

Find $\frac{1}{4}$ Starting from 1, Assessment Variation

Sample task from achievethecore.org

By Illustrative Mathematics and Student Achievement Partners

GRADE LEVEL Third

IN THE STANDARDS 3.NF.A.2

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Helps students understand fractions in relation to whole numbers (3.NF.A.2a).
- Requires students to understand that the fraction $\frac{1}{4}$ is formed by partitioning the whole (1) into 4 equal parts (3.NF.A.1).
- Uses a number line to demonstrate students' understanding of fractions as numbers.
- Encourages students to think carefully about the placement of the unit fraction $\frac{1}{4}$ as 0 and 1 are arbitrarily placed on the number line.

In the classroom:

- Can lead into related discussions such as, "Find x starting from y", where x and y are fractions.
- Allows students to demonstrate conceptual understanding through a brief conceptual problem with low computational difficulty.
- Provides an example of how students might encounter such a task in a computer-based environment.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go [here](#) to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.
- Provides opportunities to support students in connecting mathematical language with mathematical representations.
- Prompts teachers to write essential ideas/concepts/language on the board as a reference for students.

MAKING THE SHIFTS¹



Focus

Belongs to the Major Work² of third grade



Coherence

Integrates fractions with whole numbers, and lays groundwork for grade 4 fraction expectations; Expands understanding of whole numbers on the number line from previous grades



Rigor³

Conceptual Understanding: primary in this task

Procedural Skill and Fluency: not targeted in this task

Application: not targeted in this task

¹For more information read [Shifts for Mathematics](#).

²For more information, see [Focus in Grade Three](#)

³Tasks will often target only one aspect of rigor.

For a direct link, go to: <http://www.achievethecore.org/page/875/find-1-4-starting-from-1-assessment-variation>

INSTRUCTIONAL ROUTINE

The steps in this routine are adapted from the [Principles for the Design of Mathematics Curricula: Promoting Language and Content Development](#).

Engage students in the [Discussion Supports Mathematical Language Routine](#) as they work through this task. This will invite all students to participate in the conversations and support rich, inclusive discussions.

Numbered Heads Together

Students count off: Each group counts off by the number of students in a group so that every group has a 1, 2, 3, 4, etc.

Pose a Question/Problem: Give students the task including a copy of a number line.

Heads Together: The students solve this task as a group, recording notes of their thinking so that they will all be able to justify and explain their answers.

Reporting: Call a random number from 1–4 to be the reporter. At this point, students are no longer allowed to talk or write to each other, but the reporting student is allowed to use his/her notes. Ask each reporter to justify the reasoning of their group. Correct answers are not revealed until every reporter has shared.

Engage students in the [Collect and Display Mathematical Language Routine](#) as a way to capture the language they use when thinking about this task. This will provide a stable, collective reference for students to refer to, build on, or make connections to while working on future tasks. This collection can be used as a model and then revised and updated as more content is learned.

As students are working on this task, circulate and listen to students talk. Record important words and phrases used along with diagrams. Add these to a visual display to use during the whole class discussion of the task. As this recording is shared, students can clarify how and why they used these words or diagrams. Ask “Which of these help our communication to be more precise?”

Help students make connections between these words. For example, “Jill used ‘partition’ and Manny used ‘divide up’.” Or “Susie said this dot *is* $\frac{1}{4}$ and Daniella said this dot *represents* $\frac{1}{4}$.”

LANGUAGE DEVELOPMENT

- Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following term: number line

Students should engage with this concept in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of this concept, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students’ articulation of concepts. This concept should be reinforced in teacher instruction, classroom discussion, and student work

ADDITIONAL THOUGHTS

As noted in the Commentary below, this task is the first in a set of three tasks. The other tasks in the set can be found here: [Find \$\frac{7}{4}\$ Starting from 1](#) and [Find 1 Starting from \$\frac{5}{3}\$](#) .

3.NF.A.2 calls for students to understand fractions as a number on the number line and to represent fractions on a number line diagram. This is part of the standards’ progression toward the rational number system (6–8, NS).

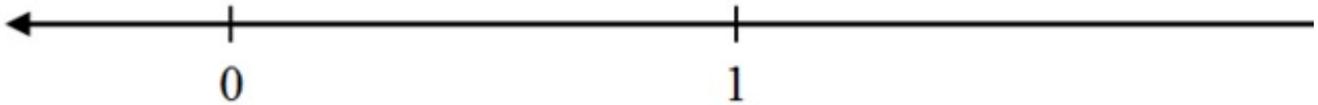
For more information on fractions on the number line, read page 3 of the progression document, *3–5, Number and Operations – Fractions*, available at www.achievethecore.org/progressions.

For more analysis on this task from an assessment perspective, please read the [Cognitive Complexity](#) section on the Illustrative Mathematics site.

For a direct link, go to: <http://www.achievethecore.org/page/875/find-1-4-starting-from-1-assessment-variation>

3.NF Find $\frac{1}{4}$ Starting from 1, Assessment Version

The number line below shows two numbers, 0 and 1.



Where is $\frac{1}{4}$ on this number line?

Commentary

This task is part of a joint project between [Student Achievement Partners](#) and Illustrative Mathematics to develop prototype machine-scorable assessment items that test a range of mathematical knowledge and skills described in the CCSSM and begin to signal the focus and coherence of the standards.

Task Purpose

This is the first of three summative assessment tasks for 3.NF.2 that progress in difficulty. Each requires that students "understand a fraction as a number on the number line" and "represent fractions on a number line diagram." Part (a) of the standard is about representing unit fractions and part (b) is about representing fractions in terms of unit fractions. The first task involves (a); the second two involve both (a) and (b) at different levels of complexity. The tasks require attention to the whole when thinking about fractions; on a number line, the whole is the interval from 0 to 1.

Cognitive Complexity

Mathematical Content

The three tasks increase in complexity:

- [3.NF Find \$\frac{1}{4}\$ starting from 1](#) asks students to construct a unit fraction on the number line given the unit interval.
- [3.NF Find \$\frac{7}{4}\$ starting from 1](#) asks students to construct a fraction greater than 1 on the number line given the unit interval.
- [3.NF Find 1 starting from \$\frac{5}{3}\$](#) asks students to work backwards from a given fraction to reconstruct the location of 1.

Mathematical Practices

Proficiency with the number line is an important component of students' ability to reason abstractly and quantitatively (MP2).

While the number line is an important tool, these tasks do not ask for strategic use of tools (MP5) because the number line is provided. An item calling for MP5 might be one that is easy if the student uses a number line, but difficult if not.

These tasks give students an opportunity to look for and make use of structure (MP7), namely, the partitioning of the unit interval.

Linguistic Demand

The linguistic demand is low.

Stimulus Material

The stimulus material is not complex, consisting of only a short written prompt and a number line.

Response Mode

The response mode might be unfamiliar to students. However, they can mess around without penalty and there is a reset option. The spacing on the selector for the tick marks should be wide enough so that students can easily pick the refinement they want. The point has a fairly large radius relative to the scale of the number line.

Additional Notes

- All of these can be turned into multiple choice tasks, as in, for example, [1NF Find 1 starting from 5/3](#). However, the multiple choice variation is less complex because the technology enhanced version asks students to construct a number line rather than select one.
- An instructional extension would be a task asking students to "Find $2/3$ starting from $1/4$." This would be more complex because it requires juggling two partitions of the unit interval, into fourths and then thirds.

Solutions

This is a one-point task; the correct answer is shown in the video. Students will get full credit as long as the point touches the correct tick mark at all.

Solutions

Solution: 1

This is a one-point task; the correct answer is shown in the video. Students will get full credit as long as the point touches the correct tick mark at all.



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