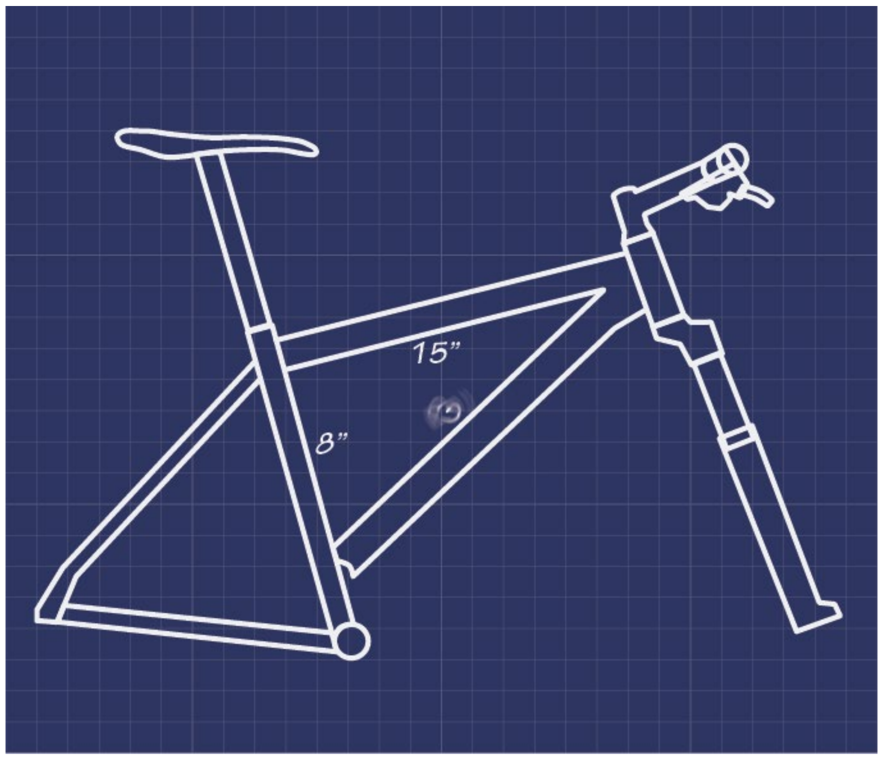
Grade 8: Blueprint

8.G.B.7 - Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

A company is using this blueprint to build bicycles.

One part of the bicycle is shaped like a right triangle. The two legs of the right triangle are 15 inches long and 8 inches long. The length of the hypotenuse is missing, because a drop of water fell on the blueprint and smeared it.

How long is the hypotenuse? The answer is a whole number.



Answer: ☐ inches

Solution

Correct if student writes 17.

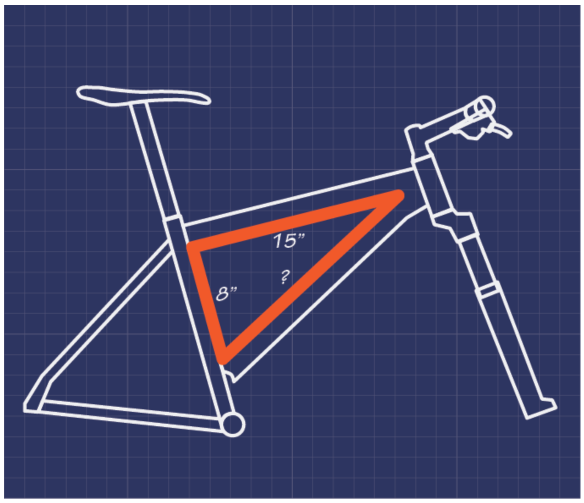
If the student doesn't know how to begin, then you might nudge them by asking if they remember the Pythagorean Theorem.

For a right triangle, the Pythagorean theorem says that the lengths of the sides are related by the equation

$$a^2 + b^2 = c^2$$

where a and b are the lengths of the two legs and c is the length of the hypotenuse (longest side).

In this problem, we are given that there is a right triangle in the diagram:



The lengths of two legs are given in inches as $a = 8$ and $b = 15$. So the Pythagorean theorem says

$$8^2 + 15^2 = c^2$$

$$64 + 225 = c^2$$

$$289 = c^2.$$

This equation determines the length c of the hypotenuse. To find the value of c if you don't have a calculator, remember we are told that c is a whole number. You can narrow down the possibilities by noticing that if c were 20, then c^2 would be too large (400) while if c were 15, then c^2 would be too small (remember 15^2 was 225). So the answer must be 16, 17, 18, or 19. Of these possibilities, only in the case of 17 does multiplying the number by itself give you an answer with a 9 in the ones place. So $c = 17$ must be the answer, as we can check by multiplying $17^2 = 17 \times 17 = 289$.

The length of the hypotenuse is 17 inches.

Elaboration on Alignment

Mathematically, the item is the most elementary kind of Pythagorean task: given the legs, find the hypotenuse. This is easier, say, than the task of finding a leg given the other two sides, and it is much easier than, say, finding the perimeter given two sides or creating a right triangle by inventively adding a line to a diagram with no apparent right triangle. The use of whole numbers also makes the task easier; asking for the hypotenuse of a triangle with legs $\sqrt{8}$ and ⅜ would make for a difficult problem. The elementary character of the problem seems appropriate given that this is the first grade in which the topic is taught, with really meaty applications not happening until high school (and likely not for all students even then). The task does require familiarity with the grade-level vocabulary *hypotenuse* and *legs*.

Logistically, the item is written in such a way as to reduce the need for a four-function calculator, or a calculator with squaring and square-root buttons. One way this is done is by informing the student that the answer is a whole number (this is possible thanks to the use of a Pythagorean triple). Giving away the fact that the answer is a whole number can help to defer logistical questions like “Do I need a calculator for this?” and “Should I round my answer?” It also could activate a guess-and-check strategy to solve $c^2 = 289$. The arithmetic would be even easier with a 3-4-5 triangle such as (15, 20, 25), but the drawback of dimensions 15” and 20” is that it would be possible to “extend the pattern” and get the right answer, 25, for the wrong reason.

The use of integers is artificial in relation to the context—a consequence of logistical considerations, not authentic issues in the situation. However, even with this distortion, the task still assesses fairly directly the use of the Pythagorean theorem itself. (The real cost of the integer constraint is on the algebraic side of the ledger, where we face an equation of the form $x^2 = q$ under the doubly artificial conditions that q is a perfect square *and we know* that it’s a perfect square.)

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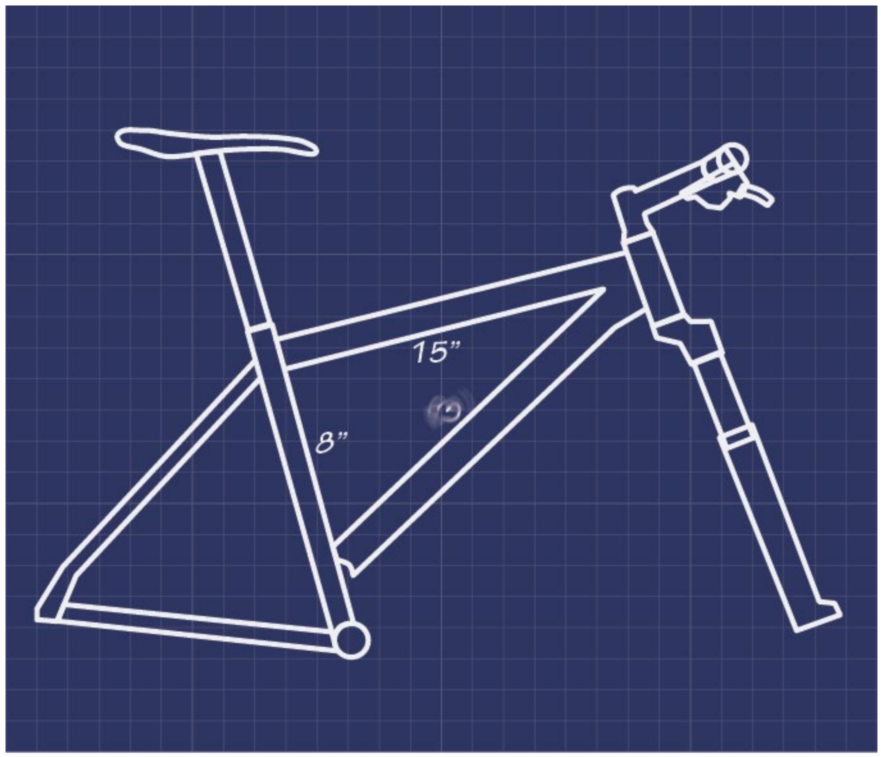

Answer: ☐ inches