

# Curious Subtraction

Sample task from [achievethecore.org](http://achievethecore.org)

By Student Achievement Partners, adapted from a video by Global Education Resources

GRADE LEVEL Second

IN THE STANDARDS 2.NBT.A.1, 2.NBT.B.5, (3.OA.D.9)

## WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Offers an opportunity to practice subtraction within 100 and work towards fluency (2.NBT.B.5).
- Reinforces understandings of the place value system (2.NBT.A).
- Engages students in looking for and expressing regularity in repeated reasoning (MP8).

In the classroom:

- Encourages students to talk about each other's thinking in order to improve their mathematical understanding.
- Can lead into related discussions about interesting mathematical phenomena (see Additional Thoughts).
- Allows for whole class, small group, or individual work.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go [here](#) to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.

## MAKING THE SHIFTS<sup>1</sup>

	Focus	Belongs to the Major Work <sup>2</sup> of second grade
	Coherence	Builds on previous work with place value (1.NBT.B); Lays foundations for grade three work with multiplication patterns and multi-digit arithmetic (3.OA and 3.NBT)
	Rigor <sup>3</sup>	Conceptual Understanding: secondary in this task Procedural Skill and Fluency: primary in this task Application: not targeted in this task <sup>1</sup>

<sup>1</sup>For more information read [Shifts for Mathematics](#).

<sup>2</sup>For more information, see [Focus in Grade Two](#).

<sup>3</sup>Tasks will often target only one aspect of Rigor.

## LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Numbers
- Digits
- Regrouping
- Subtracting
- Place Value
- Tens
- Ones
- Difference

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work.

ELLs may need support with the following vocabulary words during the classroom discussion:

- Investigate
- Result
- Same

## TASK

### TASK: ACTIONS

The teacher sets the stage by letting students know that today they will investigate an interesting thing that happens when subtracting certain numbers. (For an illustration of this, please see video, start–1:17.)

The investigation begins with the teacher displaying the digits 1 through 9 on the board. S/he chooses two digits and asks the class what possible two-digit numbers they can make. (For an illustration of this, please see video, 1:17–2:16.)

The teacher then asks students to find the difference between the numbers they created. (The problem and the difference should be recorded on the subtraction template.) The teacher leads students through a second example, making sure to choose digits that will have the same result as the first example. (For an illustration of this, please see video, 2:16–5:10.)

The teacher then asks the students if the result will always be the same when subtracting two two-digit numbers made from the same pair of digits and challenges them to find out. Students can work independently or with partners using the subtraction template. As the students work, the teacher has the opportunity to encourage students to try different digit combinations and support any students who are subtracting incorrectly. After enough time has passed, the teacher has the students put their results (subtraction templates or equations) on the board. (For an illustration of this, please see video, 5:10–6:55.)

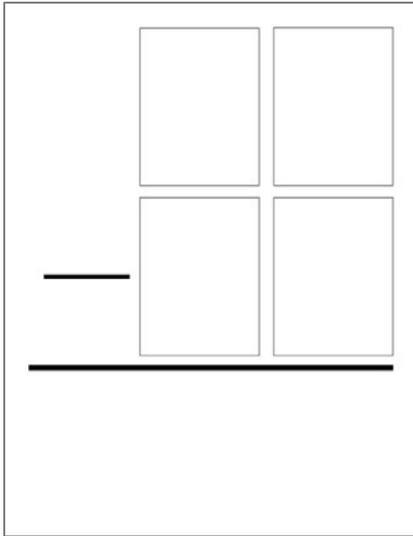
Once the results are all on the board, the class discussion begins. (For an illustration of this, please see video, 6:55–13:56) Follow up questions the teacher may ask include, but are not limited to:

- Were all of our answers the same?
- What are all the differences that our class found?
- Do you think we missed any possible differences?

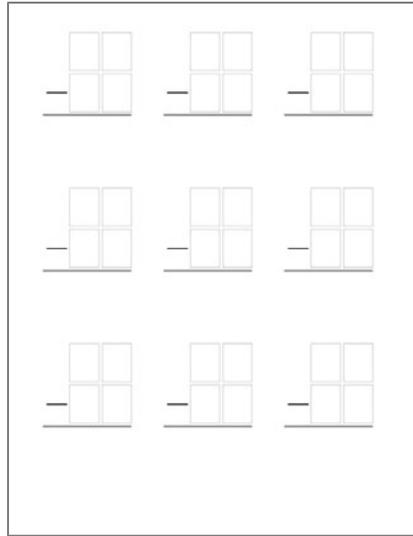
For a direct link, go to: <http://achievethecore.org/page/907/curious-subtraction-task>

- Is there anything special about the differences we found?
- With your partner, discuss one or two interesting things you wrote down about the results of this investigation.

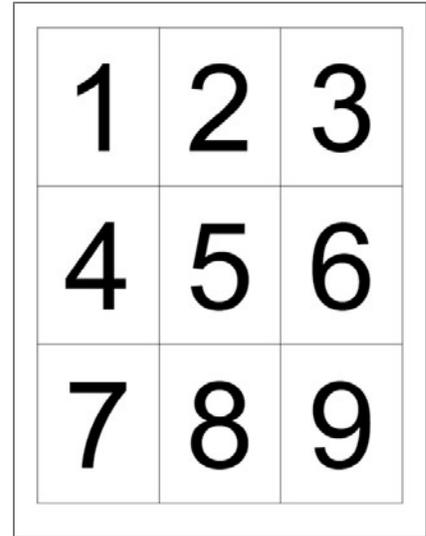
TASK: MATERIALS



Two-Digit Subtraction Print  
Template (large)

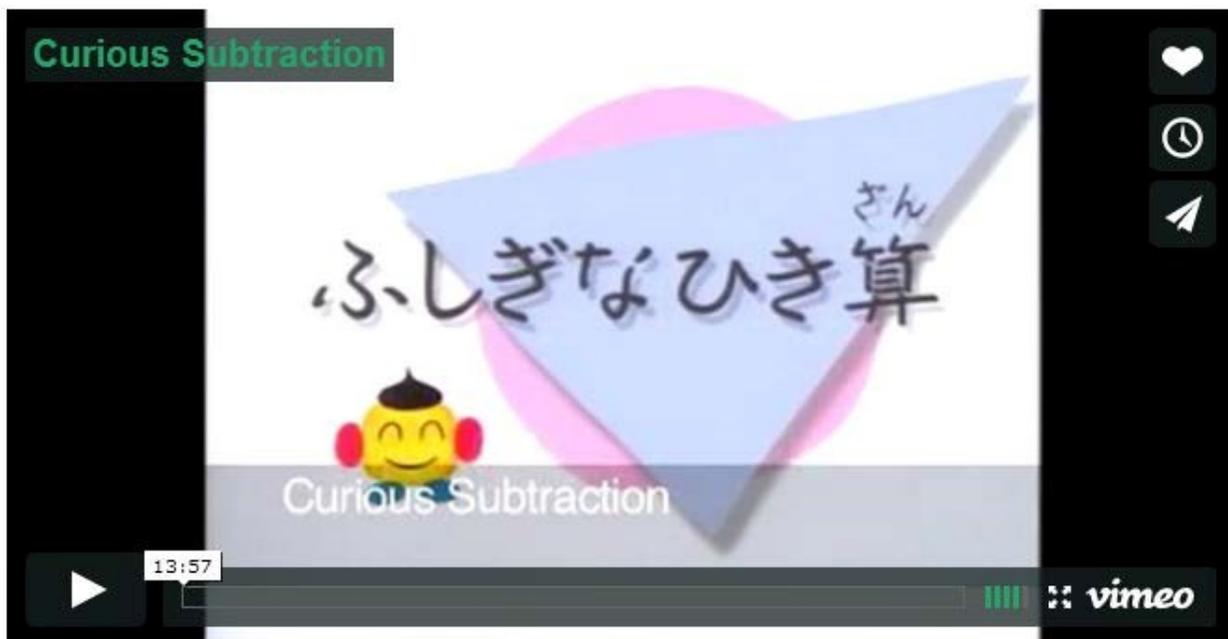


Two-Digit Subtraction Print  
Template (small)



Digits 1 –9 Print Template

## TASK: ADDITIONAL RESOURCES



Curious Subtraction (K–2), by Mr. Hiroshi Tanaka, University of Tuskuba Elementary School

This video serves as an illustration of this task in action. The original video can be viewed [here](#).

## COMMENTARY AND SOLUTION

### COMMENTARY

The joint purpose of this task is to provide students an opportunity to practice two-digit subtraction with regrouping and to engage students in looking for and expressing regularity in repeated reasoning (MP8). Throughout grade 2, students need many opportunities to practice subtraction in order to achieve fluency subtracting within 100 by the end of the grade. For students in grade 2, it is sufficient for them to recognize that differences of 9 will stem from numbers whose digits differ by 1, subtractions resulting in 18 will stem from numbers whose digits differ by 2, etc.

### SOLUTION

Having worked extensively with place value in grade 1 (1.NBT.B), students should come to grade 2 able to make both possible two-digit numbers from any given two digits.

The answers for the subtraction problems are dependent upon the numbers chosen, but are always multiples of 9 (e.g., if the digits 5 and 9 are selected, the numbers 59 and 95 are possible. When subtracting 59 from 95, the result is 36, a multiple of 9.).

The students should notice that when their subtraction result is 9, the difference of the two digits used is 1 (the numbers are consecutive or adjacent). When the result is 18, the difference is 2 (the numbers are “neighbors”). And so on. Some students may also notice that the digits in the results have a sum of nine if they are added together.

For a direct link, go to: <http://achievethecore.org/page/907/curious-subtraction-task>

### ADDITIONAL THOUGHTS

This task engages students in thinking about relationships between numbers and operations and can lead to investigations of other mathematical phenomena (e.g., reverse subtraction with three-digit numbers, adding two-digit reverse numbers). This task can be adapted or extended for students working above grade level. Identifying the arithmetic patterns in the numbers used and the solutions to the subtractions extends to grade three (3.OA.D.9). Students in grade three should note that the solutions are always multiples of 9 and that the difference will always be 9 times the positive difference between the two digits.

This task can also be adapted to eighth grade by asking students to algebraically prove that the solutions will always be a specific multiple of nine, depending on the digits selected. For example, when choosing two digits,  $a$  and  $b$ , one can make two distinct, two-digit numbers:  $ab$  and  $ba$ . To see why the difference between these will always be a multiple of 9 when subtracting the smaller from the larger, consider:

$$\begin{aligned}(10a+b)-(10b+a) &= 10a+b-10b-a \\ &= 10a-a+b-10b \\ &= 9a-9b \\ &= 9(a-b)\end{aligned}$$

The difference between the two numbers will be 9 times the positive difference between the two digits in the numbers.

For more information on the progression of place value understanding and properties of operations in grade 2, read pages 8–10 of the progression document, [K–5, Number and Operations in Base Ten](#).