Grade 8: Movie Flop

8.EE.C.7b: Solve linear equations in one variable. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.

> **7.EE.B.4a**. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
>
> **7.EE.B.4b**. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

The movie was so boring, Fernanda walked out of the theater after seeing only a quarter of it. Fifteen minutes later, Beatrice walked out after seeing a third of it. How long was the movie?

Answer: ☐ minutes

Solution

Correct if student writes 180.

Some students might write an equation to help them solve the problem. To see how this approach might go, say that the unknown quantity m is the length of the movie, in minutes. Then Fernanda walked out of the theater after $\frac{1}{4} m$ minutes. Beatrice walked out of the theater after $\frac{1}{3} m$ minutes. That was fifteen minutes later, or in other words, the difference between the two times was fifteen minutes:

$$\frac{1}{3} m - \frac{1}{4} m = 15.$$

This equation gives the answer to the problem. One can check that the equation is true when the value 180 is substituted for m.

The equation can be solved in several ways, for example as follows:

Factor the left-hand side:

$$\left(\frac{1}{3} - \frac{1}{4}\right)m = 15.$$

Simplify:

$$1/12\ m = 15.$$

Divide both sides by $1/12$ to find

$$m = 15 \div 1/12$$

$$m = 180.$$

The movie was 180 minutes long.

Another way to solve the original equation would be to multiply both sides of the equation by 12. This produces a new equation,

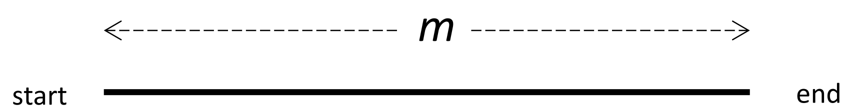
$$4m - 3m = 180$$

which simplifies to $m = 180$.

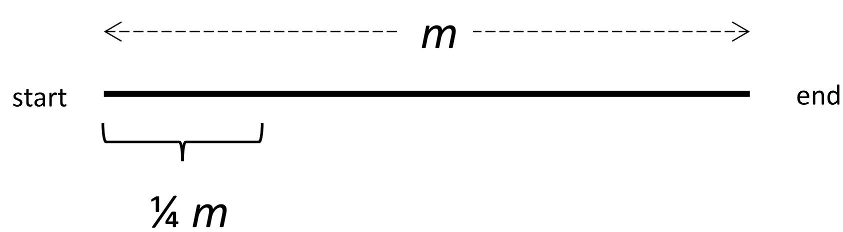
Some students might solve the problem *without* using a variable, by thinking about the numbers in a way that mirrors the algebra approach. Using a pencil and paper to do the scratch work, they could think: "Well, the difference between a third of the movie and a quarter of the movie was 15 minutes...but the difference between a third and a quarter is a twelfth, so in other words, a twelfth of the movie was 15 minutes long. And if a twelfth of the movie was 15 minutes long, then the entire movie was 15 × 12 minutes long. That's 180 minutes."

If a student struggles to solve this problem, try analyzing the problem in a freer way. Start by making a guess that the movie is 120 minutes long. In that case, ask the student when Fernanda walked out (after how long). Answer: After 30 minutes, because 120 ÷ 4 = 30. Then ask when Beatrice walked out. Answer: After 40 minutes, because 120 ÷ 3 = 40. So, it seems Beatrice walked out 10 minutes after Fernanda did (because 40 – 30 = 10). That isn't right, because the problem tells us that Beatrice walked out 15 minutes after Fernanda did. What this means is that our initial guess of 120 minutes was too short, because the difference between a third and a fourth of 120 minutes wasn't big enough. From here one could try a larger guess or translate the growing intuitions into an equation $\left(\frac{1}{3} - \frac{1}{4}\right) m = 15$.

Another idea to help a student work on the problem could be to ask them to draw a segment, imagining its length to stand for the unknown duration of the movie:

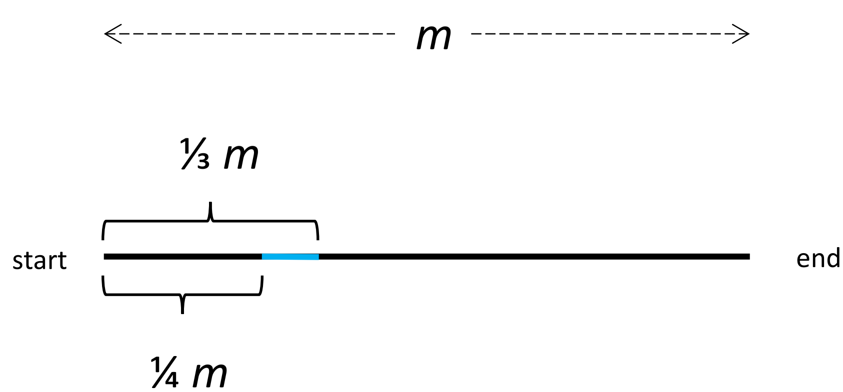


Fernanda leaves a quarter of the way into the movie. Here is how we can show this fact on the diagram:



The width of the bracket shows the length of time Fernanda watched the movie, a quarter of the total.

Beatrice leaves a third of the way into the movie. Here is how we can show this fact on the diagram:



The width of the top bracket shows the length of time Beatrice watched the movie, a third of the total. The top bracket is wider than the bottom bracket, because $\frac{1}{3}$ is greater than $\frac{1}{4}$. The difference is shown as a blue section. One way to express this difference is by subtracting the two widths shown. The result of doing this subtraction is $\frac{1}{3} m - \frac{1}{4} m$.

Meanwhile, reading the words in the problem statement, another way to express the difference is as a numerical value, 15 minutes. And so, because $\frac{1}{3} m - \frac{1}{4} m$ and 15 are two ways of naming the same quantity, we may write an equation that sets them equal:

$$\frac{1}{3} m - \frac{1}{4} m = 15.$$

This is the equation that was solved above.

Finally, some students might use algebra with a different unknown quantity. For example, let x be the unknown amount of time, in minutes, that Fernanda stays in the theater. Then according to the problem statement, Beatrice stays in the theater for $x + 15$ minutes. Now since Fernanda stays in the theater for a quarter of the movie, it must be that $4x$ is the number of minutes in the entire movie. And since Beatrice stays in the theater for a third of the movie, it must be that $3(x + 15)$ is the number of minutes in the entire movie. And so, because $4x$ and $3(x + 15)$ are two ways of naming the same quantity, we may write an equation that sets them equal:

$$4x = 3(x + 15).$$

This equation can be solved to find the value $x = 45$. That is, Fernanda stayed in the theater for 45 minutes. Since this was a quarter of the movie, the full length of the movie was $4 \times 45 = 180$ minutes.

Elaboration on Alignment

Most students completing an eighth-grade course of study will likely attempt to use algebraic methods for this problem. The problem is set up to make it likely for the student to engage in some amount of symbol manipulation involving the distributive property or other properties.

The numbers in the problem are all easy to work with, whether mentally or on scratch paper, although at the expense of a whiff of unrealism (popular movies aren't typically 3 hours long; the issue is mitigated by identifying boredom as the reason for the walkouts). The use of a continuous variable is what allows for fractions to enter the problem; both of these circumstances raise the difficulty. On the other hand, the fact that the numbers work out so cleanly means that the variable can still be thought of as a discrete quantity (a countable number of minutes).

The presence of fractions assesses comfort with rational numbers in algebra and also brings properties to the foreground. For example, if the equation were based on whole numbers, as in

$$37m - 32m = 15,$$

it would be very natural for students to think "37 of something minus 32 of something is 5 of that thing," so they would write

$$5m = 15$$

without realizing that what they were really doing was applying the distributive property:

$$37m - 32m = (37 - 32)m = 5m.$$

When the coefficients are fractions, it is harder to think, "⅓ of something minus ¼ of something is $1/12$ of that thing," so we probably rely on the properties more explicitly:

$$\tfrac{1}{3}m - \tfrac{1}{4}m = (\tfrac{1}{3} - \tfrac{1}{4})m.$$

Similarly, faced with

$$5m = 15$$

it is routine for students to divide both sides by 5. It seems to take more courage to recognize that

$$\frac{1}{12} m = 15$$

has exactly the same form as $5m = 15$, and therefore exactly the same method, which is to divide both sides by the coefficient:

$$m = 15 \div \frac{1}{12}.$$

But this is what it looks like when students "understand fractions as numbers" (cf. grade 3).

The alternative approach described in the solution, of guessing 120 minutes and seeing what happens, is a classic method called "the method of false position." Here is Roger Howe using it in his piece "From Arithmetic to Algebra":

> **14: Cows and chickens**
>
> In a farmyard, there are cows and chickens.
> There are 50 heads and 120 feet.
> How many cows are there? How many chickens?
>
> **Arithmetic Solution:**
>
> Chickens have two feet, and cows have four feet. Each has one head.
> If there were only chickens, there would be $50 \times 2 = 100$ feet.
> But there are 120 feet.
> The extra 20 feet must come from cows.
> Each cow has 2 more feet than a chicken, so there are $\frac{20}{2} = 10$ cows, and 50 - 10 = 40 chickens.

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Name: ______________________________

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Answer: [ ] minutes