Grade 7: Snow Vehicle

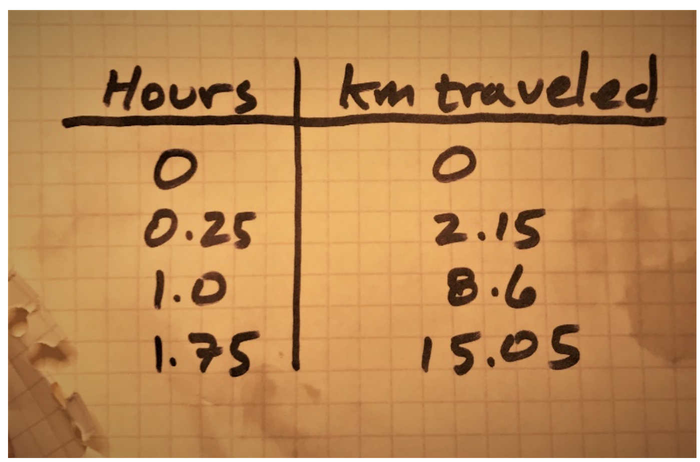
7.RP.A.2 - Recognize and represent proportional relationships between quantities.

7.RP.A.2b -Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

A team of scientists drove a snow vehicle from their camp to the South Pole. Along the way, one scientist wrote down how far the vehicle had traveled at different times.

Here are the distances and times the scientist wrote.

If the speed of the vehicle was constant, then what number from the table completes the following sentence correctly? Choose a number from the table.



Hours	km traveled
0	0
0.25	2.15
1.0	8.6
1.75	15.05

The speed of the vehicle was ☐ km/hr.

Solution

Correct if student writes 8.6, 8.60, 8.600, 08.6, 008.6, or 08.60.

The speed of the vehicle was 8.6 km/hr. This can be seen from the third row of the table, which shows that the vehicle traveled 8.6 km in 1 hour. No other number in the table equals the speed of the vehicle in units of km/hr.

Potential strategies

- Finding the row that shows 1 in the Hours column is the easiest way to determine the speed.
- Some alternative approaches could include:
  - The second row of the table says that the vehicle travels 2.15 miles in a quarter-hour; so, in a whole hour, it travels four times as far, or $4 \times 2.15 = 8.6$ km. That reproduces our earlier answer of 8.6 km/hr.
  - Using the fourth row, divide 15.05 km by 1.75 hours, and the result once again is 8.6 km/hr.
  - Note: if a student calculated the speed by dividing or multiplying, point out that the value can be found without a calculation, thanks to the row of the table that shows 1 in the Hours column.

Extension

- A follow-up question you could ask might be, "If the distance from camp to the South Pole was 21.5 km, and they drove at a speed of 8.6 km/hr the whole way, then how many hours did the trip last?"
  - Answer: Applying the idea that "distance = rate × time," we have $21.5 = 8.6 \times T$; solve for $T = 21.5/8.6 = 2.5$ hours. One could check this answer by taking advantage of the second row of the table. This row says that in 0.25 hours we travel 2.15 km; therefore, in ten times as many hours, we'll cover ten times as much distance. That is, in 2.5 hours, we travel 21.5 km.

Elaboration on Alignment

The problem doesn't ask "What is the speed of the vehicle?" Instead, it asks for which number from the table is the speed of the vehicle. The point, in this case, is <u>not</u> to calculate. As a result, the problem is hard to get wrong (there are only so many numbers from the table you could choose, and most of them are silly). But at the same time, the problem is less rote and more conceptual than "What is the speed?"

In a proportional relationship, there are two quantities (call them X and Y) that can each vary in value, but that vary in such a way that the ratio Y/X does <u>not</u> vary. In this problem, X is the quantity of time and Y is the quantity of distance. There are several different values of X (0, 0.25, 1.0, 1.75) and several different values of Y (0, 2.15, 8.6, 15.05), so the quantities both vary. But the quantities *vary together* ("co-vary") in such a way that the ratio of the values Y/X doesn't change. The value of this constant ratio is 8.6.

Said another way, Y is always 8.6 times as much as X. This is the "law" of X and Y, in the given situation. Another way to say it is that the "unit rate," or "constant of proportionality," between Y and X, is 8.6.

What does the "law" of these two quantities say in the special case where X is 1?

	Y (for any value of X) is 8.6 times as much as X.
Therefore,	Y (for $X = 1$) is 8.6 times as much as 1.
In other words,	Y (for $X = 1$) is 8.6.
Or more generally,	Y (for $X = 1$) tells the constant of proportionality.

So to find the unit rate / constant of proportionality, all you have to do is look at the Y value in the row of the table for $X = 1$. (On a graph in the XY coordinate plane, look at the Y-coordinate of the point with X-coordinate equal to 1.) That's the point of this problem. Compare to the grade 7 expectation of "Explain[ing] what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate." In the snow vehicle problem, we're doing something like that, but using numerical data in context rather than using a graph in the abstract setting of the coordinate plane.

By the way, if we know the unit rate, that's helpful because we can use it to solve problems. For example, since the snow vehicle travels 8.6 km in 1 hour, in 2.7 hours it travels 2.7 times as far, or $2.7 \times 8.6 = 23.22$ km. But in this problem, we aren't asked to extrapolate or interpolate the values in the table—all we're asked to do is identify the value of the unit rate itself, by finding the number of kilometers when the time is 1 hour. It seems simple, and the way the problem is written it should be hard to get it wrong, but there is a lot going on here.

In grade 7, it is expected that students will work with unit rates associated with ratios of fractions (here expressed in decimal notation).

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Name: ______________________________

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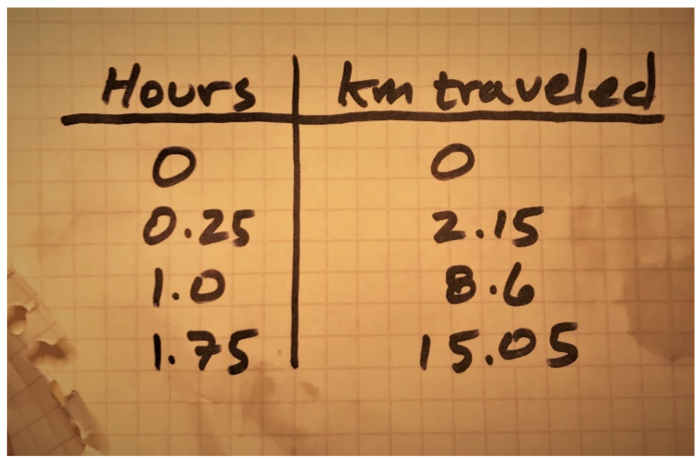

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