Grade 6: Planting Corn

6.RP.A.3 - Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

> **6.RP.A.3b -** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

A farmer uses a tractor to plant corn quickly in the springtime.

The farmer plants 216 acres every 12 hours.

How can you find the number of acres planted in n hours?

Answer: Multiply n by this number: ☐

Solution

Correct if student writes 18.

The farmer plants corn at a rate of 18 acres per hour (216 ÷ 12 = 18).

To find the number of acres planted in n hours, multiply the number of hours by the rate:

$$\text{Number of acres planted in } n \text{ hours} = \text{rate per hour} \times \text{number of hours}$$

$$= 18 \times n$$

The answer is 18.

Here's another way to think of it. In 1 hour, which is $^1/_{12}$ of the given time, the farmer will plant $^1/_{12}$ of the given acreage, which is $^1/_{12} \times 216 = 18$ acres. But if the farmer plants 18 acres in 1 hour, then in n hours, the farmer plants n times as much. The expression for "n times as much as 18" is $n \times 18$, or $18 \times n$.

Another way to find the answer is to try some specific values for n and see what multiplier works.

- For example, try $n = 2$ hours. This is one-sixth of 12 hours, and in a sixth of the time, the farmer plants a sixth of the acreage, which is 36 acres. To get 36 from 2, multiply 2 by 18.
- What if $n = 6$ hours? In half the time, the farmer plants half the acreage, or 108 acres. To get 108 from 6, multiply 6 by 18.
- $n = 12$ hours: the farmer plants 216 acres. To get 216 from 12, multiply 12 by 18.
- $n = 1$ hour: in one-twelfth the time, the farmer plants one-twelfth the acreage, which is 18 acres. To get 18 from 1, multiply 1 by 18.
- $n = 16$ hours: if the time is one-and-a-third as much as 12, then the acreage is one-and-a-third as much as 216, which is 216 + 72 = 288 acres. To get 288 from 16, multiply 16 by 18.
- In every case, whatever the number of hours n, the number of acres is $18 \times n$.

Another way to find the number of acres planted in n hours is to "set up a proportion":

$$\frac{216}{12} = \frac{\text{number of acres planted in } n \text{ hours}}{n}$$

Many students are taught to solve proportions by cross-multiplying, an approach that certainly has its uses. But before cross-multiplying, let's first complete the division on the left-hand side:

$$18 = \frac{\text{number of acres planted in } n \text{ hours}}{n}$$

If we *now* cross-multiply, we obtain a useful formula:

$18 \times n$ = number of acres planted in n hours.

This again shows that the answer to the problem is 18.

Many students in grade 6 will analyze proportionality problems by making a table of specific values and using the values in the table to generalize the relationship. For example, a student might think, "Well, 216 acres in 12 hours is the same rate as 432 acres in 24 hours, or 108 acres in 6 hours, or 54 acres in 3 hours, or 18 acres in 1 hour." These values can be recorded in a table:

Number of hours	Number of acres
1	18
3	54
6	108
12	216
24	432
n	?

What is the relationship between the values in the left column and the values in the right column? Each value in the right column is a multiple of 18. In each case, the number that multiplies 18 is just the corresponding number from the left column:

$18 = 1 \times 18$
$54 = 3 \times 18$
$108 = 6 \times 18$

$216 = 12 \times 18$
$432 = 24 \times 18$

To continue this relationship for the last row of the table, we should fill in the "?" with the expression $n \times 18$:

$? = n \times 18.$

This is another way to obtain the answer 18.

Note that in middle school, products like $18 \times n$ are usually written without the multiplication sign as $18n$.

Elaboration on Alignment

This proportionality problem isn't one of the classic "three-number" variety, such as

"216 acres in 12 hours, how many hours for 54 acres?"
"216 acres in 12 hours, how many acres in 16 hours?"

Those problems are good, especially if you can ask half a dozen of them in a row concerning a single situation. But the problem of the farmer planting corn isn't about using proportionality to extract particular findings. The problem is about the functional thinking inherent in a proportional relationship. That's why it names the two variable quantities in the problem (so-many number of hours, and so-many number of acres planted in so-many number of hours). And that's why the answer to the problem is a formula, not a number.

(For logistical reasons, the input answer is numerical, but the number specifies the coefficient in an expression, not a quantity of acres or hours.)

The problem of the farmer planting corn actually is a three-number problem, in a sense; it's just that one of the "numbers" is a variable quantity. That is, instead of a proportion like

$$\frac{216}{12} = \frac{54}{n}$$

we have

$$18 = \frac{\text{however-many acres}}{n}$$

This just makes explicit what is probably going on in the brain anyhow when we write a proportion with three numbers. As the student is setting up a proportion like $216/21 = 54/n$, when they get to the 54, they are probably thinking "OK, now however-many acres goes here." So, in the present problem, we are just making *that idea* the thing that is to be written, rather than any specific number.

The numbers 216 and 12 were chosen because of the many divisibilities they offer. This not only makes the unit rate a whole number, but it also allows an informative table to be populated using only whole-number values.

Because the curriculum for proportional relationships generally misrepresents the material, few students will emerge from grade 6 understanding proportional relationships. To make the problem easier, a sentence-frame is provided: “Multiply *n* by this number.” This allows students to work on the problem even if they haven’t been taught what a proportional relationship is (namely, a relationship or situation in which two variable quantities vary together in such a way that one quantity is always a fixed multiple of the other quantity).

Many students in grade 6 try to use additive thinking for rate problems, especially at the start of the subject. For example, to find the number of acres planted in 3 hours, they don’t multiply 3×18, but instead add $18 + 18 + 18$. They typically generate a table of *consecutive n* values (1, 2, 3, 4, ...), which makes it impossible *not* to see an additive pattern in the numbers (“To get the next value, I add 18”). This can be a good way to generate data and comprehend the situation itself. However, proportional relationships are multiplicative relationships. That’s why the solution above uses multiplicative thinking only. Note that in cases where the independent variable is continuous, additive thinking will not get you fully into the situation. (Across the state, there are a thousand farmers; in a 16-hour day, how many acres will be planted across the state? If 6.73 kg of water evaporated in 9.18 hr, how many kg evaporated in 6 hr?)

This problem, and the RP domain itself, are “aiming at” $y = mx + b$, in which the unit rate is central and is *multiplied* by the independent variable. (The “offset” b in $y = mx + b$ is where additive thinking enters into a linear functional relationship.)

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Name: ______________________________

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Answer: Multiply n by this number: [ ]