Grade 7: Piggy Bank

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

> **7.EE.B.4a** Solve word problems leading to equations of the form $px + q = r$ and $p(x+ q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Right now I have $6.75 in my piggy bank. If I add $2.25 per week to the bank, and I never take out any money, then how many weeks from now will I have $45.00 in my piggy bank?

Answer: $\boxed{}$ weeks

Solution

Correct if student writes 17.

If I add $2.25 per week to the bank, and I never take out any money, then I will have $45.00 in my piggy bank 17 weeks from now.

Some students will find this answer by using algebra. That approach could look like:

Let W be the unknown number of weeks. After W weeks, I will have $45.00 in my piggy bank.

The number 45.00 is one way to express the amount of money in the piggy bank after W weeks. There is another way to express the amount of money in the piggy bank after W weeks. Consider that in a single week, I add $2.25 to my piggy bank. That means the amount of money added at the end of W weeks will be W times as much as that, or $2.25W$. Added to the initial 6.75, the amount of money in the piggy bank after W weeks will be $2.25W + 6.75$. This is a formula, one that tells how much money I will have based on how long I wait.

On the one hand, the amount in the piggy bank after W weeks is 45.00, and on the other hand, the amount in the piggy bank after W weeks is $2.25W + 6.75$. Because $2.25W + 6.75$ and 45.00 are two ways of naming the same quantity, we may write an equation that sets them equal:

$$2.25W + 6.75 = 45.00$$

This equation can be seen as asking a question: for what value of W does our formula $2.25W + 6.75$ give the desired value 45?

To find the answer, one can first subtract 6.75 from both sides of the equation, then divide both sides of the equation by 2.25. This gives

$$W = (45.00 - 6.75) \div 2.25$$

$$= 38.25 \div 2.25$$

$$= 17.$$

The quotient $38.25 \div 2.25$ can be found with a calculator, or by long division. One could also multiply numerator and denominator by 4 to rewrite the quotient as $153 \div 9$, and then calculate that quotient.

One could even multiply both sides of the original equation by 4, producing a new equation

$$9W + 27 = 180.$$

Solve this equation as

$$W = (180 - 27) \div 9$$

$$= 180 \div 9 - 27 \div 9$$

$$= 20 - 3$$

$$= 17.$$

This problem can also be solved without algebra by thinking about the situation numerically. That approach could look like:

I want to have \$45.00, but right now I only have \$6.75, so I need more money, specifically \$45.00 – \$6.75 more, which is \$38.25. How long will it take me to build up \$38.25? At a rate of \$2.25 per week, it will take $38.25 \div 2.25 = 17$ weeks.

One could also note that after a single week, the piggy bank will hold \$9.00, so \$36 is still needed. At a rate of \$2.25 per week that will take another $36 \div 2.25 = 36 \div 2\frac{1}{4} = 36 \div {}^{9}/_{4} = 36 \times {}^{4}/_{9} = 16$ weeks, for a total of 17 weeks.

The algebraic approach and the arithmetic approach are consistent; the same operations of subtraction and division were eventually carried out in both approaches. Students who find the algebraic solution more comfortable could be encouraged to think through the arithmetic solution; students who find the arithmetic solution more comfortable could be encouraged to practice the algebraic solution. The algebraic method generalizes to problems that are too difficult to be solved by numerical thinking—students will see plenty of problems like that in later grades.

Elaboration on Alignment

Decimal quantities are used because they are appropriate to the context and because they make the problem more directly about multiplicative thinking and the idea of a rate. Even if a student advances the clock one week to replace $6.75 with $9.00, the remainder of the analysis is fundamentally multiplicative thinking; it would be tedious to finish the problem by writing out a table, and it would be difficult to get the final answer right by skip-counting 2.25's. The arithmetical steps in the problem should probably be carried out with a calculator, for the sake of accuracy, but the numbers in the problem are such that pencil-and-paper work or mental approaches are also possible.

The final answer is a whole number. Fractions of a week would only make sense if the money were being deposited continuously, like water from a tap, or at any rate only if the money were being deposited with greater than weekly frequency.

It is natural to interpret the given rate of $2.25/wk as reflecting weekly deposits, as in the situation of a weekly allowance. But the phrasing of the problem makes it immaterial just when these deposits will be made (tomorrow? A week from now? Did I just make a deposit, or am I just about to?). It doesn't really matter, if "I add $2.25 per week" to a known starting amount.

Problems with structure $ax + b = c$ are often of two kinds:

(1) Initial amount + (rate × interval) = final amount;
(2) Number satisfying a logical constraint (e.g., "Some nickels and three dimes make a total of $1.70; how many nickels are there?" or "A piece of wire 13.7 cm long is formed into a circle with 2.2 cm left over. To the nearest hundredth, what is the diameter of the circle?)

The piggy bank problem is of type (1), rather than type (2), because together with the problems in other grades, a problem of type (1) makes for a straighter through-line in terms of the content progression from elementary grades up through middle school to grade 8.

The solution commentary provides both perspectives. On the one hand, W is interpreted as an unknown number, in the style of (2), and in the style of (2) we produce our equation by naming a single quantity in two different ways. However, along the lines of (1), we also characterize $2.5W + 6.75$ as a formula for a time-dependent quantity (here W is interpreted not as an unknown number but a variable quantity), and the equation is given the functional interpretation $f(W) = 45$.

Learn More

See a digital version of this task at Learning Heroes' Readiness Check:
https://bealearninghero.org/readiness-check

Name: ______________________________

Right now I have $6.75 in my piggy bank. If I add $2.25 per week to the bank, and I never take out any money, then how many weeks from now will I have $45.00 in my piggy bank?

Answer: $\boxed{}$ weeks