Box of Clay

Sample task from achievethecore.org
Task by Illustrative Mathematics, annotation by Student Achievement Partners

GRADE LEVEL Fifth

IN THE STANDARDS 5.MD.C

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Centers on the fundamental volume concept of the unit cube (5.MD.C.3).
- Connects volume (5.MD.C) to previous multiplicative ideas of times-as-much.

In the Classroom:

- Gives students the opportunity to develop the method they will use to solve the problem (drawing boxes, using snap cubes, applying the volume formula) (MP5).
- Encourages students to talk about each other's thinking, in order to improve their mathematical understanding.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Elicits evidence of thinking both verbally and in written form.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.
- Provides students with support in negotiating written word problems through multiple reads and/or multi-modal interactions with the problem.

MAKING THE SHIFTS[1]

Focus	Belongs to the Major Work[2] of grade five
Coherence	Connects volume to multiplication and addition
Rigor[3]	Conceptual Understanding: primary in this task Procedural Skill and Fluency: secondary in this task Application: primary in this task

[1] For more information read Shifts for Mathematics.
[2] For more information, see Focus in Grade Five.
[3] Tasks will often target only one aspect of rigor.

For a direct link, go to: http://www.achievethecore.org/page/617/box-of-clay-task

INSTRUCTIONAL ROUTINE

Engage students in the Three Reads Instructional Routine. This routine is designed to develop students' ability to make sense of problems by deconstructing the process of reading mathematical situations. Over time, students will internalize this process, thereby creating a heuristic for reading and making sense of mathematical story problems.

The task is read three times. Each read has a different purpose.

> First read: teacher reads the task to the class and asks students to be thinking about what the task is about. Students offer responses: "This problem is about _____."
>
> Second read: problem is projected. One student reads task out loud and students follow along thinking about what is the question that is being asked. Students turn and talk to their partner and rephrase the question(s) in their own words. Questions are shared in the full group. Teacher records.
>
> Third read: problem is still projected. Students read problem out loud in their partner groups and begin to name the important information. Important information is shared out by students and recorded by the teacher.

From here, choose whether students will solve problem independently or with their partners.

LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Centimeter
- Twice
- Width
- Length
- Grams

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work

ADDITIONAL THOUGHTS

This task could be extended by adding a follow-up with larger numbers. That would enhance the connection to the fluency expectations in the Standards for multi-digit multiplication (5.NBT.B.5).

For more information about the expectations for volume in Grade 5, read pages 26 and 27 of the progression document, *K-5 Geometric Measurement*, available at www.achievethecore.org/progressions.

5.MD Box of Clay

Task

A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box has twice the height, three times the width, and the same length as the first box. How many grams of clay can it hold?

Commentary

This task provides an opportunity to compare the relative volumes of boxes in order to calculate the mass of clay required to fill them. These relative volumes can be calculated geometrically, filling the larger box with smaller boxes, or arithmetically using the given dimensions.

This purpose of this task is to help students understand what happens when you scale the dimensions of a right rectangular solid. At some point students need to understand that if you (for example) double the length, width, and height of a rectangular solid, then the volume increases by a factor of $2 \times 2 \times 2 = 8$. Before they get to the point of generalizing this phenomenon, they should think about the effects of scaling the different dimensions by different factors, as they do in this task.

This is a high-level instructional task for fifth graders, but it fits squarely within the scope of work that students are doing to understand the volume of right rectangular solids with whole-number side-lengths in fifth grade. Depending on how comfortable students are with visualizing rectangular solids, they might benefit from having snap-cubes or some other physical model to help them solve this problem.

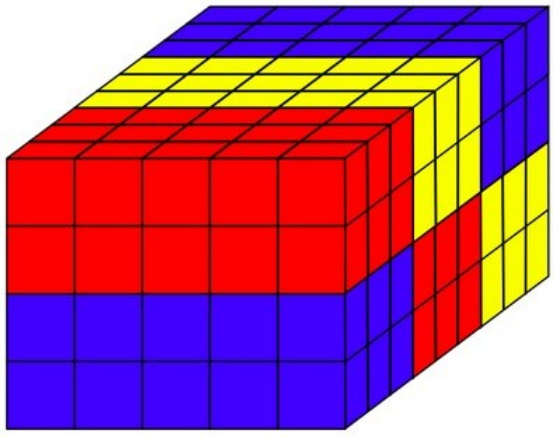
This task was adapted from problem #3 on the 2012 American Mathematics Competition (AMC) 12A Test. The responses to the multiple choice answers for the problem had the following distribution:

Choice	Answer	Percentage Answers
(A)	120	2.08
(B)	160	3.60
(C)	200	5.15
(D)	240	83.82
(E)	280	0.99
Omit	--	4.33

Of the 72,238 students who participated, 28,268 or 39% were in 12th grade, 34,124 or 47% were in 11th grade, 4,615 or 6% were in 10th grade, and the remainder were below 10th grade.

Solution: 1 Geometric visualization

The second box has 3 times the width and the same length as the first, smaller box. So we can fit three of the smaller boxes inside the second box to make one layer which will be 2 cm high. The second box is 2 times as high as the smaller one so we can add one more layer of three smaller boxes to fill the second box.



This means that it takes 6 small boxes to fill the large box so the large box holds six times as much as the small box.

Since the small box holds 40 grams of clay, the large box holds $6 \times 40 = 240$ grams of clay.

Solution: 2 Arithmetic comparison of volumes

The first box is 2 centimeters high, 3 centimeters wide, and 5 centimeters long so it has volume

$$2\text{cm} \times 3\text{cm} \times 5\text{cm} = 30 \text{ cubic centimeters}$$

and it holds 40 grams of clay. The second box is 4 centimeters high, 9 centimeters wide, and 5 centimeters long so its volume is

$$4\text{cm} \times 9\text{cm} \times 5\text{cm} = 180 \text{ cubic centimeters.}$$

Since the volume of the second box is $180 \div 30 = 6$ times bigger, it can hold 6 times as much clay. So the second box can hold $6 \times 40 = 240$ grams of clay.