

Super Fill

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TASK: Every year, most of the trash produced by over 330 million people in the United States ends up in landfills. In order to meet the need for additional landfill space, suppose the United States designated 200 acres of land for a new landfill (called Super Fill) on land where local regulation requirements limit the depth of the pit to 150 feet. The purpose of this task is to estimate the time it will take to completely fill up the landfill.

The full student-facing task can be found at the beginning of page 10.

STANDARDS: In working on this task, students have an opportunity to engage in mathematical modeling by

- making assumptions;
- applying their knowledge of shapes and their properties and knowledge of density;
- applying their knowledge of linear functions to build functions that model municipal solid waste (MSW) consumption;
- applying their knowledge of non-linear functions to approximate MSW composting and recycling rates over time; and
- strategically using tools, such as spreadsheets and graphing software, to create mathematical models to support their thinking.

Throughout the task, students are asked to reflect on the assumptions they've made in developing their model.

The following standards may be addressed while solving this task: HS.N-Q.A.1, HS.G-MG.A.1, HS.G-MG.A.2, HS.G-MG.A.3, HS.F-BF.A.1, HS.A-REI.B.3

IMPLEMENTATION SUPPORT: The task is intended to take two class sessions to complete, in addition to an optional extension assignment.

Launch: Begin with a short discussion about trash by asking students, "What happens to our trash after it is picked up?" After eliciting some student responses, share more about landfills and how they work. [This website](#) and others like it can give students a visual of what a landfill looks like and how it works.

Day 1 and Day 2: Students work in pairs or small groups to complete the task. Day 2 should allow for time for students to share their models, solutions, and assumptions with other groups.

Optional Extension Prompts:

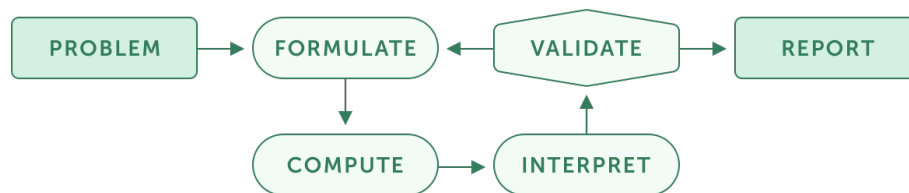
Option 1: Given a landfill like Super Fill in your community, how long will it take to reach capacity?

Option 2: Localize the prompt by inviting students to apply the given constraints to their neighborhood/city and determine how long it would take for their neighborhood/city to fill such an area. (This will require accessing local data. A teacher may choose to gather this data ahead of time or build in the time for students to do so within the scope of this task.)

Example prompts:

- What does trash production look like in our community? How much MSW do we produce? Where does our trash go?
- What is one of our closest landfills? What is its capacity? How close are we to filling it?
- Based on what you know about our community, when do you anticipate our landfill will be at capacity? What can we do to extend the time it would take to fill our landfill?

COMMENTARY: This task scaffolds the modeling and problem solving for students, and it includes all stages of the modeling cycle represented by the graphic below:



Problem:

Students need to make sense of the **context** of the task. The task launch offers students the opportunity to gain an understanding of landfills in the United States. Students will also need to **make sense** of the dimensions of the landfill. Question 1 supports students to visualize the size of land that covers 200 acres with a depth of 150 feet. Students will also need to make sense of the data found within the resource page.

Formulate:

Students can **formulate** more than one model in this task. In parts *a* and *b* of question 2, they use information from the resource page to create mathematical models that best predict a) the amount of MSW produced over time, and b) the amount of MSW sent to landfills over time. Students select the appropriate tool. They may choose paper and pencil or a spreadsheet; Desmos or hand-held graphing calculators with statistical capabilities can enhance the experience. Students can also use recent data on municipal solid waste when it becomes available on the EPA website.

In part *a*, Figure 1 in the resource page offers two options for students: they may use the Total MSW data or the Per capita generation data to create their scatterplot. They need to accurately identify the dependent variable and formulate the function that best models the data. Students familiar with residuals can use the magnitude of residuals to select the best model. In part *b*, students have several options: they may use the difference between the sets of data in Figure 1 and Figure 2 and create a function that they use to predict the amount of MSW that is sent to landfills. They can also use their model from the first part along with information in Figure 3 to find the amount sent to landfills. In formulating the solution to question 4, students use the conversion factors from Figure 4 to convert tons to cubic feet, acres to square feet, cubic yards to cubic feet, then find the time it would take to fill up the landfill. Attention to precision and accuracy in unit conversion are hallmarks of the formulation stage.

Compute:

Students need to correctly **compute** based on the models they created in questions 2 and 4. Calculators are recommended to maintain accuracy with computations, some of which involve large numbers. Students are more likely to compute with accuracy when they pay attention to units in their calculations. This task offers an opportunity for students to revisit the work of grade 8 around scientific notation (8.EE.B.4).

Interpret:

Students need to consider assumptions they made in questions 2 and 4 to **interpret** their answers in context using correct units, and determine whether the values they found are overestimates or underestimates of the true amounts.

Validate:

Students should **validate** their results by comparing their answers with those of students who used an approach that differs from theirs. They might also need to do research to decide whether their answers are reasonable. If there are discrepancies, students need to reformulate or identify and correct possible computation errors before they report their results, make recommendations, and propose solutions to the trash problem.

Report:

Students need to **report** their findings and make recommendations in question 6.

SOLUTION:

1. Solutions will vary depending on the size of the classroom. A successful response demonstrates accurate conversions.

#1 Sample Response 1:

My classroom dimensions are 15 feet by 25 feet by 15 feet.

I know 640 acres = 1 mile² and 1 mile = 5280 feet

$$\frac{640 \text{ acres}}{5280^2 \text{ ft}^2} = \frac{200 \text{ acres}}{x}$$
$$x = 8,712,000 \text{ ft}^2$$

Volume of Super Fill

$$8,712,000 \times 150 \approx 1.3 \times 10^9 \text{ ft}^3$$

How many classrooms fit into Super Fill?

$$\frac{1.3 \times 10^9}{15 \times 25 \times 15} = 232,320 \text{ classrooms}$$

#1 Sample Response 2:

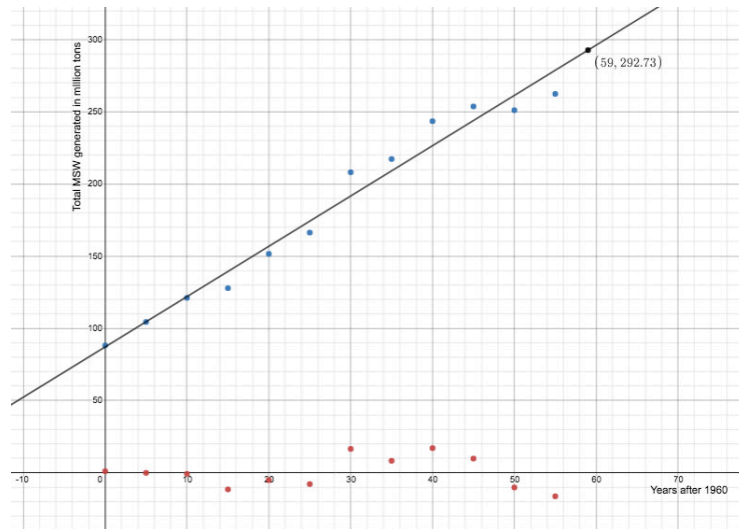
Our classroom is 20 feet wide, 25 feet long, and 10 feet high. That gives a volume of 5,000 cubic feet.

Super Fill is 150 feet deep over an area of 200 acres. 1 acre = 43,560 square feet. The volume of Super Fill is 200 × 43,560 square feet × 150 feet = 1,306,800,000 cubic feet. Dividing that by 5,000 cubic feet in a possible classroom: 261,360 classrooms could fit inside the Super Fill landfill.

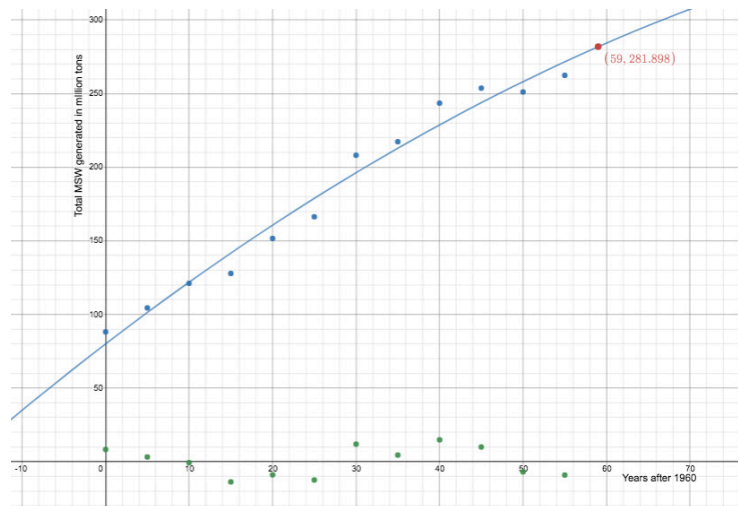
- 2a. Students successfully create a model that predicts the MSW for the current year.

#2a Sample Response 1:

Using the data in figure 1, a linear model for the amount of MSW generated over time from 1960 to 2015 could be $W(t) = 3.485t + 87.119$, where t = years after 1960 and $W(t)$ is measured in million tons (see graph below). This model has negative residuals for values of t at 50 and 55, which suggests that $W(59) = 292.73$ million tons, the prediction with this model for 2019 is an overestimate of the true amount of municipal waste generated in 2019.



A quadratic model for the data is $W(t) = -0.0157t^2 + 4.35t + 79.9$ might be a better model because the residuals near $t = 59$ are smaller in absolute value. This model predicts a lower overestimate $W(t) = 281.898$ million tons for the year 2019 (see graph below).



#2a Sample Response 2:

Amount of MSW produced in US in 2020:

Average rate of change from 1990 to 2015:

$$\frac{262.4 - 208.3}{2015 - 1990}$$

= 2.164 million tons per year

$$\begin{aligned} \text{In 2020: } & 262.4 + 5(2.164) \\ & = 273.22 \text{ million tons in 2020} \end{aligned}$$

#2a Sample Response 3:

I used the Total MSW generation data from Figure 1 to predict a value for 2020.

Defining x as the year and y as the total amount of MSW generated, in millions of tons, the linear regression function $y = 3.4849x - 6743.3$ has an R^2 value of 0.9703. This formula predicts that in 2020, there will be about 296.2 million tons of MSW produced in the United States.

- 2b. Figure 3 names that in 2015, 52.5% of MSW was sent to landfills. Students may make assumptions or gather additional data to determine the percentage sent to landfills in the current year. They will compute that percentage of their answer to 2a to determine the amount of MSW sent to landfills.

#2b Sample Response 1:

Assuming the percentage of waste sent to landfills remained 52.5% of total MSW in the US in 2019 (Figure 3), and using the predicted 281.898 million tons of MSW in 2019, there would be $281.898 \times 52.5\% \approx 148$ million tons of MSW sent to landfills in 2019.

#2b Sample Response 2: [note - this response includes both the solution to 2b and some assumptions made]

Figure 3 shows that in 2015, 52.5% of MSW was sent to landfills. In Figure 2, we can see that the percentage of MSW sent to landfills has increased for many years, but the rate of increase has slowed down in a way that suggests an asymptote. To estimate the y -value of the asymptote, I tested different values in increments of 0.1% to find which limit generated the strongest correlation. With an asymptote at $y = 37.6\%$, I found the strongest correlation, with $R^2 = 0.9864$. This suggests that the percentage of MSW recycled or composted will not soon exceed 37.6%, and therefore, the percentages given in Figure 3 for 2015 will not change significantly by 2020. Optimistically hoping for some reduction in landfill use in 2020, I estimate that 51% of MSW will be sent to landfills in 2020. 51% of 296.2 million tons is about 151.1 million tons of MSW that is expected to be sent to landfills in 2020.

#2b Sample Response 3:

Figure 3 says 52.5% sent to landfills, so $273.22(0.525) = 143.44$ million tons

- 2c. Students articulate the assumptions they made to create the model and determine the amount of MSW sent to landfills in the current year.

#2c Sample Response 1:

Assumptions: 1) the percentage of MSW that ends up in landfills remained unchanged since 2015, and 2) the US population continued to generate waste at an increasing rate.

#2c Sample Response 2:

I assumed that the relatively steady rate of increase in MSW generation rates (total) since 1990 (Figure 1) would continue through 2020, and that the percentage landfilled would remain unchanged from 2015 (Figure 3) to 2020.

3. Students successfully use the data from Figure 4 to convert the weight of the amount of MSW they determined would be sent to a landfill this year to an approximate volume. There are options students can make that will need to be named as part of their assumptions. Students must also use that volume to compute how long it will take to fill Super Fill. Precision and attention to units are critical here.

#3 Sample Response 1:

Assuming that Super Fill is a large landfill with best management practices, from the last line in Figure 4, approximately 1,700 to 2,000 lbs, or an average of 1,850 lbs = 0.925 tons of compacted MSW will have a volume $\approx$ 1 cubic yard = 27 cubic feet. Assuming that in 2019, 148 million tons will go to the landfill, the compacted volume of the waste will be 148 million tons $\times$ 27 cubic feet / 0.925 tons = 4,320 million cubic feet.

Each acre covers an area equal to 43,560 square feet. Assuming the landfill is not graduated, its volume would be 200 acres $\times$ 43,560 ft²/acre $\times$ 150 ft = 1,306,800,000 ft³.

Since 0.925 tons of compacted waste is equivalent to 27 cubic feet in volume, 148 million tons $\times$ 27 cubic feet per 0.925 tons = 4,320 million cubic feet of waste was destined to landfills in 2019.

Our landfill holds 1,306.8 million cubic feet of compacted waste, and 1,306.8 million cubic feet $\div$ 4,320 million cubic feet per year = 0.3025 years = 3.63 months = 15.73 weeks. Assuming the trash that goes to the landfills stays the same, it would take about 16 weeks to fill up the 200-acre landfill!

The solution assumes that the landfill is perfectly shaped with no sloping sides so that its volume would be calculated by multiplying the area of the base by the depth. This solution also assumes that the landfill is deemed full if it is level with the ground.

#3 Sample Response 2:

To convert 143.44 million tons to volume:

From Figure 4, using the last line average 1850 lbs = 1 cubic yard.

$$143.33 \times 10^6 \text{ tons} \times \frac{2000 \text{ lbs}}{1 \text{ ton}} = 2.87 \times 10^{11} \text{ pounds}$$

$$2.87 \times 10^{11} \div 1850 = 1.55 \times 10^8 \text{ yards}^3$$

$$1.55 \times 10^8 \text{ yards}^3 \times 27 = 4.19 \times 10^9 \text{ ft}^3$$

$$\text{Volume of Super Fill is } 1.3 \times 10^9 \text{ ft}^3$$

$$\text{Amount of MSW sent to Super Fill this year is } 4.19 \times 10^9 \text{ ft}^3$$

$$1.3 \times 10^9 \text{ ft}^3 \div 4.19 \times 10^9 \text{ ft}^3 = 0.31 \text{ years}$$

$$0.31 \text{ years} \times 12 \text{ months} = 3.7 \text{ months}$$

I assumed all the waste would be compacted using best management practices and that the US would continue to produce landfill waste at the same rate for the next 3+ years.

#3 Sample Response 3:

I have estimated that 151.1 million tons of MSW will be sent to landfills in the US in 2020. According to Figure 4, one cubic yard (27 cubic feet) of compacted MSW weighs 1,700-2,000 pounds. Using 2,000 pounds for simplicity, that means every ton takes up 27 cubic feet, so 151.1 million tons would take up 4,079.7 million cubic feet. I calculated that Super Fill has a capacity of 1,306,800,000 cubic feet. Dividing that capacity by 4,079.7 million cubic feet yields a quotient of about 0.32 – it would only take about 4 months to fill Super Fill!

In answering this, I assumed that the final section in Figure 4, "MSW – Landfill" is the one that describes the waste that would end up in Super Fill. I would need to do more research to understand the difference between the three sections of Figure 4. I used the maximum possible weight of a cubic yard of compressed MSW because it made the calculations simple. Using a smaller weight per cubic yard, such as the minimum value of 1,700 pounds, or a median value of 1,850 pounds, would mean that the 151.1 million tons of waste would take up even more space, and fill Super Fill even faster.

4. Students share recommendations that will decrease landfill use. These could include decreasing landfill waste and increasing recycling and composting. Students may share solutions that can be done by individuals or large-scale changes to packaging and production.

#4 Sample Response 1:

Increase recycling, create community gardens with composting facilities, campaign for reducing/eliminating the use of plastic bags, organize collections for usable and worn clothing, distribute reusable shopping bags, replace grocery plastic bags with biodegradable ones, organize events at school that raise awareness about trash. For example, ask students to carry their waste or keep track of the type and amount of waste they generate during a one-week period, and award a prize to the person with the least amount.

#4 Sample Response 2:

Some recommendations I'd make is that we need to be better consumers of materials. I've heard other countries produce a lot less waste. They take their own bags with them to shop and bring their own containers to refill, as opposed to buying a new container when an item runs out, like shampoo or drinks. Increasing recycling efforts would help, but from the data in this problem it looks like we have hit a plateau. I'm not sure what it would take to motivate folks to do more.

#4 Sample Response 3:

To extend the time to fill the landfill, you would have to slow the rate at which people produce landfill waste. Individuals and companies could be incentivized and educated to reduce, recycle, and reuse.

#4 Sample Response 4:

To extend the time it would take to fill the landfill, we should work to find ways to increase our rate of recycling, composting, and energy-recovery combustion. Making these clean waste management methods a national priority will have a significant environmental impact in the long run.

Name: _____

Super Fill

Every year, most of the trash produced by over 330 million people in the United States ends up in landfills. In order to meet the need for additional landfill space, suppose the United States designated 200 acres of land for a new landfill (called Super Fill) on land where local regulation requirements limit the depth of the pit to 150 feet. The purpose of this task is to estimate the time it will take to completely fill up the landfill.

1. Given the dimensions of your classroom, approximately how many of your classrooms would fit in the Super Fill Landfill? Show your calculations.
2. Municipal Solid Waste (MSW), commonly known as trash or garbage in the United States, is waste that consists of everyday items discarded by the public. Use the information provided on the Resource Page to create a mathematical model that could predict:
 - a. The amount of MSW produced in the US during this year.
 - b. Using that model, determine the amount of MSW sent to landfills during this year.
 - c. What are some assumptions you made when creating this model and determining the amount of MSW sent to landfills this year?
3. If all the municipal solid waste in the US this year was destined to the new landfill Super Fill, approximately how long will it take until the landfill is completely full? What assumptions did you make in answering question 3? What impact do you think these assumptions have on your estimate?
4. What recommendations would you make to extend the time it would take to fill the landfill?

Landfill Task

Resource Page

Figure 1: Municipal Solid Waste (MSW) Rates, 1960 to 2015¹

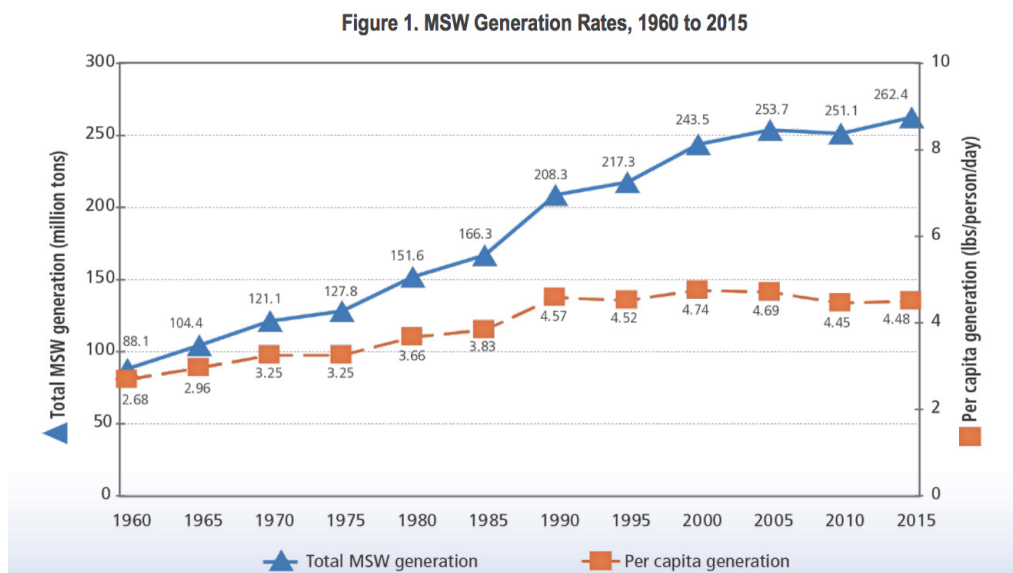
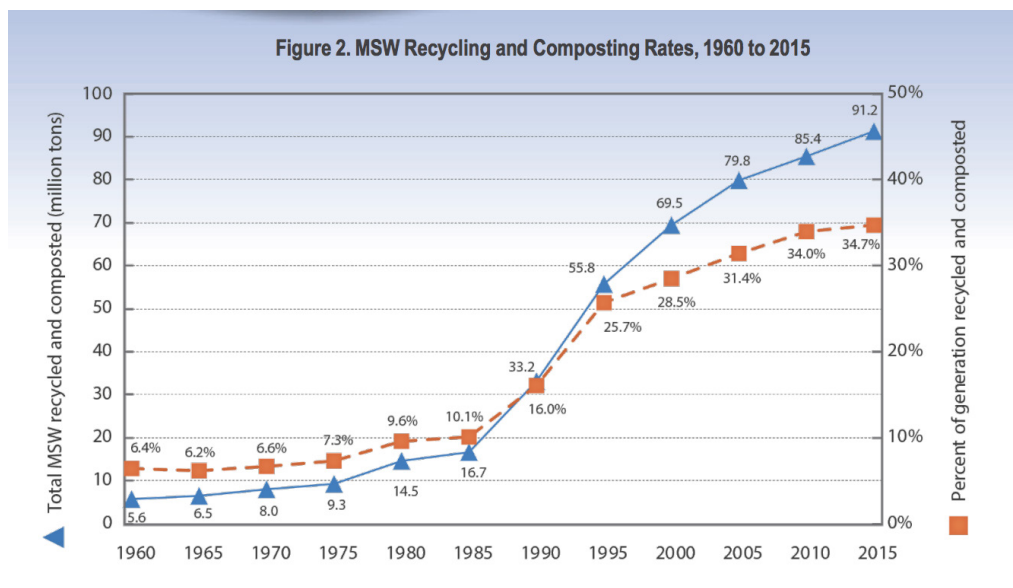


Figure 2: Municipal Solid Waste (MSW) Recycling and Composting Rates, 1960 to 2015¹



Sources:

- https://www.epa.gov/sites/production/files/2018-07/documents/2015_smm_msw_factsheet_07242018_fnl_508_002.pdf
- Volume-to-Weight Conversion Factors U.S. Environmental Protection Agency Office of Resource Conservation and Recovery April 2016

Figure 3: Management of Municipal Solid Waste (MSW) in the United States, 2015¹

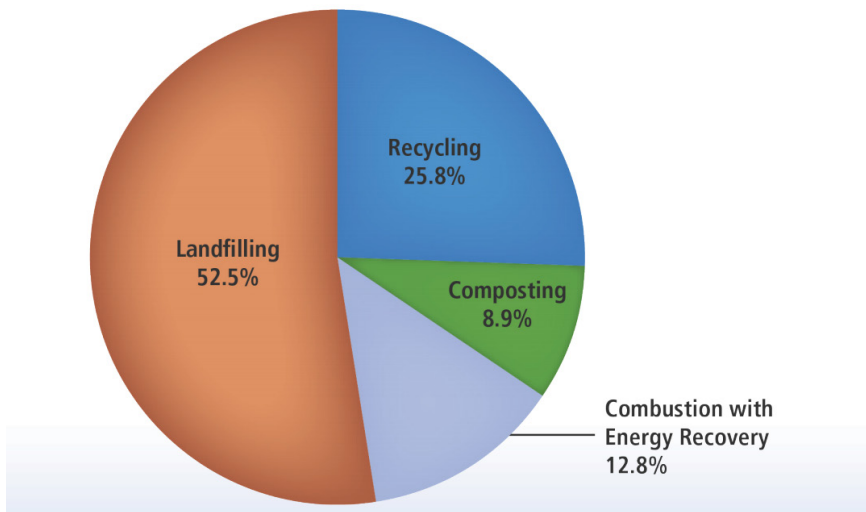


Figure 4: Standard Volume-to-Weight Conversion Factors²

Category	Recyclable Materials	Volume	Estimated Weight (lbs)	Source
Municipal Solid Waste	MSW- Commercial			
	Commercial- dry waste	cubic yard	56-73	16, 8
	Commercial- all waste, uncompacted	cubic yard	138	21
	Mixed MSW- Residential, Institutional, Commercial			
	Uncompacted	cubic yard	250-300	14
	Compacted	cubic yard	400-700	14
	Mixed MSW- Multi-family uncompacted	cubic yard	95	21
	MSW- Landfill			
	Compacted- MSW Small Landfill with Best Management Practices	cubic yard	1,200- 1,700	17
	Compacted- MSW Large Landfill with Best Management Practices	cubic yard	1,700- 2,000	17

Sources:

1. https://www.epa.gov/sites/production/files/2018-07/documents/2015_smm_msw_factsheet_07242018_fnl_508_002.pdf

2. [Volume-to-Weight Conversion Factors U.S. Environmental Protection Agency Office of Resource Conservation and Recovery April 2016](#)