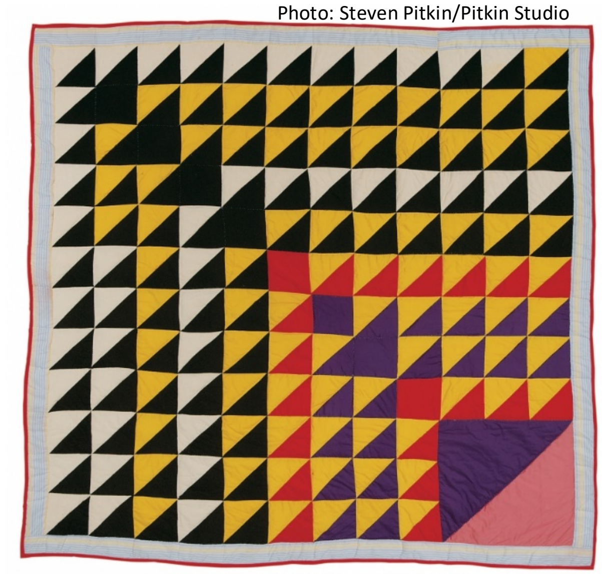
Grade 6: Quilt Triangles

6.GA.A.1 - Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

This is a quilt by Lucy T. Pettway (1921–2004), an American artist.

In making the quilt, Pettway used many small, yellow triangles. There is also a large, pink triangle in the lower-right corner.

Photo: Steven Pitkin/Pitkin Studio



Suppose that Pettway had decided to remove the large, pink triangle and replace it with a large, yellow triangle made of the small, yellow triangles. How many more of the small, yellow triangles would Pettway have needed in order to do that?

Answer: ☐

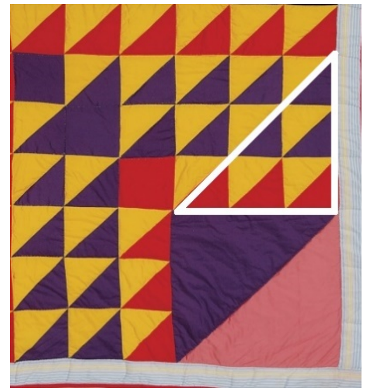
Solution

Correct if student writes the number 9 in the box.

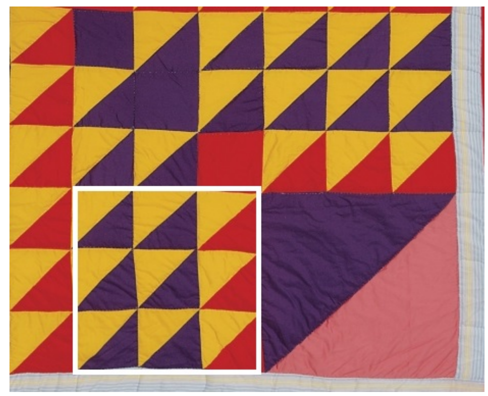
About 9 of the small, yellow triangles would be needed to make a large, yellow triangle to replace the large, pink triangle. There are a number of different ways to see this. Ask the student to justify the answer they gave, so that you can see how they thought about the problem.

Here are some possible approaches. Some of them involve more mathematical technique, and others less. Some of them involve more mathematical reasoning, and others less.

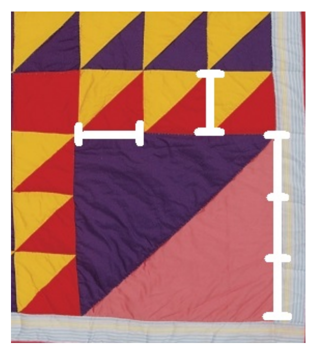
- *Tile a large triangle with small triangles.* Here is a close-up of the bottom-right corner of the quilt. A group of small triangles has been outlined in white. Each of these small triangles is about the same size as a small yellow triangle, and together the small triangles make a large triangle that is about the same size as the pink triangle. Now if you count the small triangles that are outlined in white, you will find that there are 9 of them. Therefore 9 of the small, yellow triangles would be needed to replace the large, pink triangle.



- *Tile a large square with small triangles.* Here is another close-up of the bottom-right corner of the quilt. Another group of small triangles has been outlined in white. Each of these small triangles is about the same size as a small yellow triangle, and together the small triangles make a square that is about double the size of the large pink triangle. Now if you count the small triangles that are outlined in white, you will find that there are 18 of them. Therefore about half that many, or 9, of the small, yellow triangles would be needed to replace the large, pink triangle.



- *Use the formula for the area of a right triangle.* If we think of one side of a small triangle as a unit of length, then the small, yellow triangle has an area of about ½ (base)(height) = ½ (1) (1) = ½ square unit. Meanwhile, the large, pink triangle has an area of about ½ (3)(3) = 4 ½ square units. The large, pink triangle has about 9 times as much area as the small, yellow triangle, because 4 ½ ÷ ½ = 9. And if you arrange the small, yellow triangles in the right way, they will cover the large, pink triangle pretty well without gaps or overlaps. So 9 of the small, yellow triangles would be needed to replace the large, pink triangle.



- *"The ratio of the triangles is the same as the ratio of the corresponding squares."* The small, yellow triangle has about half the area of a small square. Likewise, the large, pink triangle has about half the area of a large square. The ratio of half the small square to half the large square is the same as the ratio of the small square to the large square; so the ratio of the pink triangle's area to the yellow triangle's area is the same as the ratio of the large square's area to the small square's area. That ratio is 9. (Think of the large square as a 3 × 3 grid of small squares.) Therefore, the large, pink triangle has about 9 times as much area as the small, yellow triangle. And if you arrange the small, yellow triangles in the right way, they will cover the large, pink triangle pretty well without gaps or overlaps. So 9 of the small, yellow triangles would be needed to replace the large, pink triangle.

Elaboration on Alignment

We'll get to the content of the task, but first let's recognize that this is a modeling problem. That's because:

- It is up to the student to choose appropriate mathematics with which to analyze the problem. The problem doesn't mention area, right triangles, ratios, tiling, or any other formal notions or techniques from the curriculum.
- The student will intentionally and productively apply mathematics to a situation in which, strictly speaking, the mathematics actually doesn't apply. The shapes being referred to colloquially here as "triangles" aren't actually triangles in the mathematical sense. Nor are their areas all the same. Nor is the area of the large, pink triangle likely to be a whole-number multiple of the average area of the small, yellow triangles.
- The stated question isn't perfectly well posed as a mathematics problem. For example, it doesn't say, "How many of the small, yellow triangles fit into the large, pink triangle without gaps or overlaps?" It doesn't say, "What is the ratio of the area of the small yellow triangle to the area of the large pink triangle?" It doesn't even say "What is the ratio...rounded to the nearest whole number."
- Instead, the task is to assist the artist with a design choice. The artist is considering an alternative. She probably will wonder how many more small, yellow triangles would be needed. In modeling, the first-order goal is to shed light on the actual situation; the artist will consider 9 to be a good answer, and therefore so do we.

Of course, much of the modeling cycle is omitted in this problem, and that's inevitable given the constraints of the use-case at hand.

As often happens with legitimate modeling tasks, the content of the problem is somewhat loosely defined. However, the content is well thought of as aligned at grade 6. One reason is that grade 6 is when students first calculate areas of right triangles. When students learn how to do this, the teacher will show them where the ½ in the formula comes from (visualize a right triangle as half of a rectangle); that embedding of a triangle in a square is likely active for the student in this problem as well. While the primitive approaches to the problem (finding a tiled large rectangle, or finding a tiled large square) don't show good evidence of meeting a standard about calculating the area of a right triangle, they do provide some evidence that the concept of area itself—heretofore defined only for rectangles—has been extended appropriately to triangles the progression demands in grade 6. Finally, there are also opportunities for ratio thinking in the problem, which is a lively topic that first appears in grade 6.

Learn More

See a digital version of this task at Learning Heroes' Readiness Check:
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Name: ______________________________

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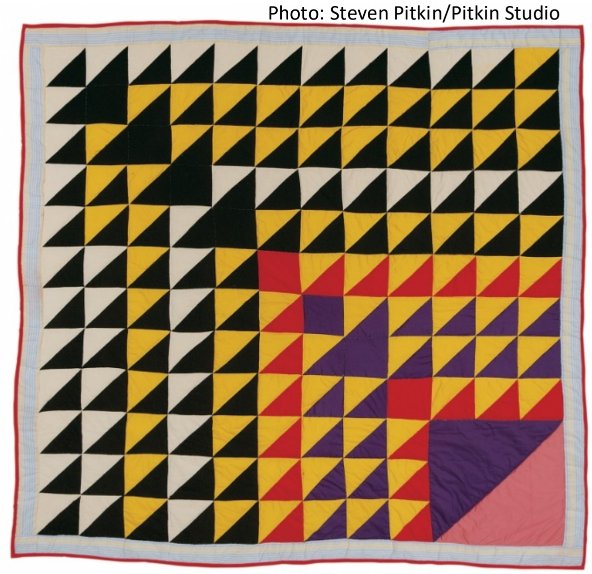

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