Grade 6: Shoelace

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6.NS.A.1 - Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Two-thirds of a shoelace is ½ meter long.
How long is the whole shoelace?

Give your answer in units of meters.

Answer: [ ] m

Solution

Correct if student writes 3/4 or 0.75.

The answer is $^3/_4$ m. (The decimal equivalent, 0.75 m, is also correct.)

One approach to the problem is to use algebra by writing and solving an equation. We could choose the letter *L* to stand for the unknown length of the shoelace (measured in meters). We know that two-thirds of this length is half a meter. In symbols, this reads

$$^2/_3\, L = {^1/_2}$$

Divide both sides of this equation by $^2/_3$ to find:

$L = {}^1/_2 \div {}^2/_3$

$= {}^1/_2 \times {}^3/_2$

$= {}^3/_4$.

Conclusion: the shoelace is three-quarters of a meter long.

There are other ways to solve the equation ${}^2/_3\, L = {}^1/_2$. Instead of dividing both sides of the equation by ${}^2/_3$, we could have multiplied both sides of the equation by 3:

$3 \times {}^2/_3\, L = 3 \times {}^1/_2$

Because $3 \times {}^2/_3$ equals 2, this gives a new equation

$2L = {}^3/_2$.

Dividing both sides of this equation by 2 gives the value of L:

$2L \div 2 = {}^3/_2 \div 2$

$L = {}^3/_2 \div 2$

$L = {}^3/_4$.

Some students might solve the problem without using algebra. They might think: "Well, ${}^2/_3$ times something is ${}^1/_2$, so the something must be ${}^1/_2 \div {}^2/_3$." Then they can go about calculating the value of ${}^1/_2 \div {}^2/_3$. (This is a more mature version of the reasoning that students used all the way back in grade 3, when they solved problems like "7 times something is 42, so the something must be $42 \div 7$.")

Some students might solve the problem in steps: "Well, ${}^2/_3$ of the shoelace is ${}^1/_2$ meter, so ${}^1/_3$ of the shoelace is half that, or ${}^1/_4$ meter. The shoelace was three times as long as that, or ${}^3/_4$ meter."

Some students might think: "Well, I could turn ${}^2/_3$ into a whole by multiplying it by one-and-a-half, so the length of the shoelace must be one-and-a-half times ${}^1/_2$ m. That's ${}^1/_2$ m + ${}^1/_4$ m = ${}^3/_4$ m."

This idea corresponds to yet another tactic for solving the equation ${}^2/_3\, L = {}^1/_2$. Multiply both sides by ${}^3/_2$:

$${}^2/_3\, L = {}^1/_2$$

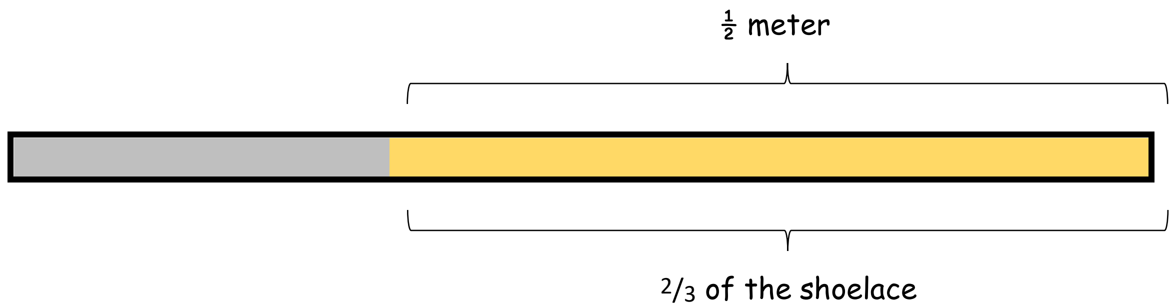
$$\Rightarrow \quad {}^3/_2 \times {}^2/_3\, L = {}^3/_2 \times {}^1/_2$$

Since ${}^3/_2 \times {}^2/_3 = 1$, this says

$$L = {}^3/_2 \times {}^1/_2$$

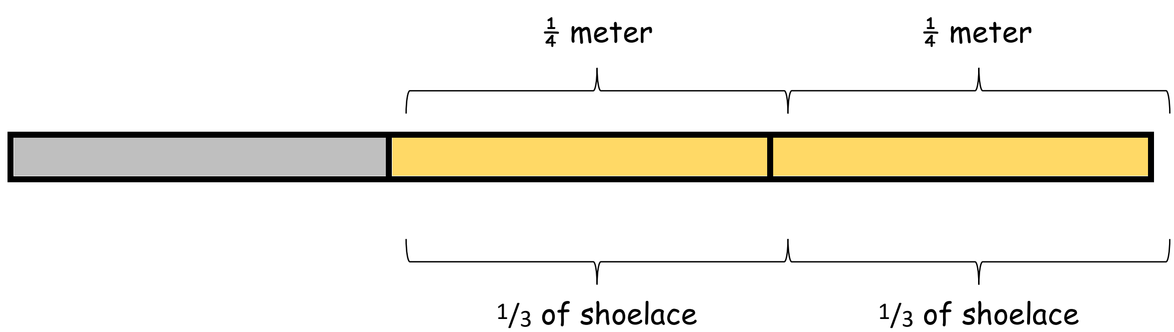
$$= {}^3/_4.$$

Some students might support their reasoning by drawing a diagram. If the student is having a hard time getting started on the problem, encourage them to draw a diagram of the problem. A diagram like this sheds light on the situation:



This diagram could make it easier to see why $2/3$ of the unknown quantity is $1/2$ meter (just compare the two different labels on the yellow section). The diagram also makes it easier to see that the whole length is equal to one-and-a-half times the yellow section, that is, $3/2 \times 1/2$ meter.

On the diagram, what if we split the yellow section in half? Then the diagram looks like this:



This picture could make it easier to see that "$1/3$ of the shoelace is $1/4$ meter"—and that the whole shoelace is $3 \times (1/4$ meter).

Finally, some students might remove the fraction $1/2$ from the problem by replacing $1/2$ meter with 50 centimeters. The problem can then be solved in ways similar to those discussed above, for example by writing and solving the equation

$$2/3\ S = 50$$

where S is the length of the shoelace measured in centimeters. The solution to this equation is $S = 75$; but in order to answer the question correctly, 75 cm must be converted to either $3/4$ m or 0.75 m.

When using the centimeters approach, the non-algebra methods discussed above also still work—for example, reasoning that since $2/3$ of the shoelace is 50 cm, then $1/3$ of the shoelace is half as much, or 25 cm, and the whole shoelace is three times as much as that, or 75 cm.

Elaboration on Alignment

This is intended to be an easy word problem whose answer is a quotient of fractions. The fractions in the problem are intended to make it easy to carry out the arithmetic steps using grade-level procedural skills, and reasonably easy to solve the problem opportunistically by applying prior-grade principles of multiplicative thinking.

The problem does require modeling a situation, but the situation and its description aim for minimum difficulty in this respect. There is a wrinkle for students to iron out, in that one number in the problem ($^1/_2$) has dimensions of length, whereas the other number in the problem ($^2/_3$) is dimension-less. The two numbers play different roles, and this has to be perceived on some level (or not misperceived).

The problem could be replaced, if desired, with a procedural task: "Solve for *x*: $^2/_3\, x = {}^1/_2$." That changes the complexion of the grade.; most of the material in the solution above could stay, but it would be much less concrete to the everyday person.

If the student works with the given measurement $^1/_2$ meter, then the solution involves finding a general quotient of fractions. If the student works with the equivalent measure 50 centimeters, then the solution involves finding the quotient of a whole number by a general fraction. Either operation exceeds grade 5 expectations and belongs to grade 6. Likewise, the algebraic approaches $^2/_3\, L = {}^1/_2$ and $^2/_3\, S = 50$ involve equations of a kind first studied in grade 6.

This is not well thought of as a proportional relationships problem. The context doesn't feature two *variable quantities*, just one *unknown value*. Also, as in previous grades, the division operation simply finds an unknown factor; it doesn't yield a third kind of quantity, as when we divide distance by time to create a new quantity speed, or divide mass by volume to create a new quantity density.

Even though no calculator is supposed to be used, some users might enter $^3/_4$ in the equivalent form 0.75. There is also a plausible solution route leading to 0.75 (if students replace $^1/_2$ m by 50 cm). Either way, because 0.75 is a mathematically correct answer, it can't be marked incorrect.

It is not unreasonable for a shoelace to be 75 cm long. Shoelaces are easily conceived of as having a length, and indeed length is the only measurable attribute of a shoelace that most people would think of. Because length is continuous, it is a good candidate for fraction arithmetic.

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Name: ____________________________

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Give your answer in units of meters.

Answer: [ ] m