High-Quality Mathematics Items Module (High School)

The pages that follow contain a selection of items from chapter assessments in High School. These items are representative of the range available in many textbook series. This activity is designed to help teachers think about how they can revise chapter tests to better align to the Standards. Each item below can be revised to more closely embody the characteristics described in the High-Quality Mathematics Items Modules.

1. Solve all of the items.
2. Take a close look at each item, thinking about what the modules explain about expectations of high-quality mathematics items. Focus on the following features:
    a. Does the item align to the aspect of rigor targeted in the Standards? (Principle 2)
    b. Does the item align to the grade-level expectations? (Principle 3)
    c. Does the item address the central concern of the identified standard? (Principle 4)
    d. If the item aligns to a Standard for Mathematical Practice, is the item appropriate to the grade? (Principle 7)
    e. Does the item type/format of the item match the content? (Principle 8)
3. Using the chart below, record your thoughts about which Alignment Principle(s) can be used to improve each item.
4. With the Alignment Principle in mind, revise the item.
5. After time for individual reflection, discuss your findings and your proposed revision with your colleagues.

Answer Key:

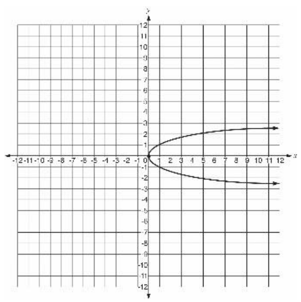
(Note: There may be other principles that could help to strengthen the alignment of the items. The ones listed here are examples.)

Standard	Item	Assessment Principle?	Sample Revised Item
HS.N-CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Which expression is equivalent to $\frac{5}{3+i}$? a) $\frac{15}{8} - \frac{5}{8}i$ b) $\frac{5}{3} - 5i$ c) $\frac{3}{2} - \frac{1}{2}i$ d) $15 - 5i$	3. Items are designed to attend to content limits articulated in the standards.	Which expression is equivalent to $(-1 + i)^3$? a) $-3i$ b) $-2 - 2i$ c) $-1 - i$ d) $2 + 2i$ Rationale: The standard limits operations to addition, subtraction and multiplication. Division of imaginary numbers is not part of the Standards. Therefore, the revised item focuses on an operation required by the standard.

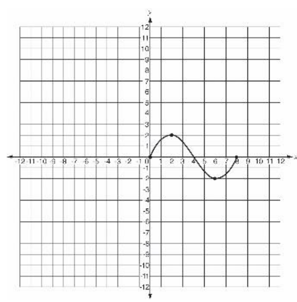
HS.F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

Use the vertical line test to determine which graph represents y as a function of x.

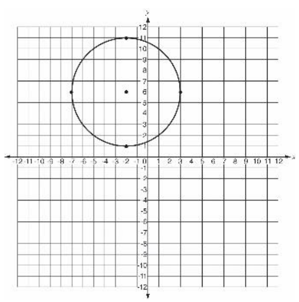
a)



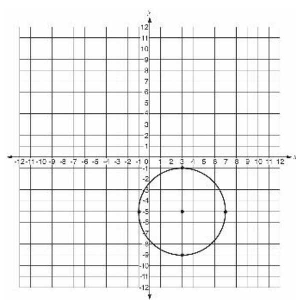
b)



c)



d)



2. Items are designed to address the aspect(s) of rigor (conceptual understanding, procedural skill, and application) evident in the language of the content standards.

The equation of a circle in the coordinate plane with center (0,0) and radius 5 is shown:

$x^2 + y^2 = 25$

Fill in the table to show an example of two ordered pairs that show this equation does **not** define y as a function of x.

x	**y**

Retrieved from Smarter Balanced HS Claim 3 Item Specifications; accessed July 2016

Rationale: The standard calls for conceptual understanding, as signaled by the verb "understand." The original item tells students a procedure to use to determine which graph represents y as a function of x. This item encourages a procedural approach to the content.

HS.G-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Which of the following shows the equation of the circle with a center located at (4, -1) and has a point on the circle at (5,3)? a) $(x-4)^2 + (y+1)^2 = 17$ b) $(x+4)^2 + (y-1)^2 = \sqrt{17}$ c) $(x-4)^2 + (y+1)^2 = \sqrt{17}$ d) $(x+4)^2 + (y-1)^2 = 17$	8. Item types are chosen to match the item's purpose and as part of the evidence required by the standards.	Use the Pythagorean theorem to find an equation in x and y whose solutions are the points on the circle of radius 2 with center (1,1) and explain why the theorem works. *Adapted from Illustrative Mathematics, G-GPE Explaining the equation for a circle* Rationale: The standard requires students to connect their previous work with the Pythagorean Theorem to their developing understanding of circles and the equations that describe them. The original item doesn't allow teachers to see whether students understand the connection between the Pythagorean Theorem and the equation of a circle. The constructed-response format of the revised item will better allow teachers to see what their students understand in relation to the standard.

<table>
<tr>
<td>HS.S-ID.B.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</td>
<td>The table shows the number of trees planted in Santa Fe since 2005.

<table>
<tr><td>Years since 2005</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr>
<tr><td># of trees planted (in thousands)</td><td>0.8</td><td>10.6</td><td>44.4</td><td>116.6</td><td>241.6</td></tr>
</table>

Write a cubic function to model the data.</td>
<td>4. Most items aligned to a single content standard should assess the central concern of the standard.</td>
<td>The table shows concentration (in milligrams per liter) of a medication in a patient’s blood.

<table>
<tr><td>Time (hours)</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr>
<tr><td>Concentration (mg/1)</td><td>0</td><td>78.1</td><td>99.8</td><td>84.4</td><td>50.1</td></tr>
</table>

Write a function to model the data. Use the function to predict the concentration of medication in the patient’s blood after 3 hours.

Rationale: The original item asks students to create a cubic function, which is not one of the types of functions emphasized in the standard. Although classroom instruction does not need to be limited to only the types of functions listed, chapter tests should focus on the functions listed, as they are at the heart of the high school standards. The revised item includes data that can be modeled with a quadratic function.</td>
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<table>
<tr>
<td>HS.A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td>
<td>Find the solution set. Justify each step and graph your solution.
<table>
<tr><td>21x + 28 < 10 - 3x</td><td>Justification</td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
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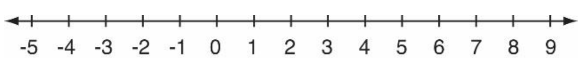
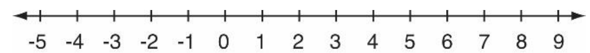
</table>
Solution set: ___________


</td>
<td>2. Items are designed to address the aspect(s) of rigor (conceptual understanding, procedural skill, and application) evident in the language of the content standards.</td>
<td>Graph the solution set to the inequality.
21x + 28 < 10 - 3x


Rationale: This standard requires procedural skill with solving equations and inequalities. In grades 6-8 students were required to make sense of the process of solving equations and inequalities. In high school, their responsibility is to fluently solve them.</td>
</tr>
</table>

MP.7 Look for and make use of structure.	Judy is working at a retail store over summer break. A customer buys a $50 shirt that is on sale for 20% off. Judy computes the discount, then adds sales tax of 10%, and tells the customer how much he owes. The customer insists that Judy first add the sales tax and then apply the discount. He is convinced that this way he will save more money because the discount amount will be larger. Is the customer right?	7. The demands of items measuring the Standards for Mathematical Practice are appropriate to the targeted grade level.	Judy is working at a retail store over summer break. A customer buys a $50 shirt that is on sale for 20% off. Judy computes the discount, then adds sales tax of 10%, and tells the customer how much he owes. The customer insists that Judy first add the sales tax and then apply the discount. He is convinced that this way he will save more money because the discount amount will be larger. a. Is the customer right? b. Does your answer to part (a) depend on the numbers used or would it work for any percentage discount and any sales tax percentage? Find a convincing argument using algebraic expressions and/or diagrams for this more general scenario. *Adapted from Illustrative Mathematics, A.SSE Taxes and Sales* Rationale: Since this item is aligned to MP.7, it requires students to look for patterns and structure in their mathematical work. In the original item, students only need to decide if a specific situation is true. The revised item raises the level of Practice by requiring students to generalize the pattern and provide an argument to justify the general case.