship (6.RP.1). Problems such as this one allow students to experience the difference between additive and multiplicative relationships. This problem in particular allows that conversation to happen via different solution methods.

POSSIBLE SOLUTION:

Jewel grew two feet ($8 - 6 = 2$) and Clyde also grew two feet ($10 - 8 = 2$), so they both grew the same amount.

POSSIBLE SOLUTION:

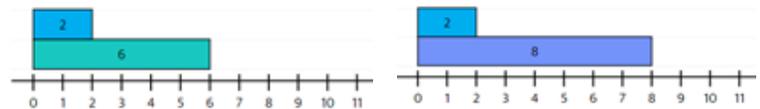
Jewel grew two feet, with a starting length of six feet, so she grew $\frac{2}{6}$ ($\frac{1}{3}$) of his original length.
Clyde grew two feet, with a starting length of 8 feet, so he grew $\frac{2}{8}$ ($\frac{1}{4}$) of his original length.
Since $\frac{2}{6}$ is greater than $\frac{2}{8}$, Jewel grew more when comparing growth to original length.

POSSIBLE STUDENT MISCONCEPTIONS:

Students who are additive thinkers may have difficulty understanding solution 2. Providing a concrete model or drawing should be helpful to these students.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

Unit cubes would be helpful to show each snakes' starting and ending length. Students can compare the two feet of growth to the original length by disconnecting the cubes and comparing that to the original length. Have students draw a picture to represent the concrete situation.



SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

If Jewel and Clyde continue to grow at the same rate, how long will each be after another year? (e.g., Jewel is growing at a rate of $\frac{1}{3}$ of her length each year, so in year 2 she will grow $\frac{8}{3}$ or $2\frac{2}{3}$ inches). Will Jewel ever be longer than Clyde? If so, when?

LANGUAGE SUPPORTS:

The homonym "feet" may confuse English language learners. Be sure that these students understand feet as the measurement unit for this problem.

TASK SOURCE:
<https://tasks.illustrativemathematics.org/content-standards/tasks/356>

WHO SOLD THE MOST CHOCOLATE?

Jared sold 194 Boy Scout chocolate bars. Matthew sold three times as many as Jared. Gary sold 297 fewer than Matthew. How many bars did Gary sell?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE

WHO SOLD THE MOST CHOCOLATE?

Jared sold 194 Boy Scout chocolate bars. Matthew sold three times as many as Jared. Gary sold 297 fewer than Matthew. How many bars did Gary sell?

TARGETED STANDARDS:
4.OA.A.2

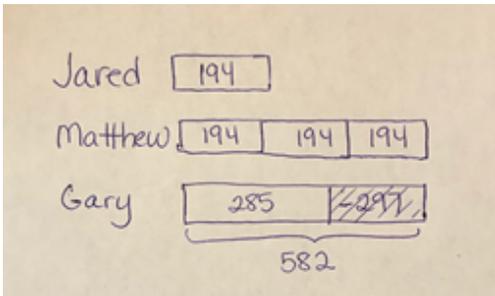
SOLUTION:
285 bars

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Students solve problems involving multiplicative comparisons in grade 4 in preparation for their work with ratios and proportions (6.RP.A.1). By working with problems related to 4.OA.A.2, students understand that multiplicative relationships (such as those encountered with ratios and proportions) act differently from additive relationships. In this two-step problem, students use multiplicative reasoning and then subtract to find the solution.

POSSIBLE SOLUTION:

Using a bar diagram



POSSIBLE SOLUTION:

Using equations

$$\text{Jared} \times 3 = \text{Matthew} \text{ so } 194 \times 3 = 582$$

$$\text{Matthew} - 297 = \text{Gary} \text{ so } 582 - 297 = 285$$

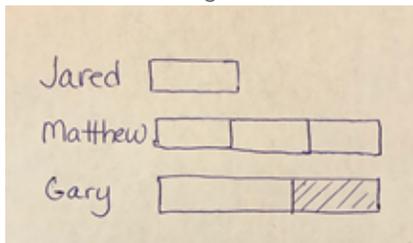
Gary sold 285 bars.

POSSIBLE STUDENT MISCONCEPTIONS:

Watch for students who first add 194 and 3 instead of multiplying. Also, students may arrive at a solution of 100 if they subtract without multiplying first.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Break the tasks into two separate problems, or provide only the first part of the problem to students who struggle.
- Provide an empty bar diagram to help students organize their thinking for this task.



SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Pose similar problems involving fractions and/or decimals, for example:

A seamstress needs $1\frac{5}{8}$ yards of fabric to make a child's dress. She needs 3 times as much fabric to make a woman's dress. How many yards of fabric does she need for both dresses?

Source: <https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-g-lesson-39>

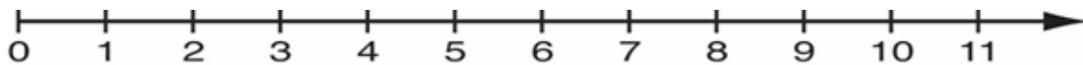
WRITE AS A FRACTION

Write $11 \div 5$ as a fraction.

Place a point that shows this number on the number line below.

SHOW YOUR WORK:

ANSWER:



TEACHER'S GUIDE WRITE AS A FRACTION

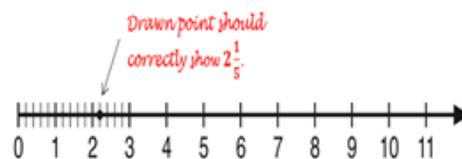
Write $11 \div 5$ as a fraction.

Place a point that shows this number on the number line below.

TARGETED STANDARDS:

5.NF.B.3

SOLUTION: Fraction: $\frac{11}{5}$ OR Equivalent



CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

In this task students, students understand a fraction as a number on the number line (3.NF.2) and can interpret the fraction as an operation of division (5.NF.B.3) and directly maps to 6.RP.A.2, in which students understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$.

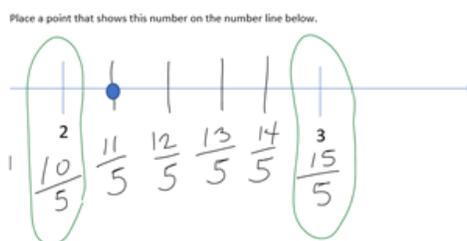
POSSIBLE SOLUTION:

Write $11 \div 5$ as a fraction.

$$2 \frac{1}{5}$$

$$\begin{array}{r} 5 \overline{) 11} \\ -10 \\ \hline 1 \end{array}$$

POSSIBLE SOLUTION:



POSSIBLE STUDENT MISCONCEPTIONS:

- Instead of recognizing $\frac{a}{b} = a \div b$, the numerator divided by the denominator, the student may try to divide the denominator by the numerator, especially when the numerator is less than the denominator.
- Students may not remember how to set up a division problem and the terms divisor, dividend, quotient and remainder and how these parts are transferred to a mixed number.

POSSIBLE SOLUTION:

Write $11 \div 5$ as a fraction. Each star represents $1/5$ since we are thinking about 11 divided by 5.



And, 5 stars equal 1 whole.

Therefore, 11 stars is a total of 2 wholes + $1/5$ or $2\frac{1}{5}$.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Emphasize the tasks as two independent tasks and practice each separately, as a division task and a graphing task.
- Scaffold both tasks by providing familiar fractions such as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{2}$, etc.
- Focus on the visual representation of fractions until the student has a conceptual understanding.
- Provide all increments on the number line, such as the fifths.
- Continued practice writing equivalent fractions: $\frac{11}{5} = 2\frac{1}{5}$ and vice versa.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

- Provide a similar task but do not give any initial increments on the number line.
- Provide tasks where $a < b$, and a student must place a point on a number line that is less than 1, but greater than 0.

LANGUAGE SUPPORTS:

- “place” as a verb (consider other terms that may be used, such as draw, sketch, etc.)
- “point” on a “number line”
- Teachers should strive to use these words in context, such as place, point, increments and equivalent, so they become familiar to students and require students to use the appropriate mathematical language. It will be important to offer synonyms, rephrasing, visual cues, and modeling of what these words mean in the specific contexts represented. Post a visual in the classroom for students to reference, such as an anchor chart.

TASK SOURCE:

<https://achievethecore.org/page/1034/multiplication-and-division-of-fractions-mini-assessment>

GREATER THAN, LESS THAN, OR EQUAL TO

Write $>$, $<$, or $=$ to make each comparison true.

A. $2 \div 3$ _____ $2 \div \frac{1}{3}$

B. $0.2 \times \frac{1}{4}$ _____ $\frac{2}{10} \times \frac{1}{3}$

C. $\frac{1}{6} \div 4$ _____ $\frac{1}{6} \times \frac{1}{5}$

SHOW YOUR WORK:

TEACHER'S GUIDE

GREATER THAN, LESS THAN, OR EQUAL TO

Write $>$, $=$, or $<$ to make each comparison true.

- a. $2 \div 3$ _____ $2 \div \frac{1}{3}$
- b. $0.2 \times \frac{1}{4}$ _____ $\frac{2}{10} \times \frac{1}{3}$
- c. $\frac{1}{6} \div 4$ _____ $\frac{1}{6} \times \frac{1}{5}$

TARGETED STANDARDS:

- 5.NF.B.3
5.NF.B.4

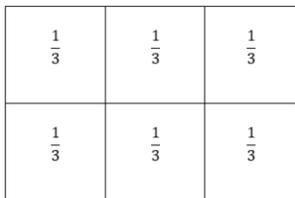
- SOLUTION:**
- a. $2 \div 3$ _____ $2 \div \frac{1}{3}$
- b. $0.2 \times \frac{1}{4}$ _____ $\frac{2}{10} \times \frac{1}{3}$
- c. $\frac{1}{6} \div 4$ _____ $\frac{1}{6} \times \frac{1}{5}$

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Prior learning stems from standards in grade 3 (3.OA.A and 3.OA.B) and grade 4 (4.MD.A.2 and 4.OA). Students will apply their understanding of multiplication and division of fractions to apply it to 6.RP.A.2, in which students understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

POSSIBLE SOLUTION:

$2 \div 3 = \frac{2}{3}$, which is less than 1
 $2 \div \frac{1}{3} = 6$ (see visual representation)



Six one-thirds equals 2 whole units.

Therefore, $\frac{2}{3}$ is less than 6.

POSSIBLE SOLUTION:

$0.2 = \frac{2}{10}$

$\frac{2}{10} \times \frac{1}{4} = \frac{2}{40}$ $\frac{2}{10} \times \frac{1}{3} = \frac{2}{30}$

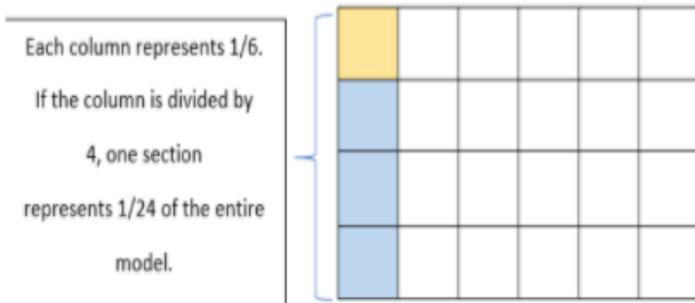
2 out of 40 is less than 2 out of 30.

Therefore, $\frac{2}{40}$ is less than $\frac{2}{30}$.

POSSIBLE STUDENT MISCONCEPTIONS:

- A.
- Instead of recognizing $\frac{a}{b} = a \div b$, the numerator divided by the denominator, the student may try to divide the denominator by the numerator, $3 \div 2$.
 - Students may see whole numbers and assume the expression on the left is larger
- B.
- Students may assume 2 out of 40 is larger than 2 out of 30 since 40 is larger than 30.
- C.
- Students may assume when dividing two numbers, the result will automatically be smaller than when you multiply two numbers.

POSSIBLE SOLUTION: One out of 24 is larger than one out of 30.



$$\frac{1}{6} \div 4 = \frac{1}{24}$$

$$\frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$$

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Ask students what it means to be greater than, less than, or equal to.
- Provide examples that use more expressions that are easily simplified, such as $1 \div 2$ compared to $2 \div 1$.
- Simplify each expression separately, prior to putting into a comparison statement
- Practice the multiplication of fractions.
- Engage students in a discussion about the relationship between the denominators of fractions.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

- Students can place their simplified expressions on a number line to have a visual understanding of the idea of comparison.

LANGUAGE SUPPORTS:

- Ensure there is a solid understanding of the concept of “greater than,” “less than,” and “equal to.”
- Ensure students have a quick recall of the symbols $<$, $>$, and $=$.

ADDITIONAL TEACHER SUPPORT:

The numbers chosen encourage students to use reasoning strategies to compare expressions without computation. Part (a) encourages students to compare division by a whole number and division by a fraction. In part (b), students can recognize 0.2 equals $\frac{2}{10}$, and then compare the effect of multiplying by $\frac{1}{4}$ and $\frac{1}{3}$. In part (c), students compare the effect of dividing by 4 and multiplying by $\frac{1}{5}$.

TASK SOURCE:

<https://achievethecore.org/page/1034/multiplication-and-division-of-fractions-mini-assessment>

JACKSON'S REASONING

Jackson claims that multiplication always makes a number bigger. He gave the following examples:

- If I take 6, and I multiply it by 4, I get 24, which is bigger than 6.
- If I take $\frac{1}{4}$, and I multiply it by 2 (whole number), I get $\frac{2}{4}$ or $\frac{1}{2}$ which is bigger than $\frac{1}{4}$.

Jackson's reasoning is incorrect. Give an example that proves he wrong, and explain his mistake using pictures, words, or numbers.

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE JACKSON'S REASONING

Jackson claims that multiplication always makes a number bigger. He gave the following examples:

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Jackson's reasoning is incorrect. Give an example that proves he wrong, and explain his mistake using pictures, words, or numbers.

TARGETED STANDARDS:

5.NF.B.5

SOLUTION:

Solutions should vary and be unique to each student (see possible student solutions based on the original information). Encourage students to communicate their thinking in multiple forms; picture, words, and/or numbers and test other possibilities.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Grade 5 work with multiplying by unit fractions, and interpreting fractions in terms of division, enables students to see that multiplying a quantity by a number smaller than 1 produces a smaller quantity. This understanding leads to the application of ratio and proportional relationships in grade 6. Students will determine a unit rate $\frac{a}{b}$ associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship.

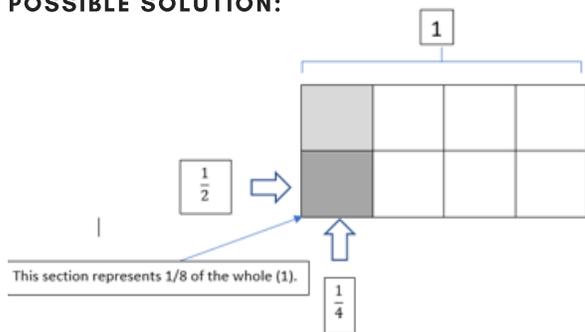
POSSIBLE SOLUTION:

If you have a whole object, which is equal to 1 and divide it into four equal sections, each section represents $\frac{1}{4}$ of the whole (1). And, then you take the $\frac{1}{2}$ section and take $\frac{1}{2}$ of it, you will have $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ of the original whole (1). One-eighth is smaller than $\frac{1}{4}$ or $\frac{1}{2}$ therefore, Jackson's reasoning is incorrect.

POSSIBLE STUDENT MISCONCEPTIONS:

- When taking half, students may think a half is represented by half of the entire diagram ($\frac{4}{8}$ or $\frac{2}{4}$). If helpful, remove some of the lines to avoid confusion.
- Students may simply read the equation (number representation) without an understanding of the concept of less than, greater than, or equal to 1 when multiplying factors.
- Students may assume when you multiply by a fraction, the product will automatically be smaller than the factors.

POSSIBLE SOLUTION:



POSSIBLE SOLUTION:

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Emphasize the visual (picture) model.
- Encourage students to communicate in verbal and written forms, in addition to the visual model using the appropriate mathematical terms. When students communicate their thinking, check for understanding as it relates to the size of the factors compared to the product.
- Use familiar fractions, such as $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{5}{2}$.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

- Pose the following probes:
- Which set of values make Jackson's statement true? Graph the values on a number line.
 - Which set of values make Jackson's claims false? Graph the values on a number line.
 - What generalizations can you make based on the two number lines?

LANGUAGE SUPPORTS:

- Scaling
- Factors
- Fractions > 1
- Fractions < 1
- Fractions $= 1$

TASK SOURCE:
<https://www.unbounded.org/math/grade-5/module-4>

FILL IN THE BLANK MULTIPLICATION

Multiply.

a. $\frac{1}{3}$ of 2 feet = _____ inches

b. $\frac{1}{6}$ of 3 yards = _____ feet

c. $(3 + \frac{1}{2}) \times 14 =$

d. $4\frac{2}{3} \times 13 =$

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE

FILL IN THE BLANK MULTIPLICATION

a. $\frac{1}{3}$ of 2 feet = _____ inches

b. $\frac{1}{6}$ of 3 yards = _____ feet

c. $(3 + \frac{1}{2}) \times 14 =$

d. $4\frac{2}{3} \times 13 =$

TARGETED STANDARDS:

5.NF.B.4.

5.MD.1

SOLUTION: A. 8 inches

B. $1\frac{1}{2}$

C. 49

D. $60\frac{2}{3}$

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

5.NF.B.4.a does not directly map to the grade 6 standards (6.RP.A.1 and 6.RP.A.2). However, 5.NF.B.4 does directly map to 5.NF.B.5, which in turn maps directly to 6.RP.A.2. In parts a and b of this task, students build on their understanding of the relationship of two quantities, such as feet and inches. This understanding is then applied to ratio and proportional relationships in grade 6. In parts c and d, students continue to build on the connection of the multiplication of fractions to the division of fractions. Students will need to apply this to the relationship between two quantities when determining the proportional relationship in grade 6.

POSSIBLE SOLUTION:

$$\begin{aligned} &\therefore (3 + \frac{1}{2}) \times 14 \\ &= (3 \times 14) + (\frac{1}{2} \times 14) \\ &= 42 + 7 \\ &= 49 \end{aligned}$$

POSSIBLE SOLUTION:

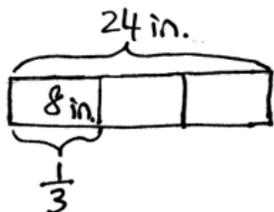
$$\begin{aligned} &4\frac{2}{3} \times 13 \\ &= (4 \times 13) + (\frac{2}{3} \times 13) \\ &= 52 + \frac{2 \times 13}{3} \\ &= 52 + \frac{26}{3} \\ &= 52 + 8\frac{2}{3} \\ &= 60\frac{2}{3} \end{aligned}$$

POSSIBLE STUDENT MISCONCEPTIONS:

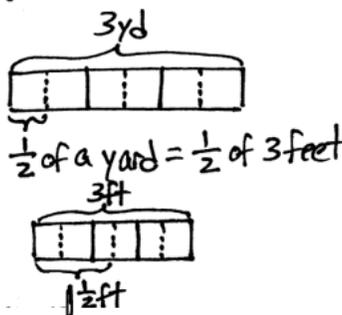
- For parts a and b, students may attempt to multiply $\frac{1}{3} \times 2$ (part a) or $\frac{1}{6} \times 3$ (part b), without converting both values to the same units.
- If students utilize the procedural method for multiplying fractions, for parts c and d, students may attempt to multiply $3\frac{1}{2}$ as $\frac{31}{2}$ or $4\frac{2}{3}$ as $\frac{42}{3}$.
- Students may need to be reminded of how to multiply a fraction by a whole number.

POSSIBLE SOLUTION:

$\frac{1}{3}$ of 2 feet = 8 inches
 2×12 inches = 24 inches



$\frac{1}{6}$ of 3 yards = 1 1/2 feet



SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

- For parts a and b, how do you know which quantities to convert (i.e. inches or feet)?
- In parts a, convert the values in terms of feet instead of inches. Would the results be the same?

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- For parts a and b, utilize actual measuring tools, such as yard sticks or rulers.
- For parts a and b, provide a tape diagram template.
- Practice additional conversions that are familiar to students.
- For parts c and d, demonstrate how the distributive property works using only whole numbers and then apply it to tasks with fractions.
- For parts c and d, consider a visual model of the distributive property, such as tape diagram or number line.

ADDITIONAL TEACHER SUPPORTS:

- Encourage students to demonstrate their understanding of conversions using a tape diagram or a visual model.
- Encourage students to share their thinking for parts c and d with specific mathematical language that demonstrates an understanding of the distributive property (without specifically naming the distributive property).

LANGUAGE SUPPORTS:

- Convert units (feet to inches)
- Mixed fractions
- Improper fraction (fraction > 1)

TASK SOURCE:

<https://www.unbounded.org/math/grade-5/module-4>

MRS. WILLIAMS' RICE CRISPY TREATS

Mrs. Williams uses the following recipe for rice crispy treats.

She decides to make $\frac{2}{3}$ of the recipe.

- 2 cups of melted butter
- 24 oz marshmallows
- 13 cups rice crispy cereal

A. How much of each ingredient will she need? Write an expression that included multiplication. Solve by multiplying.

B. How many fluid ounces of butter will she use? (Use your measurement conversion chart, if you wish.)

C. When the rice crispy treats have cooled, Mrs. Williams cuts them into 30 equal pieces. She gives two-fifths of the treats to her son and takes the rest to school. How many treats will Mrs. Williams take to school? Use any method to solve.

TEACHER'S GUIDE

MRS. WILLIAMS' RICE CRISPY TREATS

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- 2 cups of melted butter
- 24 oz marshmallows
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- A. How much of each ingredient will she need? Write an expression that included multiplication. Solve by multiplying.
- B. How many fluid ounces of butter will she use? (Use your measurement conversion chart if you wish.)
- C. When the rice crispy treats have cooled, Mrs. Williams cuts them into 30 equal pieces. She gives two-fifths of the treats to her son and takes the rest to school. How many treats will Mrs. Williams take to school? Use any method to solve.

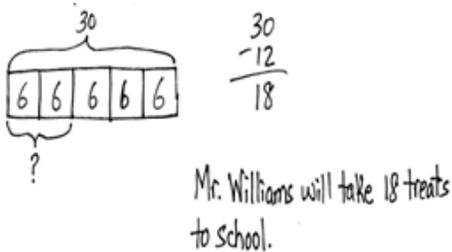
TARGETED STANDARDS:
 5.NF.B.4.
 5.NF.B.6
 5.MD.1

SOLUTION: A. $1\frac{1}{3}$ cups of melted butter
 16 oz marshmallows
 $8\frac{2}{3}$ cups rice crispy cereal
 B. $10\frac{2}{3}$ fl oz
 C. 18 treats

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

5.NF.4.a does not directly map to the grade 6 standards (6.RP.1 and 6.RP.2). However, 5.NF.4 does directly map to 5.NF.5, which in turn maps directly to 6.RP.2. Students continue to build on the connection of the multiplication of fractions to the division of fractions. Students will need to apply this to the relationship between two quantities when determining the proportional relationship in grade 6.

POSSIBLE SOLUTION:



POSSIBLE SOLUTION:

1 cup = 8 ounces $1\frac{1}{3} \times 8 = (1 \times 8) + (\frac{1}{3} \times 8)$
 $= 8 + \frac{8}{3}$
 She will use $10\frac{2}{3}$ fluid ounces of butter.
 $= 8 + 2\frac{2}{3}$
 $= 10\frac{2}{3}$

POSSIBLE STUDENT MISCONCEPTIONS:

- Students may not remember $2 = \frac{2}{1}$.
- Practice converting from an improper fraction to a mixed number.
- Require students to communicate their solutions within the context of the task.
- Students may need to be reminded that $1\frac{1}{3} = 1 + \frac{1}{3}$.
- Students may assume she takes 12 to school based on the diagram. Remind students to answer the correct question; how many does she take to school?

POSSIBLE SOLUTION:

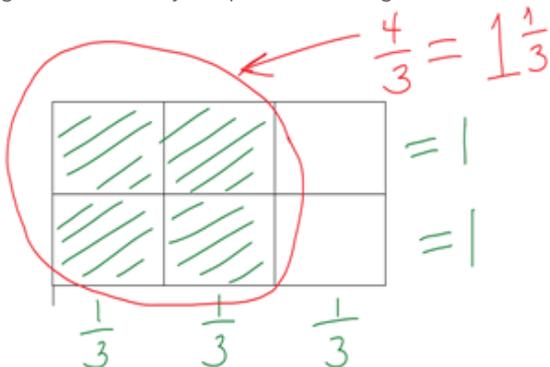
Butter: $\frac{2}{3} \times 2 \text{ cups} = \frac{2 \times 2}{3} = \frac{4}{3} = 1\frac{1}{3} \text{ cups}$
 Marshmallows: $\frac{2}{3} \times 24 \text{ oz.} = \frac{2 \times 24}{3} = \frac{48}{3} = 16 \text{ oz.}$
 Cereal: $\frac{2}{3} \times 13 \text{ cups} = \frac{2 \times 13}{3} = \frac{26}{3} = 8\frac{2}{3} \text{ cups}$
 she will need $1\frac{1}{3}$ cups of butter, 16 ounces of marshmallows, and $8\frac{2}{3}$ cups of rice crispy cereal.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

If Mrs. Williams lived in a country that used the metric system, how much of each ingredient would she use to make crispy rice treats?

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Provide a conversion chart
- Encourage students to justify their thinking with a visual model such as: $\frac{4}{3} \times 2$



LANGUAGE SUPPORTS:

- Define the difference between fluid ounces and ounces.
- Provide real-life examples of a cup and ounce.

TASK SOURCE:
<https://www.unbounded.org/math/grade-5/module-4>

BAKING RAISIN COOKIES

The baker needs $\frac{5}{8}$ cups of raisins for one batch of cookies. How many raisins will he need for 7 batches of cookies?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE BAKING RAISIN COOKIES

The baker needs $\frac{5}{8}$ cups of raisins for one batch of cookies. How many raisins will he need for 7 batches of cookies?

TARGETED STANDARDS:
4.NF.B.4

SOLUTION:
The baker needs $4\frac{3}{8}$ cups of raisins.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:
Rationale for 6.RP.A connection: In standard 6.RP.A.3, students are asked to use ratio and rate reasoning to solve real-world problems. Students' previous mastery with multiplying and adding fractions progresses to a student's ability to build ratios and multiply ratios successfully by rates in grade 6.

The baker needs $\frac{35}{8}$ cups of raisins.

POSSIBLE SOLUTION:

$$7 \times \frac{5}{8} = \frac{35}{8} = 4\frac{3}{8} \text{ cups of raisins}$$

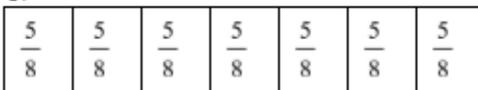
A student who has mastered 4.NF.B.4 will recognize this problem involves multiplying a whole number by a fraction and apply multiplication accurately to the problem. Students may leave the answer as a fraction greater than one or create the equivalent mixed number representation.

If a student leaves the answer as $\frac{35}{8}$ ask the prompt, "How many whole cups are in $\frac{35}{8}$?"

POSSIBLE SOLUTION:

$$\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} = \frac{35}{8} = 4\frac{3}{8}$$

Or



$$\frac{35}{8} = 4\frac{3}{8} \text{ cups of raisins}$$

A student who shows this method understands adding fractions with like denominators using a visual model. If a student uses this method ask the prompt, "How else can you solve this problem?" or "What other operations can we explore to find the identical answer?"

POSSIBLE STUDENT MISCONCEPTIONS:

$$\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} = \frac{35}{56}$$

The above example could be evidence the student has not mastered the 3rd grade standard of identifying fractions as numbers or understanding addition as joining parts of the same whole (grade 4 standard) If this answer is shown, prompt the student with, "Let's estimate, should our answer be greater or less than one cup of raisins?" Then, "Does your fraction represent more or less than one cup?"

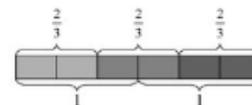
SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Explore this problem with whole numbers to check for understanding of multiplication.
- Introduce visual models, (for example a tape diagram) to represent addition and connect multiplication.
- Use a measuring cup to model the problem to the whole group (if time permits)
- Replace $\frac{5}{8}$ with $\frac{1}{2}$ and recheck for understanding.
- Fraction Mini Assessment can be implemented to identify students' baseline understanding of fractions.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

5.NF.B.4 Task

Makayla said, "I can represent $3 \times \frac{2}{3}$ with 3 rectangles each of length $\frac{2}{3}$."



Connor said, "I know that $\frac{2}{3} \times 3$ can be thought of as 3 of $\frac{2}{3}$. Is 3 copies of $\frac{2}{3}$ the same as 3 of 3?"

- Draw a diagram to represent $\frac{2}{3}$ of 3.
- Explain why your picture and Makayla's picture together show that $3 \times \frac{2}{3} = \frac{2}{3} \times 3$.
- What property of multiplication do these pictures illustrate?

LANGUAGE SUPPORTS:

- Read the problem aloud.
- Be sure to define "batch." This word could create a barrier for students to understand what the problem is asking.

TASK SOURCE:
<https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-g-lesson-37/file/29251> (page 487)

THE RELAY TEAM

Eight students are on a relay team. Each run $1\frac{3}{4}$ kilometers. How many total kilometers does their team run?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE THE RELAY TEAM

Eight students are on a relay team. Each runs $1\frac{3}{4}$ kilometers. How many total kilometers does their team run?

TARGETED STANDARDS:
4.NF.B.4

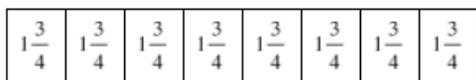
SOLUTION:
The relay team ran 14 kilometers.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Rationale for 6.RP.A connection: In standard 6.RP.A.3, students are asked to use ratio and rate reasoning to solve real-world problems. Students' previous mastery with multiplying and adding fractions (and mixed numbers) progresses to a student's ability to build ratios and multiply ratios successfully by rates in 6th grade.

POSSIBLE SOLUTION:

$$8 \times 1\frac{3}{4} = 8 + \frac{24}{4} = 8 + 6 = 14 \text{ kilometers}$$



A student who has mastered 4.NF.B.4 will recognize this problem involves multiplying a whole number by a mixed number and apply multiplication accurately to the problem. In the above possible solution the student used the distributive property to multiply the mixed number.

POSSIBLE SOLUTION:

$$8 \times 1\frac{3}{4} = 8 \times \frac{7}{4} = \frac{56}{4} = 14$$

In the above example the student created an equivalent representation of $1\frac{3}{4} = \frac{7}{4}$. Next, the student displayed mastery of multiplying a whole number by a fraction and the mastery of creating equivalent fractions, specifically showing 14. This answer displays the understanding of whole numbers as a fraction as well.

POSSIBLE STUDENT MISCONCEPTIONS:

If a student has not mastered equivalent fractions they may produce inaccurate representations of $1\frac{3}{4}$. Another possible misconception could occur if a student attempts to solve this problem as repeated addition (visual in Possible Student Solution 1). If the student recognizes the total of the whole numbers as 8 Is they could misconceive the total of $8\frac{3}{4}$ as $\frac{24}{32}$.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Prompting Question:
 - Look at the problem, estimate your answer. Will your answer be more or less than 16? Explain your reasoning
- Students who struggle with the above prompt:
 - Create a visual representation of the eight runners. If possible, use physical representations to recreate a tape diagram with eight representations of 1 and eight representations of $\frac{3}{4}$. Ask the student to move the pieces around and organize them together.
- Example of a leading question with the above representation: How many wholes do four $3/4$ s equal?

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

- [5.NF.B.6 - Making Cookies by Illustrative Mathematics](#)

LANGUAGE SUPPORTS:

- Read the problem aloud.
- Have a conversation around what a "relay team" is. What does it look like? Possibly apply the problem to track and field.

TASK SOURCE:
<https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-g-lesson-38/file/29261> (page 498)

SOLVING MULTIPLICATION PROBLEMS

Solve using any method.

A. $7 \times \frac{3}{4}$

B. $9 \times \frac{2}{5}$

C. $60 \times \frac{5}{8}$

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE

SOLVING MULTIPLICATION PROBLEMS

Solve using any method.

A. $7 \times \frac{3}{4}$

B. $9 \times \frac{2}{5}$

C. $60 \times \frac{5}{8}$

TARGETED STANDARDS:
4.NF.B.4

SOLUTION:

A. $\frac{21}{4}$ or $5\frac{1}{4}$

B. $\frac{18}{5}$ or $3\frac{3}{5}$

C. $\frac{300}{8}$ or $37\frac{4}{8}$ or $37\frac{1}{2}$

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

Rationale for 6.RP.A connection: In standard 6.RP.A.3, students are asked to use ratio and rate reasoning to solve real-world problems. Students' previous mastery with multiplying and adding fractions progresses to a student's ability to build ratios and multiply ratios successfully by rates in 6th grade.

POSSIBLE SOLUTION:

Algorithm Method:

A. $7 \times \frac{3}{4} = \frac{21}{4} = 5\frac{1}{4}$

**Fraction greater than one or mixed number are acceptable solutions

B. $9 \times \frac{2}{5} = \frac{18}{5} = 3\frac{3}{5}$

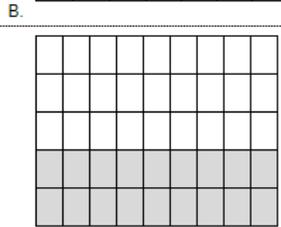
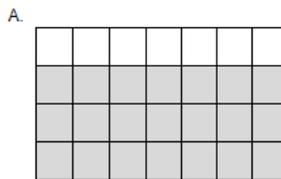
**Fraction greater than one or mixed number are acceptable solutions

C. $60 \times \frac{5}{8} = \frac{300}{8} = 37\frac{4}{8} = 37\frac{1}{2}$

**Fraction greater than one or mixed number are acceptable solutions

POSSIBLE SOLUTION:

Visual Representation Method (One example below applies to A & B) - Students can also utilize tape diagrams for A & B. Area Model:



POSSIBLE STUDENT MISCONCEPTIONS:

One common misconception with multiplication is some students believe that both numbers need a common denominator in order to multiply. Although if equivalent fractions are mastered, this will yield an accurate answer, encourage students to think about estimating the final answer for multiplication. Attempt to deduce their reasoning to realize that common denominators are not needed.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Ask students if the fraction in the problem is greater than or less than one.
- Create a visual representation for the first two problems to help the student visualize what is being asked.
- Add context to the numbers to create a real world connection for the student.

ADDITIONAL TEACHER SUPPORTS:

- The teacher can use the visual in problem B to have a conversation around problem C and the different strategies to multiple fractions. They can debate which strategy can be more effective with larger numbers like 60.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Decide whether the expression in column A or B is larger. Check the correct box for each row.

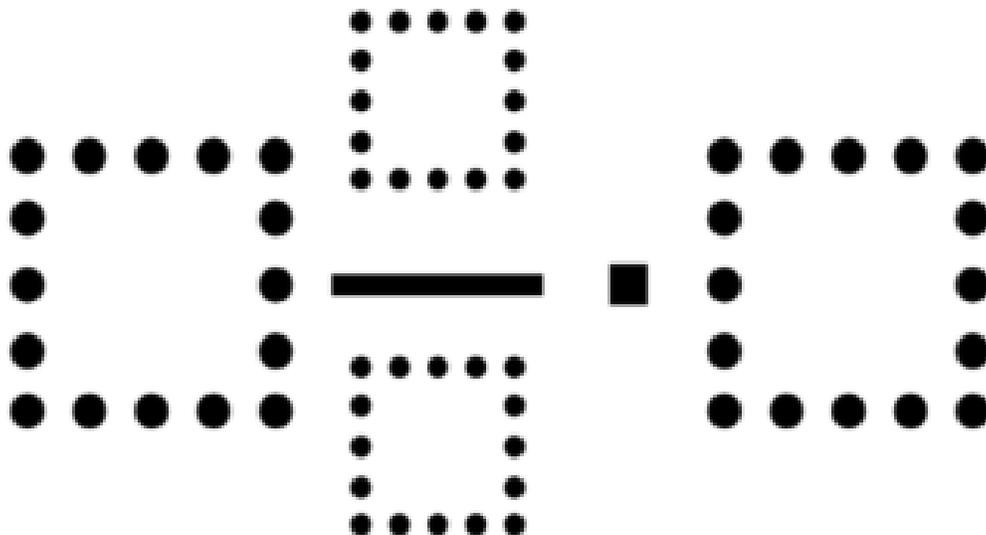
A	B	Which is larger?				
$\frac{5}{17} \times \frac{2}{3}$	$\frac{5}{17} \times \frac{3}{2}$	<table border="0"> <tr> <td>A</td> <td>B</td> </tr> <tr> <td><input type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> </table>	A	B	<input type="checkbox"/>	<input type="checkbox"/>
A	B					
<input type="checkbox"/>	<input type="checkbox"/>					
$\frac{27}{25} \times 36$	$\frac{30}{32} \times 36$	<table border="0"> <tr> <td>A</td> <td>B</td> </tr> <tr> <td><input type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> </table>	A	B	<input type="checkbox"/>	<input type="checkbox"/>
A	B					
<input type="checkbox"/>	<input type="checkbox"/>					
$\frac{63}{54}$	$\frac{63}{54} \times \frac{62}{54}$	<table border="0"> <tr> <td>A</td> <td>B</td> </tr> <tr> <td><input type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> </table>	A	B	<input type="checkbox"/>	<input type="checkbox"/>
A	B					
<input type="checkbox"/>	<input type="checkbox"/>					
$9 \times \frac{1}{3}$	$\frac{1}{3}$	<table border="0"> <tr> <td>A</td> <td>B</td> </tr> <tr> <td><input type="checkbox"/></td> <td><input type="checkbox"/></td> </tr> </table>	A	B	<input type="checkbox"/>	<input type="checkbox"/>
A	B					
<input type="checkbox"/>	<input type="checkbox"/>					

TASK SOURCE:

<https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-g-lesson-36/file/29241> (page 483)

MULTIPLYING MIXED NUMBERS BY WHOLE NUMBERS

Using the digits 1 through 9 at most one time each, fill in the boxest to make the smallest (or largest) product.



SHOW YOUR WORK:

TEACHER'S GUIDE

MULTIPLYING MIXED NUMBERS BY WHOLE NUMBERS

Using the digits 1 through 9 at most one time each, fill in the boxes to make the smallest (or largest) product.

TARGETED STANDARDS:
4.NF.B.4

SOLUTION:
The current best answer for largest product is $8\frac{7}{1} \times 9$.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

In standard 6.RP.A.3, students are asked to use ratio and rate reasoning to solve real-world problems. Students' previous mastery with multiplying and adding fractions (and mixed numbers) progresses to a student's ability to build ratios and multiply ratios successfully by rates in 6th grade.

The current best answer for smallest product is $2\frac{3}{9} \times 1$.

POSSIBLE SOLUTION:

**Solutions will vary depending on the students' placement of the digits and accuracy of the multiplication. **

POSSIBLE STUDENT MISCONCEPTIONS:

Students will automatically attempt to use the four largest digits to create the largest product. Conversely, students will attempt to use the four smallest digits to create the lowest solution.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Prompt the students to start by creating two equations. Have the students compare the products for both of those solutions. Prompt the student, "What do you notice about your choices?" Encourage the student to attempt a third solution.
- For students who struggle with multiplication of a mixed number and a whole number, briefly review multiplying whole numbers. Deduce the students reasoning to a whole number and a fraction less than one. Finally, make a connection to a mixed number and a whole number. If possible, have the student create an equivalent fraction greater than one for the improper number they chose.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Write $>$, $=$, or $<$ to make each comparison true.

a. $2 \div 3$ _____ $2 \div \frac{1}{3}$

b. $0.2 \times \frac{1}{4}$ _____ $\frac{2}{10} \times \frac{1}{3}$

c. $\frac{1}{6} \div 4$ _____ $\frac{1}{6} \times \frac{1}{5}$

LANGUAGE SUPPORTS:

- Clarify "digits."

TASK SOURCE:

<https://www.openmiddle.com/multiplying-mixed-numbers-by-whole-numbers/>

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RHONDA'S EXERCISE PLAN

Rhonda exercised for $\frac{5}{6}$ of an hour for five days. How many total hours did Rhonda exercise?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE RHONDA'S EXERCISE PLAN

Rhonda exercised for $\frac{5}{6}$ of an hour for five days. How many total hours did Rhonda exercise?

TARGETED STANDARDS:
4.NF.B.4

SOLUTION:

$$\frac{25}{6} = 4 \frac{1}{6} \text{ hours}$$

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

In standard 6.RP.A.3, students are asked to use ratio and rate reasoning to solve real-world problems. Students' previous mastery with multiplying and adding fractions progresses to a student's ability to build ratios and multiply ratios successfully by rates in 6th grade.

POSSIBLE SOLUTION:

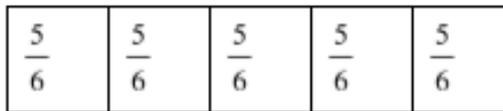
$$5 \times \frac{5}{6} = \frac{25}{6}$$

$$\frac{25}{6} = 4 \frac{1}{6} \text{ hours}$$

POSSIBLE SOLUTION:

$$\frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{25}{6}$$

Or



$$\frac{25}{6} = 4 \frac{1}{6} \text{ hours}$$

POSSIBLE STUDENT MISCONCEPTIONS:

$$\frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{25}{30}$$

The above example could be evidence the student has not mastered the grade 3 standard of identifying fractions as numbers or understanding addition as joining parts of the same whole (grade 4 standard).

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Explore this problem with whole numbers to check for understanding of multiplication.
- Introduce visual models (for example a tape diagram) to represent addition and connect multiplication.
- Use minutes in the context of the problem.
- Replace $\frac{5}{6}$ with $\frac{1}{2}$ and recheck for understanding.
- Fraction Mini Assessment can be implemented to identify students' baseline understanding of fractions.

<https://achievethecore.org/page/3233/understanding-a-fraction-as-a-number>

LANGUAGE SUPPORTS:

Ask students for examples of exercising to help ensure understanding of the word "exercise" by all students.

ADDITIONAL TEACHER SUPPORTS:

- When reading $\frac{25}{6}$, do not refer to this fraction as "improper." Instead discuss this fraction as greater than one. How many wholes are represented in this fraction greater than one? What does the remainder represent, " $\frac{1}{6}$ of an hour."

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Draw a diagram to show $\frac{2}{3} \times \frac{3}{4}$.

What is the product of $\frac{2}{3} \times \frac{3}{4}$?

TASK SOURCE:

<https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-g-lesson-36/file/29241> (page 478)

IDENTIFY CONNECTIONS BETWEEN NUMBER PAIRS

For each pair of points below, think about the line that joins them. For which pairs is the line parallel to the x-axis? Circle your answer(s). Without plotting them, explain how you know.

A. $(3.2, 7)$ and $(5, 7)$

B. $(8, 8.4)$ and $(8, 8.8)$

C. $(6\frac{1}{2}, 12)$ and $(6.2, 11)$

TEACHER'S GUIDE

IDENTIFY CONNECTIONS BETWEEN NUMBER PAIRS

For each pair of points below, think about the line that joins them. For which pairs is the line parallel to the x-axis? Circle your answer(s). Without plotting them, explain how you know.

TARGETED STANDARDS:

5.G.A.1

SOLUTION:

A. (Circled = yes), the y value is identical in each point, thus creating a parallel

relationship to the x axis.

B. (Not circled = no), the y value differs in each point.

The x value is the same, thus creating a perpendicular relationship to the x-axis.

C. (Not circled = no), the y value differs in each point.

This would create an intersecting line with the x-axis.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

5.G.A.1 is a related standard to 5.G.A.2. 5.G.A.2 directly supports standard 6.RP.A.3. In 6.RP.A.3, students “graph the pairs of values displayed in ratio tables on coordinate axes”. The graph of such a collection of equivalent ratios lies on a line through the origin, and the pattern of increases in the table can be seen in the graph as coordinated horizontal and vertical increases.” Therefore, it is vital the foundational skills of graphing presented in 5.G.A.1 & 5.G.A.2 are mastered prior to exploring ratio tables on coordinate axes. Note: 5.G.A.2 Represents real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

POSSIBLE SOLUTION:

(Circled) - y values are identical, creates a horizontal line that is parallel to the x axis.

(Not circled) - y values are not identical. This example has the x values identical, this creates a perpendicular relationship to the x axis.

(Not circled) - y values are not identical. This relationship creates intersecting lines with the x axis.

POSSIBLE STUDENT MISCONCEPTIONS:

(Circled) - y values are identical, creates a horizontal line that is parallel to the x-axis.

(Not circled) - y values are not identical. This example has the x values identical, this creates a perpendicular relationship to the x-axis.

(Not circled) - y values are not identical. This relationship creates intersecting lines with the x-axis.

LANGUAGE SUPPORTS:

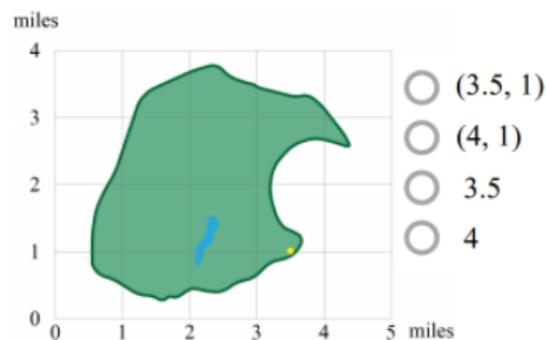
- Words to focus on w/ ELL students: points/parallel translations
- Vocabulary support for all students: points /parallel/x-axis

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

- Tier 1 - *Offer students grid paper or a pre-made coordinate grid that the students can use as a visual aid.
- Tier 2 - *Offer students a coordinate grid that is pre-labeled to help them identify the points and create a visual of the two points connected.
- Tier 3 - *Offer students a coordinate grid that is pre-labeled to help them identify the points and create a visual of the two points connected. Label the x and y axis.
- *Have students define parallel lines and explore what the vocabulary means in context to another line.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

This is a map of an island. There is a coordinate system on the map. The yellow dot shows where there is a lighthouse. What are the coordinates of the lighthouse?



Supports 5.G.A.2, which directly supports 6.RP.A.3

ADDITIONAL TEACHER SUPPORTS:

Questions to promote student discourse: “I noticed that B and C were not circled. What types of relationships would these two pairs of points create with the x axis? Defend your reasoning with evidence.” “Who can provide an original pair of points that would be parallel to the x-axis?” “Who can provide an original pair of points that would be at least 25 units from the y-axis AND parallel to the x-axis?”

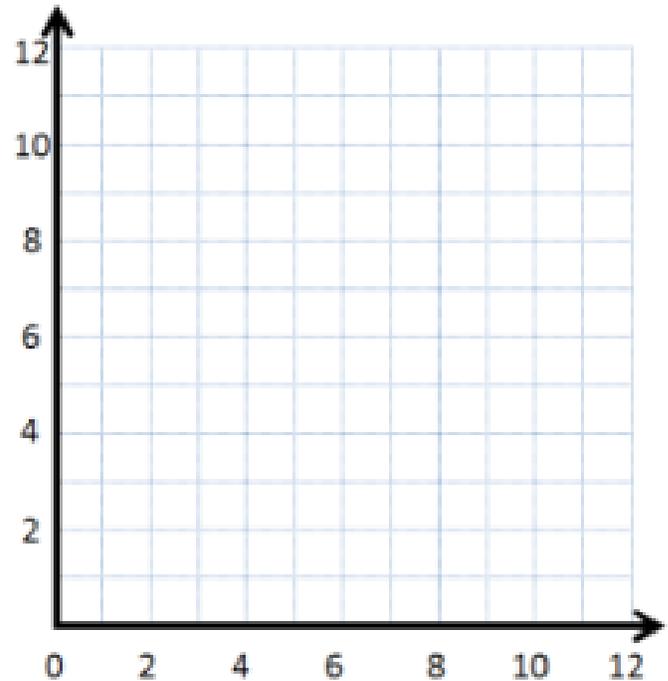
TASK SOURCE:

<https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-lesson-5/file/69636> (Page 80)

PLOTTING ON A COORDINATE PLANE

Complete the chart. Then, plot the points on the coordinate plane.

x	y	(x, y)
0	4	
2	6	
3	7	
7	11	



1. Use a straightedge to draw a line connecting these points.
2. Write a rule to show the relationship between the x - and y -coordinates for points on the line.
3. Name two other points that are also on this line.

ANSWER:

TEACHER'S GUIDE PLOTTING ON A COORDINATE PLANE

Complete the chart. Then, plot the points on the coordinate plane.

TARGETED STANDARDS:

5.G.A.1

SOLUTION:

***See Possible Student Solution section for completed table, rule for the table and suggested points on the line.**

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

This problem focuses on standard 5.G.A.1. Standard 5.G.A.1 is a related standard to 5.G.A.2. Standard 5.G.A.2 directly supports standard 6.RP.A.3. In 6.RP.A.3, students “graph the pairs of values displayed in ratio tables on coordinate axes. The graph of such a collection of equivalent ratios lies on a line through the origin, and the pattern of increases in the table can be seen in the graph as coordinated horizontal and vertical increases.” Therefore, it is vital the foundational skills of graphing presented in 5.G.A.1 & 5.G.A.2 are mastered prior to exploring ratio tables on coordinate axes.

POSSIBLE SOLUTION:

x	y	(x, y)
0	4	(0, 4)
2	6	(2, 6)
3	7	(3, 7)
7	11	(7, 11)

1. Use a straightedge to draw a line connecting these points.
2. Write a rule to show the relationship between the x- and y-coordinates for points on the line.
y value = x value + 4
3. Name two other points that are also on this line.
answers will vary

POSSIBLE STUDENT MISCONCEPTIONS:

- Since the coordinate plane is not labeled, some students could graph the inverse of the points listed in the table. This would create a successful rule for the misrepresented points.
- Students may look at the pattern of the given four points in the table and state there is no rule since the points are not consecutively listed in the table.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Have the teacher or volunteer student label the axis prior to starting the task.
- Ask questions like, “What do you notice about the x values in the table? What do you notice about the y values in the table?” prior to launching independent work.
- Clarify student’s ability to decode/define the mathematical vocabulary needed to master this task (point, plot, coordinate plane, x coordinate, y coordinate, straightedge).

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

- <https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-b-lesson-7/file/69681>
 - Page 112, Problem No. 2

ADDITIONAL TEACHER SUPPORTS:

For students who defend that there is no rule in the table based on the four points given, have those students explore part 3 of the problem in relation to the x values “in between,” the given x values (ie: 1, 4, 5, & 6) or create a new table where all values from 0 through 6 are listed for x. After that table is complete have the students compare the original table to the new table.*

LANGUAGE SUPPORTS:

Focus on vocabulary:
 x coordinate
 y coordinate
 Straightedge
 Point
 Plot
 Coordinate plane

TASK SOURCE:

<https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-b-lesson-7/file/69681> (page 111)

PLOTTING BASED ON RULES

Complete the table for the given rules.

Line *e*

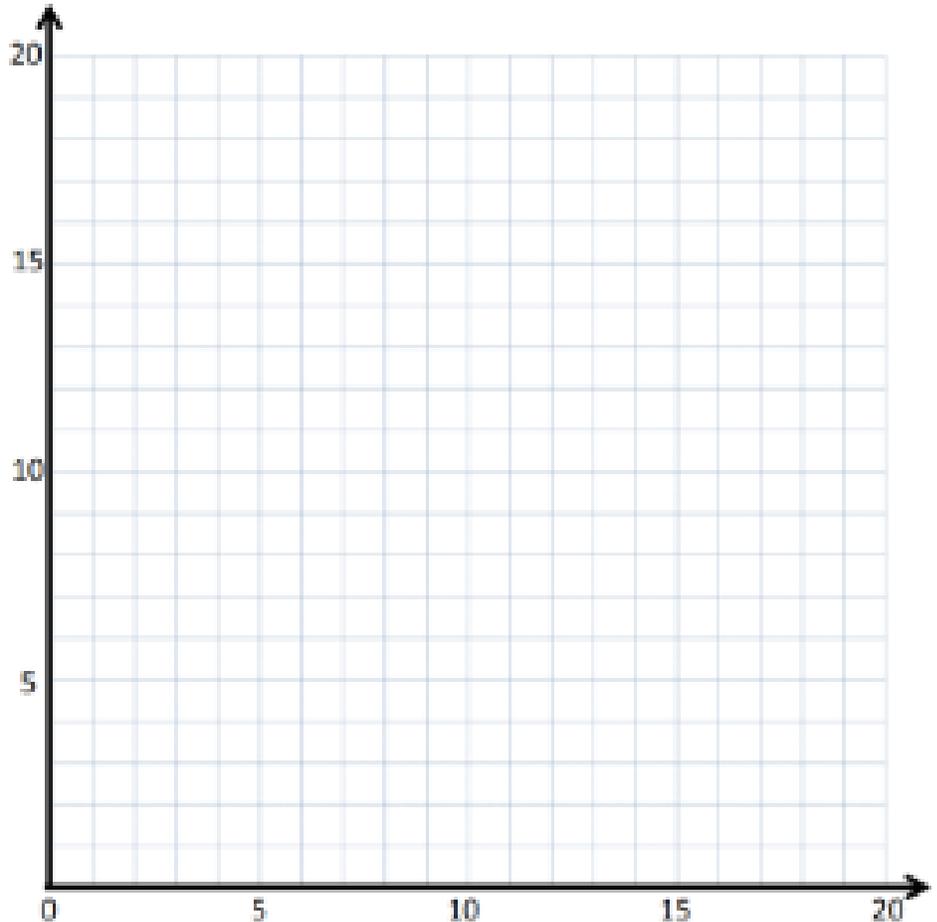
Rule: *y* is twice as much as *x*

<i>x</i>	<i>y</i>	(<i>x</i> , <i>y</i>)
0		
2		
5		
9		

Line *f*

Rule: *y* is half as much as *x*

<i>x</i>	<i>y</i>	(<i>x</i> , <i>y</i>)
0		
6		
10		
20		



A. Construct each line on the coordinate plane above.

B. Compare and contrast these lines.

C. Based on the patterns you see, predict what line *g*, whose rule is "*y* is 4 times as much as *x*" would look like? Draw your prediction in the plane above.

TEACHER'S GUIDE PLOTTING BASED ON RULES

Complete the table for the given rules.

TARGETED STANDARDS:

5.G.A.1

SOLUTION:

See "Possible Student Solution" section for completed table, lines, and one example of a possible prediction from a student.

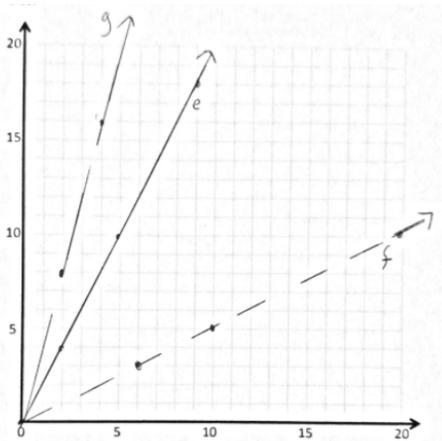
Possible Student Observation:

- Line e is steeper than Line f.
- Line f is less steep than Line e.
- Line e "goes up" faster than Line f.
- Line f is "flatter" than Line e. Both lines are straight.
- Both lines go "up to the right."

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

This problem focuses on standard 5.G.A.1. Standard 5.G.A.1 is a related standard to 5.G.A.2. Standard 5.G.A.2 directly supports standard 6.RP.A.3. In 6.RP.A.3, students "graph the pairs of values displayed in ratio tables on coordinate axes. The graph of such a collection of equivalent ratios lies on a line through the origin, and the pattern of increases in the table can be seen in the graph as coordinated horizontal and vertical increases." Therefore, it is vital the foundational skills of graphing presented in 5.G.A.1 & 5.G.A.2 are mastered prior to exploring ratio tables on coordinate axes.

POSSIBLE SOLUTION:



Line e ——— Rule: y is twice as much as x

x	y	(x,y)
0	0	(0,0)
2	4	(2,4)
5	10	(5,10)
9	18	(9,18)

Line f - - - Rule: y is half as much as x

x	y	(x,y)
0	0	(0,0)
6	3	(6,3)
10	5	(10,5)
20	10	(20,10)

line g

x	y
0	0
2	8
4	16

→ possible table to check prediction

POSSIBLE STUDENT

MISCONCEPTIONS:

- Students may graph the coordinate pairs (y,x) if there is not mastery of a coordinate pair being represented as (x,y) .
- Students may have mastered the coordinate pair of (x,y) but may still have unfinished learning on how to graph a point and misrepresent the coordinate pairs at inaccurate locations.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Read the "rule" aloud. Ask the student(s) to share the first y value and assess understanding.
- For students who struggle with graphing, perhaps label each interval to assist with accurate horizontal and vertical distance.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

5.G Meerkat Coordinate Plane Task
<http://tasks.illustrativemathematics.org/content-standards/5/G/A/2/tasks/1516>

ADDITIONAL TEACHER SUPPORTS:

After line g is graphed, consider asking the following to increase student discourse:

- What do you notice about all three lines?
- How does line g compare to line f?
 - How many times greater are the y values in the table for line g than in line f (if a table for line is generated with at least one non-zero x value identical to the table for line f)?
- Who can create a rule for a table that would produce the "least steep" line on the coordinate grid if graphed?

LANGUAGE SUPPORTS:

Vocabulary support - "rule"

TASK SOURCE:

<https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-b-lesson-9/file/69711> (page142)

COORDINATE PLANE Q&A

Use the coordinate plane to answer the questions.

A. Use a straightedge to construct a line that goes through points A and B. Label the line g.

B. Line g is parallel to the _____-axis and is perpendicular to the _____-axis.

C. Draw two more points on line g. Name them C and D.

D. Give the coordinates of each point below:

A: _____

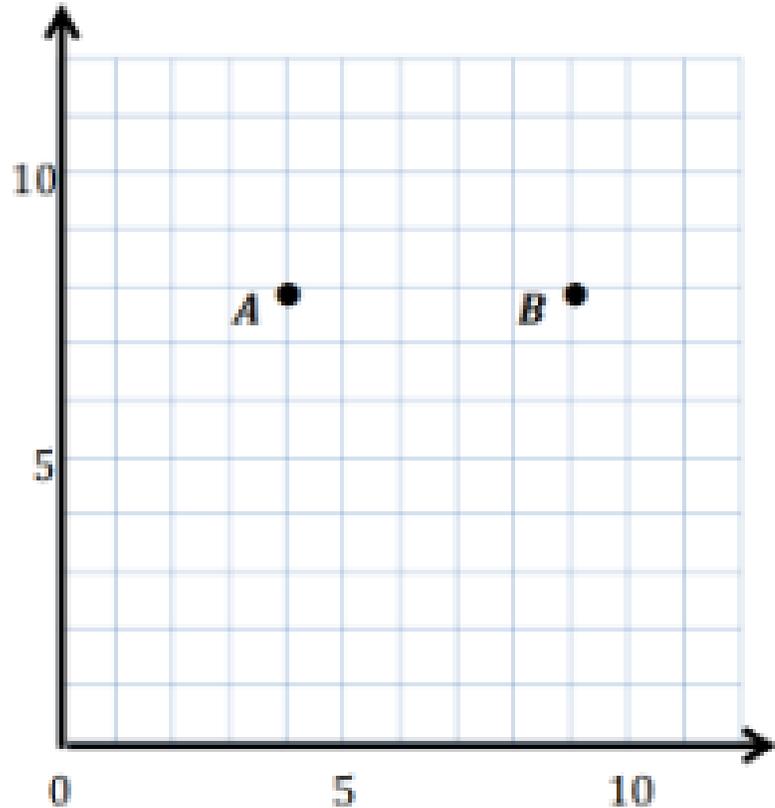
B: _____

C: _____

D: _____

E. What do all the points on line g have in common?

F. Give the coordinates of another point that falls on line g with an x-axis coordinate greater than 25.



TEACHER'S GUIDE COORDINATE PLANE Q&A

Use the coordinate plane to answer the questions.

TARGETED STANDARDS:

5.G.A.1

SOLUTION:

See possible student solution.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

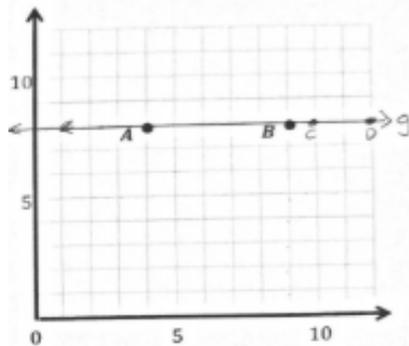
This problem focuses on standard 5.G.A.1. Standard 5.G.A.1 is a related standard to 5.G.A.2. Standard 5.G.A.2 directly supports standard 6.RP.A.3. In 6.RP.A.3, students “graph the pairs of values displayed in ratio tables on coordinate axes. The graph of such a collection of equivalent ratios lies on a line through the origin, and the pattern of increases in the table can be seen in the graph as coordinated horizontal and vertical increases.” Therefore, it is vital the foundational skills of graphing presented in 5.G.A.1 & 5.G.A.2 are mastered prior to exploring ratio tables on coordinate axes.

- Answers or parts of answers to C, D, and F will vary.
 - For part C, all points must be plotted on line g.
 - For part D, coordinates c and d will vary.
 - For part F, the value of x should be greater than 25 and the y value should equal 8.

POSSIBLE SOLUTION:

- a. Use a straightedge to construct a line that goes through points A and B. Label the line g.
- b. Line g is parallel to the x-axis and is perpendicular to the y-axis.
- c. Draw two more points on line g. Name them C and D.
- d. Give the coordinates of each point below.

A: (4, 8) B: (9, 8)
C: (10, 8) D: (12, 8)



- e. What do all of the points on line g have in common? *y value of the coordinate pair is 8. or Distance of 8 from the x-axis*
- f. Give the coordinates of another point that falls on line g with an x-coordinate greater than 25. *(100, 8)*

POSSIBLE STUDENT MISCONCEPTIONS:

- Students may confuse the x- and y- axis.
- Students may confuse, mix up or not be aware of the terms, “perpendicular,” and “parallel.”
- Students may identify the points “backwards,” displaying (8,4) for point A, (8,9) for B, etc.

LANGUAGE SUPPORTS:

Possible Vocabulary Supports:

- coordinate plane
- coordinates
- parallel
- perpendicular
- construct
- straightedge

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Develop the vocabulary needed to show mastery of this standard (e.g., perpendicular, parallel, coordinates).
- Provide or have students label intervals for every unit on the x- and y- axis.
- Provide a label for the x- and y- axis, if appropriate. Provide line g on the coordinate axis for students to use if using a straightedge could create inaccurate points.

ADDITIONAL TEACHER SUPPORTS:

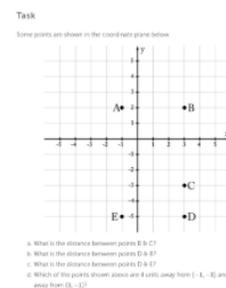
Possible extension question to check for understanding of perpendicular and/or parallel line definitions:

- Name at least two points, that when a line is constructed through, would create a perpendicular line to line g
- Name at least two points, that when a line is constructed through, would create a parallel line to line g.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

This task is grade level appropriate - if a student completes the Do Now quickly with 100% accuracy, this task should be used to help the student explore supported standard on level (6th grade). This task includes all four quadrants of a coordinate grid.

6.NS Distances between Points
Provided by Illustrative Mathematics



- a. What is the distance between points A and C?
- b. What is the distance between points B and D?
- c. What is the distance between points D and B?
- d. Which of the points shown above are 4 units away from (-4, -2) and 3 units away from (0, -3)?

TASK SOURCE:

<https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-lesson-5/file/69636> (page 79)

PEARL'S STICKERS

Pearl buys 125 stickers. She gives 53 stickers to her little sister. Pearl then puts 9 stickers on each page of her album. If she uses all of her remaining stickers, how many pages does Pearl put stickers on?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE PEARL'S STICKERS

Pearl buys 125 stickers. She gives 53 stickers to her little sister. Pearl then puts 9 stickers on each page of her album. If she uses all of her remaining stickers, how many pages does Pearl put stickers on?

TARGETED STANDARDS:
3.OA.A.3
4.OA.A.2

SOLUTION:
Pearl will need 8 pages to put the stickers on.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

This problem is standard 3.OA.A.3 (maps to 3.OA. A.1 and 3.OA.A.2 on Coherence Map) and is connected to 6.RP.A. Students' previous work with multiplication and division of equal groups connects to their understanding of ratios and rates. In this particular problem, students must find how many stickers are remaining, and then use that information to solve how many pages would be needed for equal groups of 9.

POSSIBLE SOLUTION:

125 stickers

53 stickers

S = number of stickers Pearl has left.

$$125 - 53 = S$$

$$S = 72$$

P = number of pages Pearl puts stickers on.

$$72 \div 9 = P$$

$$P = 8$$

Pearl puts stickers on 8 pages.

My answer is reasonable because $9 \times 8 = 72$ and $72 + 53 = 72 + 50 = 122$ and that is close to 125.

POSSIBLE SOLUTION:

$$\begin{array}{r} 125 \\ - 53 \\ \hline 72 \end{array}$$

$$72 \div 9 = 8$$

8 pages are needed

POSSIBLE STUDENT MISCONCEPTIONS:

- Students may try to divide 125 into equal groups of 9; if so prompt with "If she gave 53 stickers to her sister, how many does she have left for her book?"
- Students may struggle with division facts. Try the inverse first if they are familiar with their multiplication facts. If not, have them draw equal groups of 9, until they reach 72.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Use simpler numbers to help illustrate the concept (e.g, 20 stickers - 10 stickers = 10 stickers. If 2 stickers were placed on each page, how many pages would be needed?)
- Focus on vocabulary
 - "gives", in this problem, means subtract.
 - "each page", in this problem, means we have to find equal groups of 9.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Students who can easily complete this problem, but still struggle with 6.RP.A.1, may benefit from a problem from standard 4.OA.A.2.

Sandra raised \$15 for the PTA and Nita raised \$45. How many times as much money did Nita raise compared to Sandra?

LANGUAGE SUPPORTS:

- See "Focus on vocabulary" at left.
- Use yes-no questions (e.g, should we add or subtract? Now, should we multiply or divide?).
- Use role-playing to act out the math or draw a visual of the scenario.

TASK SOURCE:

<https://www.engageny.org/resource/grade-3-mathematics-module-3-topic-e-lesson-18/file/35031> (page 231)

MARIA'S RIBBONS

A. Maria cuts 12 feet of ribbon into 3 equal pieces so she can share it with her two sisters. How long is each piece?

B. Maria has 12 feet of ribbon and wants to wrap some gifts. Each gift needs 3 feet of ribbon. How many gifts can she wrap using the ribbon?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE MARIA'S RIBBONS

A. Maria cuts 12 feet of ribbon into 3 equal pieces so she can share it with her two sisters. How long is each piece?

B. Maria has 12 feet of ribbon and wants to wrap some gifts. Each gift needs 3 feet of ribbon. How many gifts can she wrap using the ribbon?

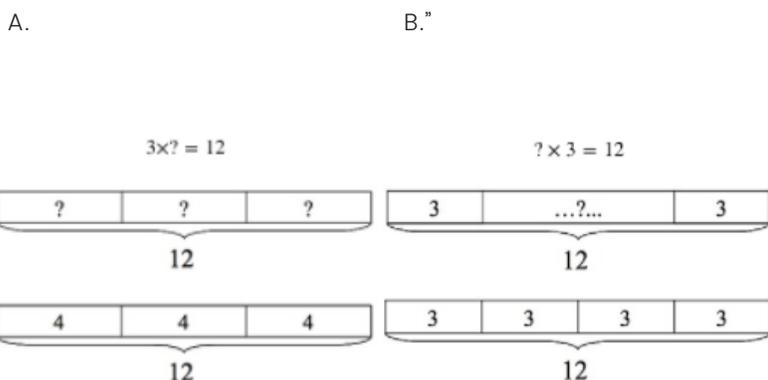
TARGETED STANDARDS:
3.OA.A.3
3.OA.B.6
4.OA.A.2

SOLUTION:
A. $12 \div 3 = 4$, so each child gets a piece of ribbon that is 4 feet long.
B. $12 \div 3 = 4$, so Maria can wrap 4 gifts.

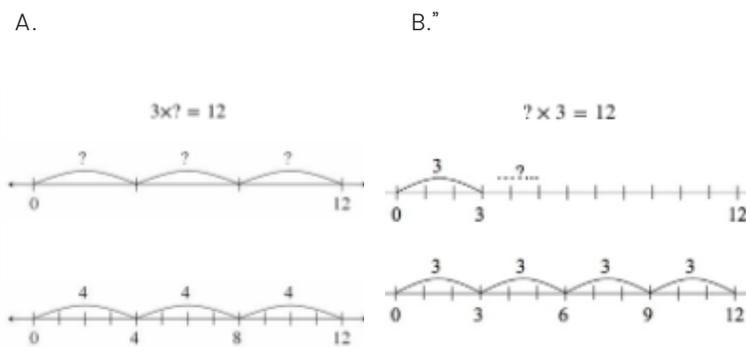
CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

This problem is standard 3.OA.A.3 (maps to 3.OA. A.1 and 3.OA.A.2 on Coherence Map) and is connected to 6.RP.A. Students' previous work with multiplication and division of equal groups connects to their understanding of ratios and rates. In this particular problem, both tasks are solved by the division problem $12 \div 3$, but what happens to the ribbon in each case is different.

POSSIBLE SOLUTION:



POSSIBLE SOLUTION:



POSSIBLE STUDENT MISCONCEPTIONS:

A. Students may incorrectly focus on Maria's two sisters only ($12 \div 2 = 6$ feet each). If so say, "Let's check again to see how many people need a piece of ribbon."

B. Students may rush and think this problem is asking for the same information as (A). If so say, "In this problem we have to find out how many, before we were looking for how much."

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Encourage the use of a tape diagram or a number line.
- For (A) say, "Maria needs to find 3 equal pieces, so we need to determine how many are in each equal group."
- For (B) say, "Maria needs to find how much ribbon each gift gets, so we need to determine how many equal groups there are."

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Students who can easily complete these problems may find it more challenging with larger numbers or fractions or decimals.

Mark has 75 feet of twine. He and his gardening club each need the same amount to tie up their plants. There are 5 members in the gardening club, including Mark. How much twine does each member need?

LANGUAGE SUPPORTS:

- Use yes-no questions (e.g., should we multiply or divide?).
- Use manipulatives (e.g., try using 12 counters while role-playing to act out each situation).
- Use a tape diagram or a number line.

ADDITIONAL TEACHER SUPPORTS:

In this case, it is particularly helpful to require students to justify their answers with a diagram. The way in which a student represents the problem can reveal whether or not he or she really understands the distinction between the two types of division problems shown here.

TASK SOURCE:
<https://tasks.illustrativemathematics.org/content-standards/3/OA/A/3/tasks/344>

BOXES OF CUPS & BAGS OF PEARS

Eighteen cups are equally packed into 6 boxes. Two boxes of cups break. How many cups are unbroken? Twenty-seven pears are packed in bags of 3. Five bags of pears are sold. How many bags of pears are left?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE

BOXES OF CUPS & BAGS OF PEARS

A. Eighteen cups are equally packed into 6 boxes. Two boxes of cups break. How many cups are unbroken?

B. Twenty-seven pears are packed in bags of 3. Five bags of pears are sold. How many bags of pears are left?

TARGETED STANDARDS:
3.OA.A.3
4.OA.A.2

SOLUTION:
A. 12 cups are not broken.
B. 4 bags of pears are left.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

This problem is standard 3.OA.A.3 (maps to 3.OA. A.1 and 3.OA.A.2 on Coherence Map) and is connected to 6.RP.A. Students' previous work with multiplication and division of equal groups connects to their understanding of ratios and rates. In these particular problems, two steps are required, but only one question is asked. While they are similar, the first problem requires division then multiplication, while the second requires division then subtraction.

POSSIBLE SOLUTION:

3. Eighteen cups are equally packed into 6 boxes. Two boxes of cups break. How many cups are unbroken?

18 cups

$18 \div 6 = 3$

$4 \times 3 = 12$

12 cups are unbroken.

5. Twenty-seven pears are packed in bags of 3. Five bags of pears are sold. How many bags of pears are left?

Sold left

27 pears

$27 \div 3 = 9$

9 bags of pears

$9 - 5 = 4$

There are 4 bags of pears left.

POSSIBLE SOLUTION:

$18 \div 6 = 3$ cups in each box

$2 \times 3 = 6$ cups break

$18 - 6 = 12$ cups not broken

9 bags sold 5 bags 4 bags left

POSSIBLE STUDENT MISCONCEPTIONS:

A. Students may focus on the middle portion of the question (2 boxes of cups break). If so, prompt them with "How many did not break?"

B. If students focus on 5 bags of three pears being sold, they may multiply 5 by 3 and get 15, then subtract 15 from 27 and get 12. This would be correct for the number of pears left, which is similar to the question structure above, but not correct because they would still need to divide by 3 to get 4 bags.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Ask students, "What is known? What is unknown?"
- Some students may need to take the extra step of identifying the portion of items that is gone (broken/sold) -see possible student solution 2.
- Encourage the use of tape diagrams and pictures.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Fifty-six jackets are shipped in boxes of 7. Three boxes are lost during shipping. Four boxes are sold. How many jackets are left?

LANGUAGE SUPPORT:

- Focus on vocabulary
 - In problem 1, "equally" means divide.
 - In problem 2, "bags of three" means we need to find out how many equal groups of 3 are in 27 (divide).

TASK SOURCE:

<https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-f-lesson-20/file/34596> (page 225)

MR. NGUYEN'S PLANTS & ANNA'S SEEDS

A. Mr. Nguyen plants 24 trees around the neighborhood pond. He plants equal numbers of maple, pine, spruce, and birch trees. He waters the spruce and birch trees before it gets dark. How many trees does Mr. Nguyen still need to water? Draw and label a tape diagram.

B. Anna buys 24 seeds and plants 3 in each pot. She has 5 pots. How many more pots does Anna need to plant all of her seeds?

TEACHER'S GUIDE

MR. NGUYEN'S PLANTS & ANNA'S SEEDS

A. Mr. Nguyen plants 24 trees around the neighborhood pond. He plants equal numbers of maple, pine, spruce, and birch trees. He waters the spruce and birch trees before it gets dark. How many trees does Mr. Nguyen still need to water? Draw and label a tape diagram.

B. Anna buys 24 seeds and plants 3 in each pot. She has 5 pots. How many more pots does Anna need to plant all of her seeds?

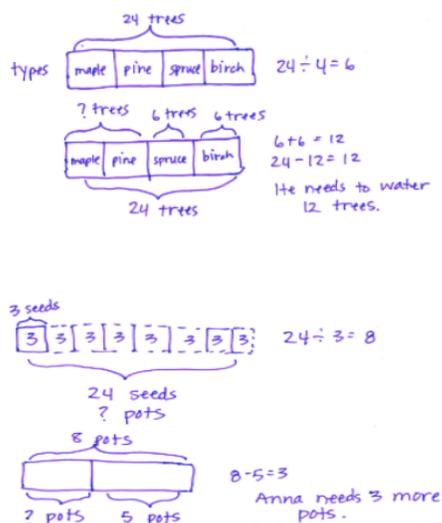
TARGETED STANDARDS:
3.OA.A.3
4.OA.A.2

SOLUTION:
A. 12 trees still need to be watered.
B. Anna needs 3 more pots.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

This problem is standard 3.OA.A.3 (maps to 3.OA. A.1 and 3.OA.A.2 on Coherence Map) and is connected to 6.RP.A. Students' previous work with multiplication and division of equal groups connects to their understanding of ratios and rates. In these particular problems, students need to first divide, then add/subtract, as well as determine what is unknown.

POSSIBLE SOLUTION:



POSSIBLE SOLUTION:

$24 \div 4 = 6$
Maple: 6, Pine: 6, Spruce: 6, Birch: 6
 $6 \text{ Spruce} + 6 \text{ Birch} = 12$ watered already
 $6 \text{ Maple} + 6 \text{ Pine} = 12$ need to be watered

24 seeds
3 seeds in each pot
5 pots $3 \times 5 = 15$ seeds planted
 $24 - 15 = 9$ seeds left
 $9 \div 3 = 3$ more pots needed

POSSIBLE STUDENT

MISCONCEPTIONS:
A. Students who struggle with reading comprehension may be challenged by only seeing one number in the problem (24). Oftentimes their strategies include locating the number in the word problem and then looking at the key words surrounding it.
B. Students may try to divide 24, total number of seeds, by 5, total number of pots, resulting in unequal groups.

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Require the use of a tape diagram.
- Ask, how many different types of trees are there? (4). OK, so how can we equally divide 24 trees into 4 groups?
- Discuss what is unknown.

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

Helen raised \$12 for the food bank last year and she raised 6 times as much money this year. How much money did she raise this year? If Ben raises twice as much as Helen raises this year, how much will Ben raise?

LANGUAGE SUPPORT:

- Read problem A to students, as it is lengthy.
- Focus on "he plants equal numbers of..."
- Require the use of a tape diagram or other pictures.

EUDORA'S RIBBONS

Eudora buys 21 meters of ribbon. She cuts the ribbon so that each piece measures 3 meters in length.

A. How many pieces of ribbon does she have?

B. If Eudora needs a total of 12 pieces of the shorter ribbon, how many more pieces of the shorter ribbon does she need?

SHOW YOUR WORK:

ANSWER:

TEACHER'S GUIDE EUDORA'S RIBBONS

Eudora buys 21 meters of ribbon. She cuts the ribbon so that each piece measures 3 meters in length. a. How many pieces of ribbon does she have? b. If Eudora needs a total of 12 pieces of the shorter ribbon, how many more pieces of the shorter ribbon does she need?

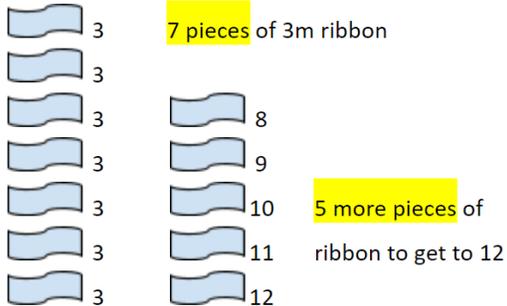
TARGETED STANDARDS:
3.OA.A.3
4.OA.A.2

SOLUTION:
5 more shorter pieces are needed to have 12 shorter pieces of ribbon.

CONNECTION TO GRADE 6 STANDARDS ON RATIO CONCEPTS AND RATIO REASONING:

This problem is standard 3.OA.A.3 (maps to 3.OA.A.1 and 3.OA.A.2 on Coherence Map) and is connected to 6.RP.A. Students' previous work with multiplication and division of equal groups connects to their understanding of ratios and rates. In these particular problem, students need to divide into equal groups and then subtract.

POSSIBLE SOLUTION:



POSSIBLE SOLUTION:



POSSIBLE STUDENT MISCONCEPTIONS:

- Students may choose the incorrect operation and think they are to multiply 21 by 3, which would give them 63.
- Students may think the second part of the problem is asking them for 12 pieces shorter than the 3 meter pieces. If so, prompt with, "the longer pieces are 21 and the shorter pieces are 3. If she needs 12 of the short pieces, how many more does she need if she already has 7?"

SUGGESTIONS FOR STUDENTS WHO STRUGGLE WITH THIS TASK:

- Encourage the use of a tape diagram.
- Ask, "How many threes are in 21?"
- Students who are less comfortable "dividing," may decide to skip count by 3 until they reach 21. Likewise, they may add on to 7 until they reach 12 as opposed to subtracting. If so, ask, "Is there a faster way? Can we use what we know...divide 21 by 3?"

SUGGESTIONS FOR STUDENTS WHO MAY BENEFIT FROM AN EXTENSION:

If the longer ribbons only came in 21 meters, after Eudora got her 5 shorter pieces, how much ribbon would be left over?

LANGUAGE SUPPORTS:

- Encourage the use of a tape diagram.
- Use yes-no questions (e.g., should we multiply or divide?).
- Focus on vocabulary
 - "Each piece," in this problem, means equal groups of 3.
 - "How many more," in this problem, means add on or subtract.