Grade 8: Simultaneous Equations

8.EE.C.8 - Analyze and solve pairs of simultaneous linear equations.

Here is a system of two equations: $\begin{cases} 3x + 2y = 5 \\ 3x + 2y = 3 \end{cases}$

Check the boxes to mark each statement True or False.

	TRUE	FALSE
The solution to the system is $x = 1$, $y = 1$.	☐	☐
The system has infinitely many solutions.	☐	☐
The system has no solutions.	☐	☐

Solution

Correct if student checks FALSE, FALSE, TRUE.

The first statement is false. The values $x = 1$, $y = 1$ do satisfy the first equation, but they don't satisfy the second equation. To qualify as a solution to the *system*, the values $x = 1$, $y = 1$ would have to satisfy *both* equations. The left bracket { emphasizes that a system is both equations at the same time.

If we solve the system by substitution, the steps might look like this:

Isolate y using the first equation: $y = {}^5/_2 - {}^3/_2\, x$

Substitute this expression for y into the second equation: $3x + 2(5/2 – 3/2\, x) = 3$

Simplify: $3x + 5 – 3x = 3$

$5 = 3.$

Of course, 5 doesn't equal 3! This contradiction tells us that the original system has no solution. So the third statement is false and the fourth statement is true.

The fastest approach to this problem is to recognize that the left-hand sides of the two equations are the same, but the right-hand sides are different. We can't very well have $3x + 2y$ equaling 5 and, at the same time, equaling 3. Since it is impossible for both equations to be true at the same time, the system has no solution. If you look at the problem this way, then you don't have to do any pencil and paper work.

Elaboration on Alignment

This is a conceptual question that touches on what it means to solve a pair of simultaneous equations (or what a system is). Each equation in a system amounts to a statement, or proposition, or assertion; the system as a whole amounts to the conjunction of all those statements. The purpose of including the left-bracket { is to tie the two equations together and emphasize that a system is all of its parts at once. The bracket is something a teacher could point to when explaining the problem.

In the first statement, the putative solution $x = 1$, $y = 1$ was presented that way, rather than in ordered-pair notation (1, 1), so as to keep the focus on the algebra. The solution to an equation in x and y is answer to the questions "What is x? What is y?" The ordered-pair notation "jumps this gun" because it is a way to link the algebra to geometry via the representation-scheme of the coordinate plane. This item isn't about the correspondence between algebraic solutions and geometric points; it is about unknowns and their values.

One way to get the item wrong is to seize on a solution to one equation that doesn't solve both equations. The problem is constructed so that we can tell at a glance that (1, 1) solves the first equation.

The problem is also constructed so that the system is easy to analyze at a glance, provided we are in the habit of looking for and making use of algebraic structure. The system amounts to the two assertions "A is 5" and "A is 3." Both can't be true at once.

The item is supposed to be just about as easy as possible given its targets. Thus, there are no coefficients involving fractions, decimals, irrational numbers, or negative signs, and both equations are given in standard form, using the stereotypical variables x and y (presented in the customary alphabetical order). There is just a smidgen of fraction work if the student pursues a rote solution strategy of elimination by substitution; otherwise the item is written in such a way as to minimize the need for a calculator.

The use of the T/F format makes guessing less likely (there's a ¼ chance of getting the item right by chance even if you know one of the answers for sure). The format also allows us to avoid having unnecessary distractors, such as a choice $x = 1$, $y = 0$ corresponding to the second equation. The fact is, a system of two simultaneous linear equations in two unknowns can turn out in one of *three* ways, not four. The three statements in the item correspond to the three possible mathematical outcomes.

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