Comparing Fractions Game

Authors: Illustrative Mathematics and Adapting Materials Project

GRADE LEVEL Third

IN THE STANDARDS 3.NF.A.3

WHAT WE LIKE ABOUT THIS TASK

Mathematically:
- Allows students to compare fractions by using common numerators, common denominators, or benchmarks.
- Encourages students to reason about the size of fractions (3.NF.A.3).

In the classroom:
- Provides resources to allow students to compare fractions with or without a visual representation of the fractions the mathematics explicit.
- Prompts students to share their developing thinking and understanding.
- Captures student attention by using an engaging context.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:
- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.

MAKING THE SHIFTS[1]

Focus	Belongs to the Major Work[2] of third grade
Coherence	Sets students up for work in grade 4 to compare a broader range of fractions with different numerators and denominators
Rigor[3]	Conceptual Understanding: primary in this task.
	Procedural Skill and Fluency: not targeted in this task.
	Application: not targeted in this task.

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Three.
[3]Taks will often target only one aspect of rigor.

INSTRUCTIONAL ROUTINE

The steps in this routine are adapted from the Principles for the Design of Mathematics Curricula: Promoting Language and Content Development.

Instructional Routine

Engage students in the Compare and Connect Mathematical Language Routine. This will support students as they identify, compare, and contrast differing mathematical approaches and representations.

Before engaging in this task (game), use the Compare and Connect strategy: "Which One Doesn't Belong?" as a warm-up.

Provide students with the following fractions:

- $^1/_6$
- $^3/_4$
- $^4/_8$
- $^2/_3$

Ask students to determine a set of three that excludes one of the fractions. Students should be supported in attending to mathematical language while describing the attributes which allow three of the fractions to be in a set. Here are a few examples:

- $^1/_6$ doesn't belong because it is a unit fraction.
- $^4/_8$ doesn't belong because it's the only one that equals $^1/_2$.
- $^1/_6$ doesn't belong because it's smaller than $^1/_2$.
- $^2/_3$ doesn't belong because the denominator is odd.

Remind the students of the language they used in this warm up. Listen for this language while students are playing the game.

LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Unit fraction
- Numerator
- Denominator
- Equal
- Greater than
- Less than

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work.

ADDITIONAL THOUGHTS

This task was created as part of the Adapting Materials Project. The goal of this project was to create a replicable process for teachers intending to adapt their materials, and to help create an environment of trust, where teachers felt empowered with the knowledge, confidence, and authority to change their own instructional materials in a way that better reflects the standards. To learn more about the work of these districts, read the "Collaborative Learning and Updating Materials" article from Aligned or access the complete case study.

For more information on the specific expectations for students working with fractions in grade 3, including the need for fractions to be referring to the same whole, read pages 3–5 in the progression document, *Number and Operations–Fractions*, available at www.achievethecore.org/progressions.

3.NF Comparing Fractions Game

Task

This activity is designed for pairs of students. They will require a set of cards (which are supplied as an attached resource, after the commentary). The goal is to compare the two fractions appearing on each card, determine if they are equivalent and, if not, which is larger. Instructions for the activity are as follows:

a. Students go through the following steps with the fraction cards:

i. The pair of students select a card.

ii. Each student individually decides whether the fractions are equal and, if not, which is greater. Then they show each other their choice.

iii. If the partners agree, they take turns explaining their reasoning. If they disagree, they discuss until reaching a consensus.

iv. Repeat 1 through 3 with a new card.

b. After 10 rounds, each pair records observations about what methods they used to compare the fractions.

IM Commentary

The goal of this task is to compare fractions with a focus on providing explanations that demonstrate deep conceptual understanding. The main comparison techniques to look and listen for (for non equivalent fractions) are

- Using a common numerator (e.g. thirds are bigger than fourths, so two thirds are bigger than two fourths).
- Using a common denominator (e.g. 1/5 is less than 2/5 because there is an extra fifth

of a whole in 2/5).

- Using a whole as a benchmark (e.g. 2/3 is less than 5/4 because 2/3 is less than 3/3 or one whole and 5/4 is larger than 4/4 or one whole).

Concerning the third method, using a benchmark to compare two fractions is explicitly mentioned in 4.NF.2. Because the meaning of a whole is fundamental to understanding a fraction, it is appropriate to use 1 as a benchmark in the third grade. The teacher may, however, choose to remove those cards containing pairs of fractions where one is larger than a whole and the other is less than a whole.

Two different sets of cards are provided as attachments, one with a picture of the two fractions being compared and one without. The pictures allow students to make a visual comparison of the fractions which is important. However, the teacher may wish for students to provide these pictures as one means of explaining their decision. Similarly, the teacher may also wish to remove cards having equivalent fractions if the goal is to work exclusively on inequalities.

Question 2 is intended to motivate a classroom discussion after students have completed the activity. In order to better prepare them for this, the teacher may suggest that students think about the strategies they are using to compare fractions as they play the game. Some methods, like drawing pictures or using fraction strips, can be used for all of the pairs of fractions. But other methods such as looking for a common numerator and common denominator are conceptually important and the teacher will want to make sure that these methods are discussed.

There are three attached resources (press ''Show attached resources'' link just below the commentary):

- Less than, equal to, greater than symbols (one of each symbol for each student)
- Set of cards without fraction pictures (one set per pair of students)
- Set of cards with fraction pictures (one set per pair of students)

Solution

a. There are four types of fractions which students will need to compare:

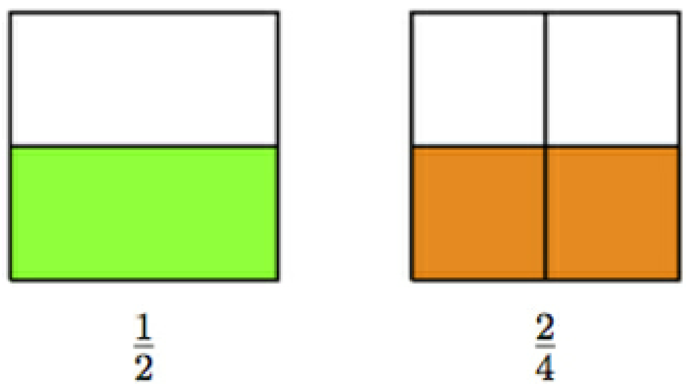
> i. Fractions having the same numerator. The denominator tells us how many equal pieces are in the whole, determining the size of each piece, and the numerator tells us how many of those pieces we have. For example, to compare $\frac{2}{3}$ and $\frac{2}{5}$, there are more fifths in the whole than thirds so fifths are smaller. This

means that $\frac{2}{5} < \frac{2}{3}$.

ii. Fractions having the same denominator. For example, we see that $\frac{2}{3} > \frac{1}{3}$ because $\frac{2}{3}$ is $\frac{1}{3}$ and an additional third so it is bigger. This relates to the reasoning described in the common numerator situation: the denominator tells us there are the same number of pieces in the whole, however one fraction has more of those pieces than the other.

iii. One fraction is less than 1 and the other fraction is larger than 1. For example, $\frac{2}{3} < \frac{3}{2}$ because $\frac{2}{3}$ is one third short of a whole while $\frac{3}{2}$ is an entire whole with an additional half added.

iv. Simple equivalent fractions such as $\frac{1}{2}$ and $\frac{2}{4}$. One way to show that these fractions represent the same quantity is with a picture:



Here the two large squares are equally sized wholes which have been divided into two equal parts (on the left) and four equal parts (on the right). The same fraction of the whole is shaded in each picture so 1/2 is equivalent to 2/4.

b. There are many important lessons to be learned from comparing these fractions including:

- If I draw a picture of the two fractions, the larger fraction will have more shaded than the smaller fraction. If the two fractions are equal, the same amount will be shaded in both.
- The denominator tells me how many pieces to cut my whole into. When the whole is cut into more pieces, the pieces are smaller (this is why 1/3 is less than 1/2).

- The numerator tells me how many equal sized pieces I have. So 3/5 is more than 2/5 because I have one extra piece.
- Fractions are built from the unit fractions so it is important to understand and be able to represent the unit fractions.
- If using the fraction cards with pictures, equal sized wholes are important when comparing fractions.
- Equivalent fractions have different sized pieces, but the same total amount shaded.
- When the numerator is a bigger number than the denominator, the fraction is greater than one whole.
- When doing mathematics, patterns emerge. These patterns support students in making conjectures, supporting their reasoning, and proving mathematical claims.