

High-Quality Mathematics Items Module (Grades 3-5)

The pages that follow contain a selection of items from chapter assessments in grades 3-5. These items are representative of the range available in many textbook series. This activity is designed to help teachers think about how they can revise chapter tests to better align to the Standards. Each item below can be revised to more closely embody the characteristics described in the High-Quality Mathematics Items Modules.

Instructions:

1. Solve all of the items.
2. Take a close look at each item, thinking about what the modules explain about expectations of high-quality mathematics items. Focus on the following features:
 - a. If the item aligns to a Supporting Work standard, does the item connect to Major Work? (Principle 1)
 - b. Does the item align to the aspect of rigor targeted in the Standards? (Principle 2)
 - c. Does the item align to grade-level expectations? (Principle 3)
 - d. Does the item address the central concern of the identified standard? (Principle 4)
 - e. If the item aligns to a Standard for Mathematical Practice, is the item appropriate to the grade? (Principle 7)
 - f. Does the item type/format of the item match the content? (Principle 8)
3. Using the chart below, record your thoughts about which Alignment Principle can be used to improve each item.
4. With the Alignment Principle in mind, revise the item.
5. After time for individual reflection, discuss your findings and your proposed revision with your colleagues.

Answer Key:

(Note: There may be other principles that could help to strengthen the alignment of the items. The ones listed here are examples.)

Standard	Item	Assessment Principle(s)?	Sample Revised Item						
<p>3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<p>Use the place value chart to add 351 and 248. Draw place value models in each column of the chart and then write the sum.</p> <table border="1" data-bbox="401 678 926 987"> <thead> <tr> <th data-bbox="401 678 627 740">Hundreds</th> <th data-bbox="627 678 812 740">Tens</th> <th data-bbox="812 678 926 740">Ones</th> </tr> </thead> <tbody> <tr> <td data-bbox="401 740 627 987"></td> <td data-bbox="627 740 812 987"></td> <td data-bbox="812 740 926 987"></td> </tr> </tbody> </table> <p>351 + 248 = ___ hundreds ___ tens ___ ones = _____</p>	Hundreds	Tens	Ones				<p>4. Most items aligned to a single content standard should assess the central concern of the standard.</p>	<p>Find the sum of 351 and 248. Show how you got your answer.</p> <p>Rationale: The central concern of the standard is about students being able to add. The standard does not dictate a specific strategy and instead emphasizes fluency as the goal. Work more focused on strategy is included in the grade 2 standards, while this item is for grade 3. The revised item will allow students to simply add the numbers and will give teachers information about how fluently students are able to find sums.</p>
Hundreds	Tens	Ones							

<p>3.NF.A.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.</p>	<p>Yesterday, Trevor rode his bike around the track 6 laps for a total of one mile. Today he wants to ride $\frac{5}{6}$ of a mile. How many laps does he need to ride around the track?</p> <p>a) 3 b) 4 c) 5 d) 6</p>	<p>2. Items are designed to address the aspect(s) of rigor (conceptual understanding, procedural skill, and application) evident in the language of the content standards.</p>	 <p>What number is represented by the point on the number line?</p> <p>a) $\frac{4}{5}$ b) $\frac{5}{4}$ c) $\frac{5}{6}$ d) $\frac{6}{5}$</p> <p>Rationale: This standard, along with the rest of the 3.NF domain, focuses on conceptual understanding. The application aspect of the original item detracts from students demonstrating conceptual understanding. The revised example shows one way to target conceptual understanding in an item.</p>
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3.NF.A.3d

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Complete the comparisons using $<$, $>$ or $=$.

- $\frac{3}{4}$ $\frac{3}{8}$
- $\frac{1}{2}$ $\frac{4}{8}$
- $\frac{3}{6}$ $\frac{2}{6}$
- $\frac{1}{4}$ $\frac{4}{6}$

3. Items are designed to attend to content limits articulated in the standards.

Complete the comparisons using $<$, $>$ or $=$.

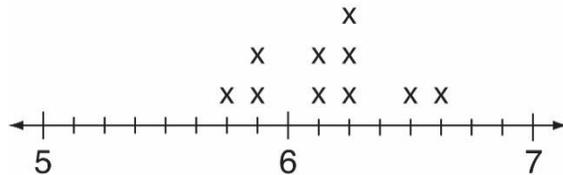
- $\frac{3}{4}$ $\frac{3}{8}$
- $\frac{1}{2}$ $\frac{4}{8}$
- $\frac{3}{6}$ $\frac{2}{6}$
- $\frac{1}{4}$ $\frac{1}{8}$

Rationale: The standard limits comparisons to those having the same numerator or the same denominator. Although students in grade 3 may be able to compare the last two fractions by reasoning about benchmarks, this is required in the NF domain in grade 4. Additionally, revisions may focus on the second comparison, which doesn't align to 3.NF.A.3d but does align to 3.NF.A.3b Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

<p>3rd grade</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Complete the multiplication table:</p> <table border="1" data-bbox="436 363 968 518"> <thead> <tr> <th>×</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> <th>7</th> <th>8</th> <th>9</th> </tr> </thead> <tbody> <tr> <td>6</td> <td>6</td> <td>12</td> <td>18</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>Would the product of 6×57 be an even or odd number? Prove your answer.</p>	×	1	2	3	4	5	6	7	8	9	6	6	12	18							<p>7. The demands of items measuring the Standards for Mathematical Practice are appropriate to the targeted grade level.</p>	<p>Complete the multiplication table:</p> <table border="1" data-bbox="1262 363 1793 518"> <thead> <tr> <th>×</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> <th>7</th> <th>8</th> <th>9</th> </tr> </thead> <tbody> <tr> <td>6</td> <td>6</td> <td>12</td> <td>18</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>Would the product of 6×57 be an even or odd number? Explain.</p> <p>Rationale: The original phrasing of the item asks students to “prove” their answer. However, students at this grade are not required to have the sophistication to show a proof. They should be using the patterns in the multiplication table to gain an idea about what the product of 6 and 57 is, but they will not be able to prove it yet mathematically.</p>	×	1	2	3	4	5	6	7	8	9	6	6	12	18						
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<p>4.NBT.4</p> <p>Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p>	<p>Solve the addition problem using two different strategies:</p> <p>$1,645 + 2,351$</p> <p>a) Use the standard algorithm.</p> <p>b) Use another strategy to show how you solved the problem</p>	<p>2. Items are designed to address the aspect(s) of rigor (conceptual understanding, procedural skill, and application) evident in the language of the content standards.</p>	<p>Solve.</p> <p>$1,645 + 2,351$</p> <p>Rationale: The standard for 4th grade requires fluency with multi-digit addition. There is no need for students to demonstrate multiple strategies as addition based on place value and properties of operations is a focus of earlier grade work in the NBT domain.</p>																																								

4.MD.B.4 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

The chart below shows the height, in feet, of some players on a football team:

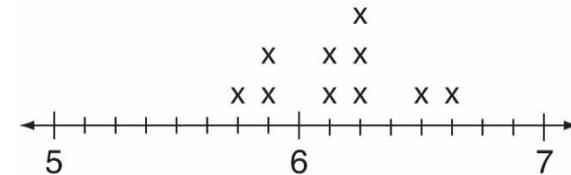


Based on the line plot, what is the typical height of this group of football players?

- a) 6 feet
- b) $6\frac{1}{8}$ feet
- c) $6\frac{1}{4}$ feet
- d) $6\frac{2}{7}$ feet

1. Most items aligned to standards in supporting clusters connect to the Major Work of the grade.

The chart below shows the height, in feet, of some players on a football team:



Based on the line plot, what is the difference between the tallest and shortest football player?

- a) $\frac{7}{8}$ feet
- b) 1 foot
- c) $1\frac{7}{8}$ feet
- d) $\frac{6}{8}$ feet

Rationale: Assessment Principle 1 states that supporting cluster work should align to the Major Work of the grade. In the case of this item, the stem should focus on the second sentence of the standard which, requires students to solve word problems - reinforcing students' work in 4th grade with fraction operations.

5.NF.B.4b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

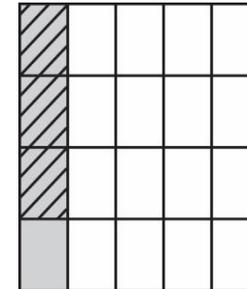
Destiny wanted to find out about what her classmates do after school. She asked $\frac{3}{4}$ of the students in her class whether they played a musical instrument. She discovered that $\frac{2}{3}$ of the students she talked to played a musical instrument. What fraction of the students in her class play a musical instrument?

To describe how to solve the problem, circle the number in each statement that completes the sentence

- First, draw a rectangular array with 3 rows and [3 4 5] columns.
- Next, shade [1 2 3] of the rows with diagonal lines.
- Next, shade [4 5 6] of the squares with diagonal lines going the other way.
- So, according to the information that Destiny found, [$\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{8}$] of the students in her class play a musical instrument.

4. Most items aligned to a single content standard should assess the central concern of the standard.

8. Item types are chosen to match the item's purpose and as part of the evidence required by the standards.



Which multiplication equation represents the model shown above?

- a) $5 \times \frac{3}{4} = \frac{1}{5}$
- b) $\frac{1}{5} \times \frac{1}{4} = \frac{4}{20}$
- c) $\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$
- d) $\frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$

Rationale: The original item assessed students' ability to describe the process of creating the model to match the multiplication expression that represents the application problem. The central concern of this standard is to connect students' understanding of whole number multiplication to multiplication of fractions. The format of choosing numbers for each blank distracts from teachers being able to understand what students understand about fraction multiplication as it relates to area models.

5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.*

Gina ate $\frac{1}{4}$ of a pizza and Tessa ate $\frac{1}{8}$ of the same pizza.

Part A: Fill in the blanks below to create fractions with a common denominator.

$$\frac{1}{4} = \frac{1}{4} \times \frac{\square}{\square} = \frac{\square}{\square}$$

$$\frac{1}{8} = \frac{1}{8} \times \frac{\square}{\square} = \frac{\square}{\square}$$

Part B: How much pizza did the girls eat together? Explain how you found your answer.

8. Item types are chosen to match the item's purpose and as part of the evidence required by the standards.

Gina ate $\frac{1}{4}$ of a pizza and Tessa ate $\frac{1}{8}$ of the same pizza. How much of the pizza did the girls eat?

Show your thinking.

Rationale: By requiring students to fill in the blank in Part A, the format of the item is distracting from gathering evidence about whether students can solve word problems using addition of fractions with different denominators. In addition, the direction to have students explain their answer is unnecessary. Requiring students to show their calculations is sufficient to understand whether students are meeting the expectation of the standard.

Note: On a summative assessment, it may make more sense to have a multiple-choice item to assess this standard. On a chapter test, it may be helpful to look at student work so that teachers can better understand how their students approach the problem.