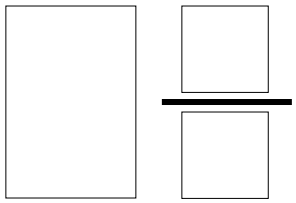
Grade 5: Forest Road

5.NF.B.6 - Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

A forest road is 4½ miles long. The road is paved along three-quarters of its length. How many miles of the road are paved?

Write the correct answer by writing a digit in each box.

 miles

Solution

Correct if student writes 3 ⅜.

Some students might draw a diagram to help them solve the problem.

The length of the paved part of the road is

¾ × 4 ½ miles = 3 ⅜ miles.

Students might calculate the product ¾ × 4 ½ in a number of ways:

One way is to write 4 ½ as a fraction, $^9/_2$, and then multiply the two fractions:

$$\frac{3}{4} \times \frac{9}{2} = \frac{27}{8}$$

Then write $^{27}/_8$ as a mixed number, 3 ⅜.

Another way to find ¾ of 4 ½ is to add three-fourths of 4 to three-fourths of ½:

¾ × 4 ½

= ¾ × (4 + ½)

= ¾ × 4 + ¾ × ½

= 3 + ⅜

= 3 ⅜ .

Still another way is to divide 4 ½ by 4, and then find 3 times as much as that:

4 ½ ÷ 4

= (4 + ½) ÷ 4

= 4 ÷ 4 + ½ ÷ 4

= 1 + ⅛

= 1 ⅛

3 × 1 ⅛

= 3 × (1 + ⅛)

= 3 × 1 + 3 × ⅛

= 3 + ⅜

= 3 ⅜.

A more complicated approach would be to divide 4 ½ by 4 and then subtract that result from 4 ½. In other words, instead of calculating ¾ × ½, we calculate

4 ½ − ¼ × 4 ½

= 4 ½ − 1 ⅛

= 3 + (½ − ⅛)

= 3 + (4/8 − ⅛)

= 3 + ⅜

= 3 ⅜.

Finally, it is possible to solve the problem by writing ¾ as 0.75 and 4 ½ as 4.5, and then multiplying $0.75 \times 4.5 = 3.375$. However, outside of this particular problem, a decimal approach won't be convenient for fraction products with denominators like 3 or 12; and a decimal approach also doesn't generalize to the kinds of products students will later see in algebra, such as $\frac{x}{4} \times \frac{1}{x}$.

If students answer the problem incorrectly, ask how they solved the problem. It's possible they wrote a wrong digit by mistake or had a hard time understanding the situation. If they couldn't make sense of the situation, ask them to draw a diagram; this might help them get started.

You could also try warming up with a related problem that uses easier numbers: "The forest road is 36 miles long. The road is paved along three-quarters of its length. How many miles of road are paved?" The answer to this problem is $\frac{3}{4} \times 36 = 27$ miles. (A quarter of 36 is 9, so three-quarters of 36 is three times as much, or 27.) The original problem is a lot like this simplified version, except the road in the original problem is 36 *eighths* of a mile instead of 36 whole miles. In the original problem, a quarter of the road's length is 9 eighths of a mile, and three quarters is three times as much, or 27 eighths of a mile.

Elaboration on Alignment

This is a word problem involving multiplication of two fractions. The thrust of the multiplication is to find a fractional part of a dimensioned quantity (the alternative would be finding the product of two dimensioned quantities).

The fractions have easy denominators that are common in everyday life (halves, fourths, eighths). The mode of entering the answer reveals that the answer is a mixed number, but knowing that the answer is a mixed number doesn't provide a significant assist in solving the problem, and the student is unlikely to enter the correct digits by chance.

A mixed number is used for the length of the road in order to agree with everyday language and also to implicate a potentially broader array of mathematics, including: the distributive property (in potentially a product sense or a quotient sense); converting between mixed number form and fractional form; and using number sense of whole number operations in tandem with number sense of fraction operations. The whole number in 4 ½ agrees with the denominator of the fraction ¾ so as to foster distributive thinking.

The problem is set up so that multiplication, if fluent, is a better strategy than the main alternative, which is the subtraction strategy 4 ½ − ¼ × 4 ½ noted above. Even if a student uses this strategy, they are still multiplying by a unit fraction.

The road is called a forest road so that it might seem natural for not all of it to be paved.

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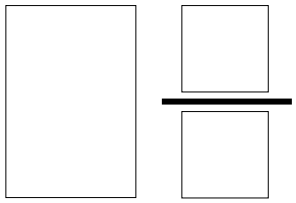 miles