A Graph of the Content Standards

A little over a year ago, I undertook to record some examples of structure in the *Common Core State Standards for Mathematics*' Standards for Mathematical Content. Some products of this study can be found online in an "atlas" document at http://tinyurl.com/ccssmstructure.

One product of the study, not published at the time, was a diagram showing connections between content standards. That graph is now included in the present document as a large-format PDF, followed by some grade-level snippets in 8 ½ × 11 format. The present document can also be accessed online at http://tinyurl.com/ccssmgraph.

Viewed from a suitable distance, the graph communicates a sense of flow and planning from kindergarten to high school. It also showcases connections. But it is almost overwhelming to look at, and it is not meant for mass communication. It was intended as a tool or reference for a thin layer of experts, including technological innovators, curriculum architects, assessment designers, learning scientists, math content experts, and future revisers of the Standards. These are the "super users"—people who need to engage with the content standards at the most detailed level to create products for the end user.

The graph isn't "final," yet I'm making it available in response to requests because it's unlikely I'll be able to do another iteration. So perhaps the graph might be thought of as a something that can evolve under anyone's intelligent direction. For example, currently the graph doesn't explicitly indicate the sort of structure captured by the cluster-level designations Major, Additional, and Supporting being used by the RTT assessment consortia. By encoding this information, renderings might be able to distinguish the standards visually based on those designations, products might behave intelligently based on those designations, etc.

Likewise, tags might be added to represent large-scale and local structures such as those discussed in the original atlas document. Then an end-user could ask to see, for example, the stream of fraction equivalence in the standards, along with its "nearest neighbors." Web-based applications could render this information and make the standards navigable by these and other structures. Other products could take these structures into account directly where helpful.

Another possible step would be to "roll up" the diagram to the cluster level, and then to the grade+domain level. The results might be simpler and more useful as mass communication about the Standards. Such an exercise might also reveal structures worth writing about.

Instead of always dealing with the entire graph, it might be useful to display certain subgraphs, such as various subgraphs generated by the set of all fraction vertices. One might portray the fraction progression as a "trunk," with roots in K–2, certain vines clinging to the trunk in 3–6, and branches spreading up into higher grades. A roll-up of such a subgraph to the cluster or domain level might also be useful for purposes of presentation or discussion. (Or it might be found that “domain-down” analysis adds more value, such as the major ties diagram for NBT, NF, and OA in the atlas document).

A longer-term goal might be to allow the graph to evolve in accordance with actual student performance data. Connections between vertices could be weighted empirically, in the manner of a Bayesian network. One might run a PageRank type algorithm and perhaps reveal empirically where intricate, difficult, and necessary things are prerequisites for intricate, difficult, and necessary things. A diagram encoded with such information could highlight these risky bits and support enhanced functionality around them in applications.

The graph is thus a tool that might be applied in many different ways, and tested and revised by its various applications.

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Further notes:

Grain size. Grain size is not uniform in CCSS, and it is not uniform in the graph either. Usually, a vertex in the graph represents a single standard. But sometimes a vertex represents a single lettered item belonging to a single standard. More rarely, a vertex is several standards or some larger group. In general, I chose to work at a very small grain size, so that rolling up would be trivial.

Arrows. An arrow from vertex X to vertex Y might be said to represent a rough judgment to the effect that a student who cannot meet the demand of X is not likely to be able to meet the demand of Y either. I only included “nearest neighbor” edges of this kind; for example, I did not draw an arrow directly from rote counting to solving quadratic equations.

Arrows from one grade into the next grade often document steps in a progression designed into the Standards. The arrow 1.NBT.2 $\rightarrow$ 2.NBT.1 is an example.

Undirected edges. In cases where two vertices “seemed connected,” yet it was not easy to say instinctively whether there was a meaningful precedence relation between them, I connected them with an undirected edge. In such cases I often felt that lessons or problem sets should treat the indicated content together.

Importantly, however, that can be true for directed arrows as well. Just because one vertex points to another, it doesn’t always mean it would be a good idea to separate the teaching and learning of those two things from one another in time. The human neural net is good at bootstrapping, and efficient curricula will leverage this rather than serializing everything. (The A-B-C groups discussed below represent a form of weak sequencing.)

A common example of this is that it is sometimes artificial or demotivating to teach computation procedures in the abstract before teaching the application of those procedures to word problems. On the other hand, there are probably cases where a teacher might wisely decide to work on a procedure in the abstract before turning to applications—or use a cycle in which a real-world problem creates a need for some new mathematics, which then becomes the abstract focus for a while. The optimal approach in any given case may depend on the nature of the particular topics at hand, even on the style of the teacher and students doing the work.

A related point is that arrows are not signposts saying "Go Here Next." For one thing, there are often several arrows emerging from a single vertex—one cannot possibly do all of those postcursors "next." Even when there is a single arrow pointing outward from a given vertex, the vertex it points to might have other precursors to consider. And as we noted above, even a purely linear chain such as $X \rightarrow Y \rightarrow Z$ might not be best approached as simply "this-then-this-then-this."

In some sense, anything might be said to connect to everything. But in general, I tried to be sparing in the drawing of both arrows and edges.

Groups. The A-B-C groups in the graph were an attempt to cluster the expectations within a grade level. Thus, during a single year of instruction, the expectations in group A might be met before those in group B, and those in group B might be met before group C. These groupings are not recommendations. Another graph might well be produced from this one by removing the group distinctions and reverting to a year-by-year timescale. That is another sort of "rolling up."

Relationship to concept maps, ontologies, etc. The graph is a very restricted object. Its vertices are standards, or parts of standards—that is to say, expectations from CCSSM. Therefore, this is not a map of mathematical topics—because a topic is just a noun, while an expectation is a complete sentence. Nor is this an ontology. Nodes in an ontology would include not only policy expectations like CCSS, but also mathematical concepts (more nouns), relationships between those concepts (e.g., "is a"), mathematical practices and like proficiencies, and so on. Nor finally is this by itself a platform for learning applications, as such a thing would include among its nodes such things as mathematical tasks, instructional videos, professional development modules, or other learning objects. However, the graph is one possible input to all such things.

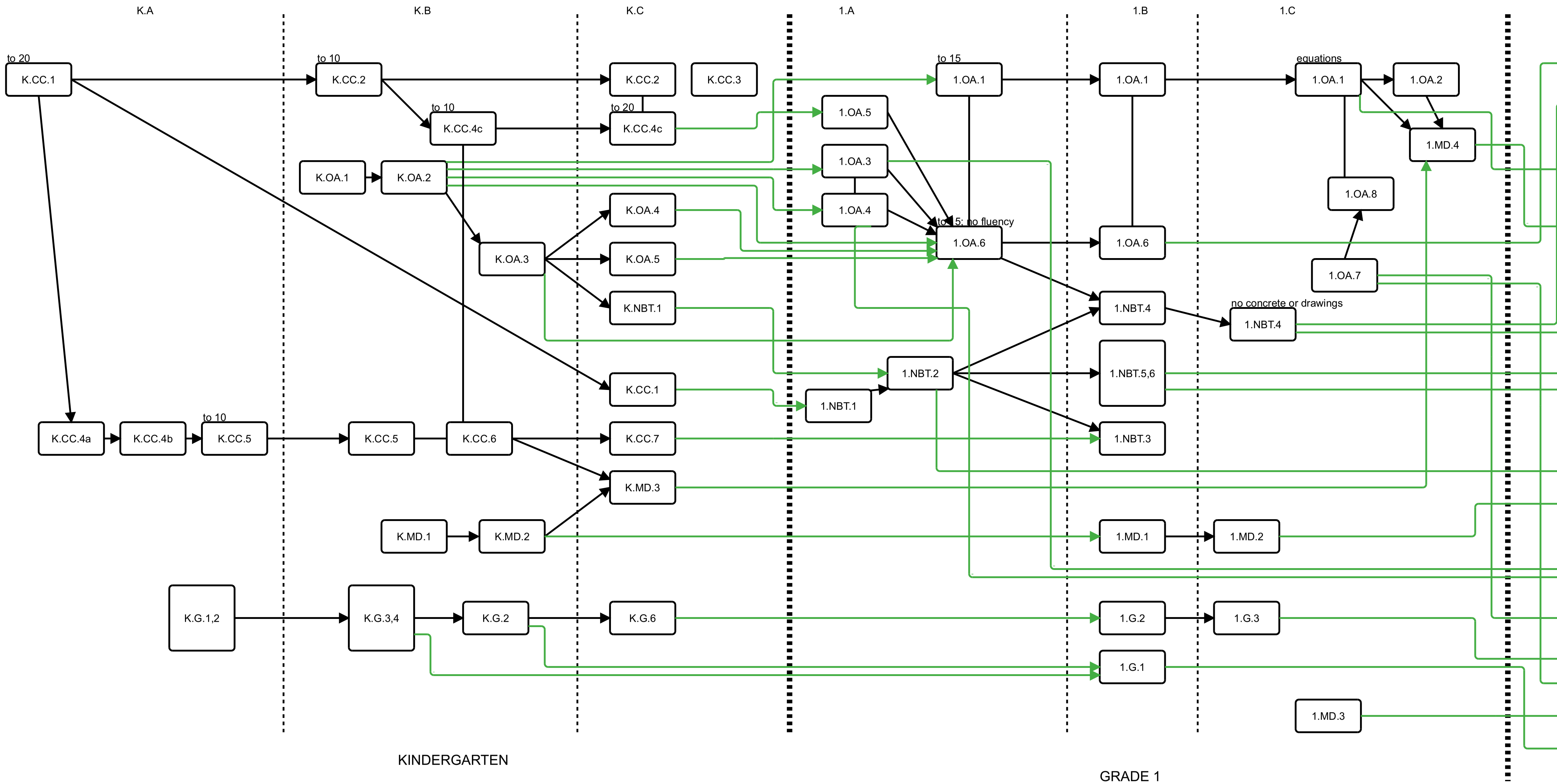
Errors. Two types of possible error are errors in the graph itself and errors in the accompanying spreadsheet data that encodes its vertices and edges, online at http://tinyurl.com/ccssmgraphdata. As for spreadsheet errors, QA was done on the spreadsheet but bugs are inevitable.

As for the graph itself, is it correct in the claims that each edge or non-edge makes about the Standards? These claims number in the tens of thousands. In the absence of data, no two experts are likely to agree on all of them, so there is no sense asking about the correctness of the graph in an absolute sense. However, even in a relative sense, because this task was so complex, the question remains whether even by my own lights I have drawn the edges exactly as I would have them. Missing edges are likely to be more common than extra edges or faulty wiring. In any case, this is another reason to think of the graph as a living document.

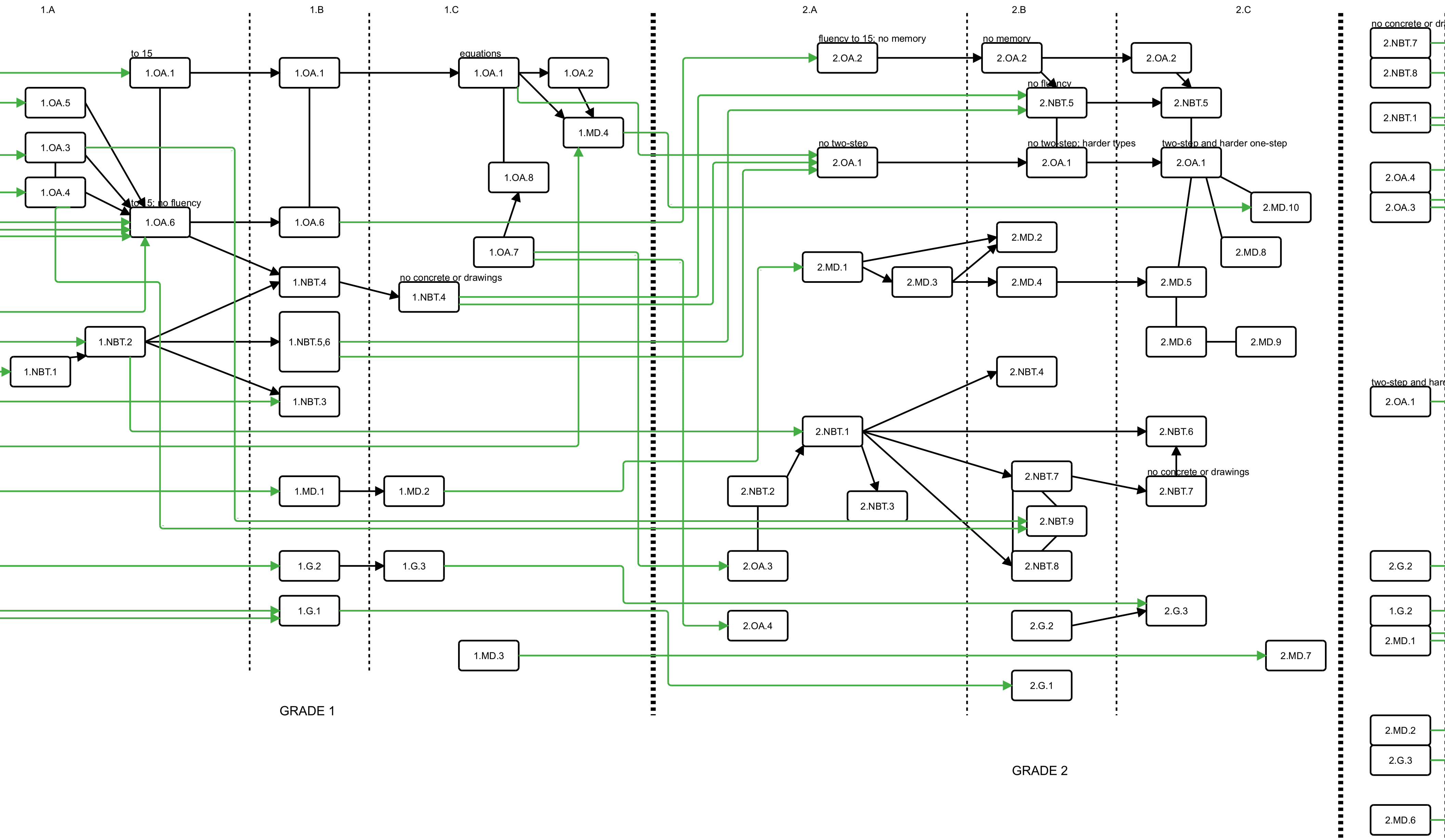
Parallel efforts. A parallel effort to record connections between content standards has been carried out at the University of California at Los Angeles, under the leadership of Heather Dallas, Executive Director and Lecturer in the UCLA Curtis Center for Mathematics and Teaching, and Markus Iseli, Senior Researcher in the UCLA Center for Research on Evaluation, Standards, and Student Testing. Interested parties may contact Dallas or Iseli for further information about this effort, which included classroom teachers among its staff. Other similar efforts may also exist.

Jason Zimba
June 7, 2012

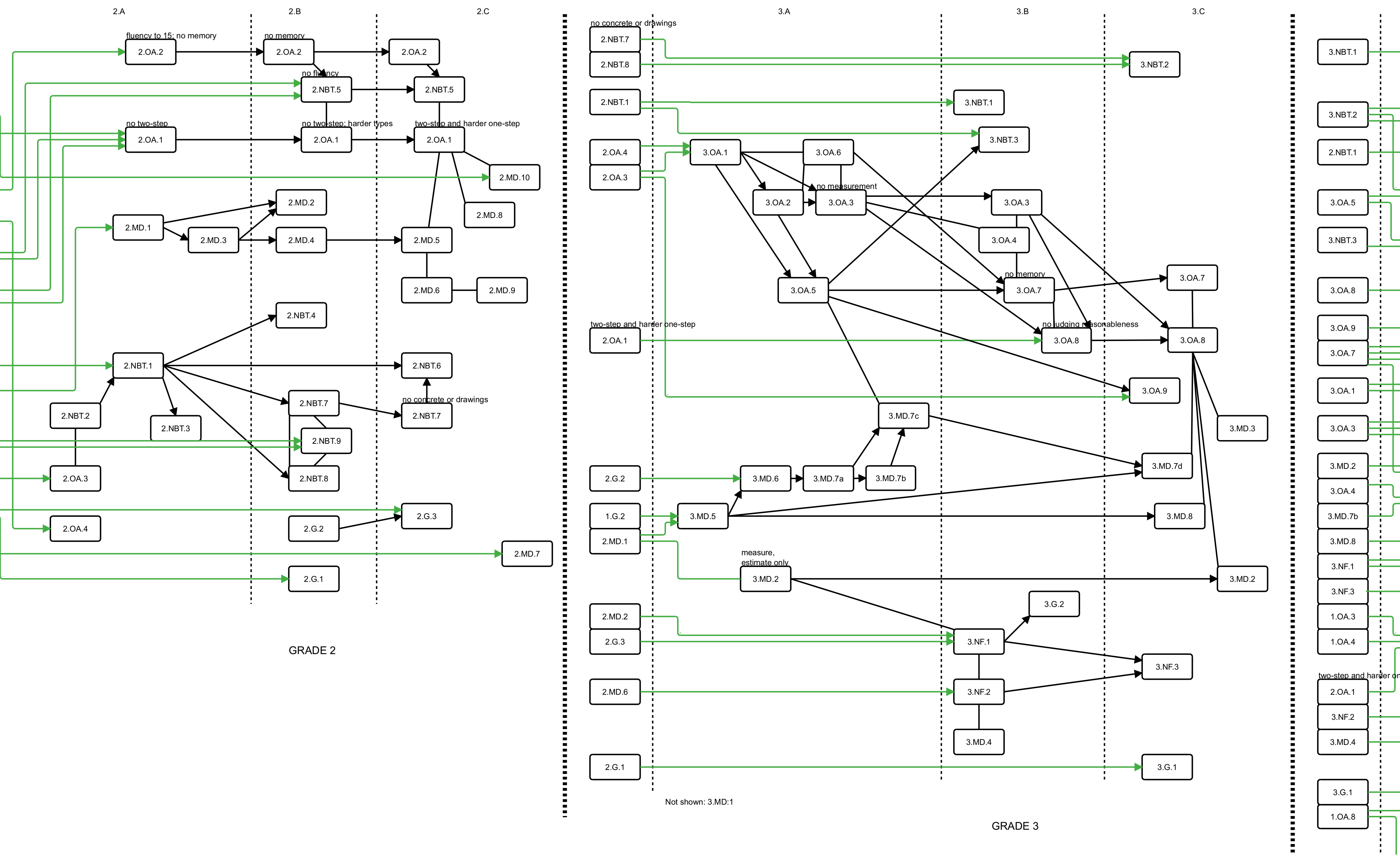

KINDERGARTEN
GRADE 1
GRADE 2
GRADE 3
GRADE 4
GRADE 5
GRADE 6
GRADE 7
GRADE 8

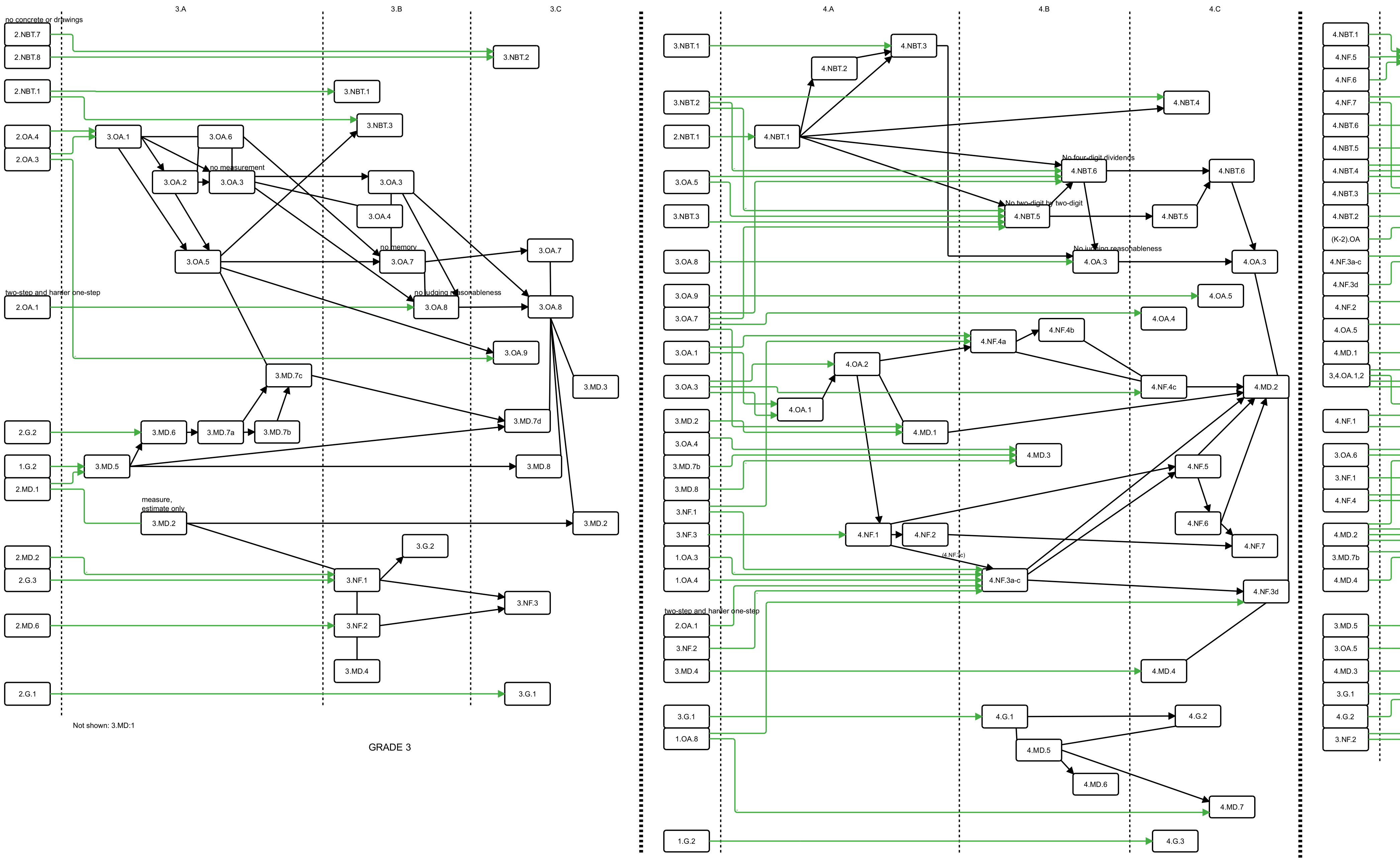


K.A
K.B
K.C
1.A
1.B
1.C
to 20
K.CC.1
to 10
K.CC.2
K.CC.2
K.CC.3
to 15
1.OA.1
1.OA.1
equations
1.OA.1
1.OA.2
to 10
K.CC.4c
to 20
K.CC.4c
1.OA.5
1.MD.4
K.OA.1
K.OA.2
1.OA.3
1.OA.8
K.OA.4
1.OA.4
to 15: no fluency
1.OA.6
1.OA.6
K.OA.3
K.OA.5
1.OA.7
K.NBT.1
1.NBT.4
no concrete or drawings
1.NBT.4
1.NBT.2
1.NBT.5,6
K.CC.1
1.NBT.1
K.CC.4a
K.CC.4b
to 10
K.CC.5
K.CC.5
K.CC.6
K.CC.7
1.NBT.3
K.MD.3
K.MD.1
K.MD.2
1.MD.1
1.MD.2
K.G.1,2
K.G.3,4
K.G.2
K.G.6
1.G.2
1.G.3
1.G.1
1.MD.3
KINDERGARTEN
GRADE 1

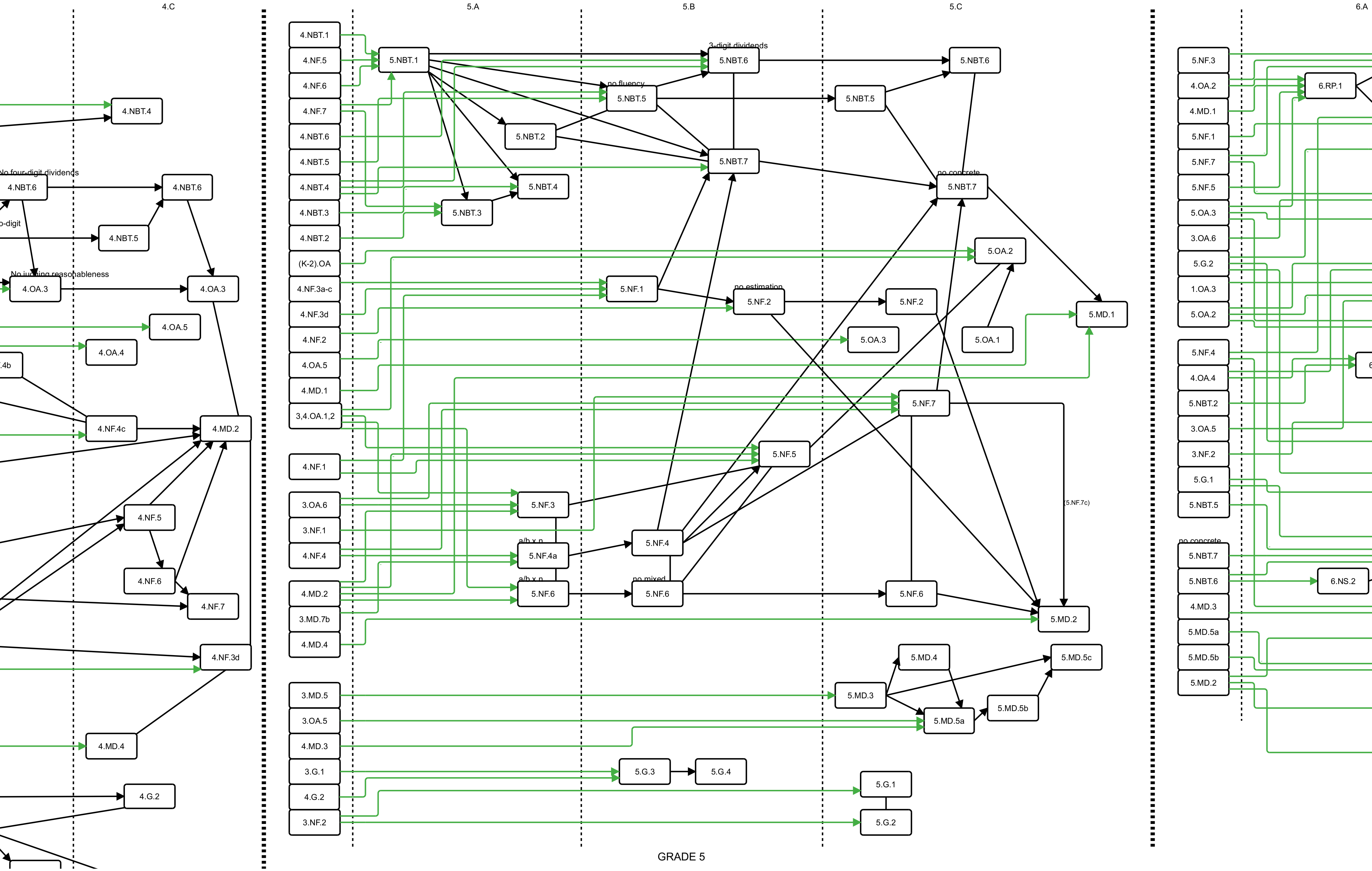


1.A
1.B
1.C
2.A
2.B
2.C
to 15
1.OA.1
1.OA.5
1.OA.3
1.OA.4
to 15; no fluency
1.OA.6
1.NBT.2
1.NBT.1
1.NBT.4
1.NBT.5,6
1.NBT.3
1.MD.1
1.G.2
1.G.1
equations
1.OA.2
1.MD.4
1.OA.8
1.OA.7
no concrete or drawings
1.MD.2
1.G.3
1.MD.3
GRADE 1
fluency to 15; no memory
2.OA.2
no two-step
2.OA.1
2.MD.1
2.MD.3
2.NBT.1
2.NBT.2
2.NBT.3
2.OA.3
2.OA.4
no memory
no fluency
2.NBT.5
no two-step; harder types
2.MD.2
2.MD.4
2.NBT.4
2.NBT.7
2.NBT.9
2.NBT.8
2.G.2
2.G.1
two-step and harder one-step
2.MD.10
2.MD.8
2.MD.5
2.MD.6
2.MD.9
2.NBT.6
no concrete or drawings
2.G.3
2.MD.7
GRADE 2
2.NBT.1
2.OA.4
2.OA.3
2.OA.1
1.G.2
2.MD.1
2.MD.2
2.MD.6

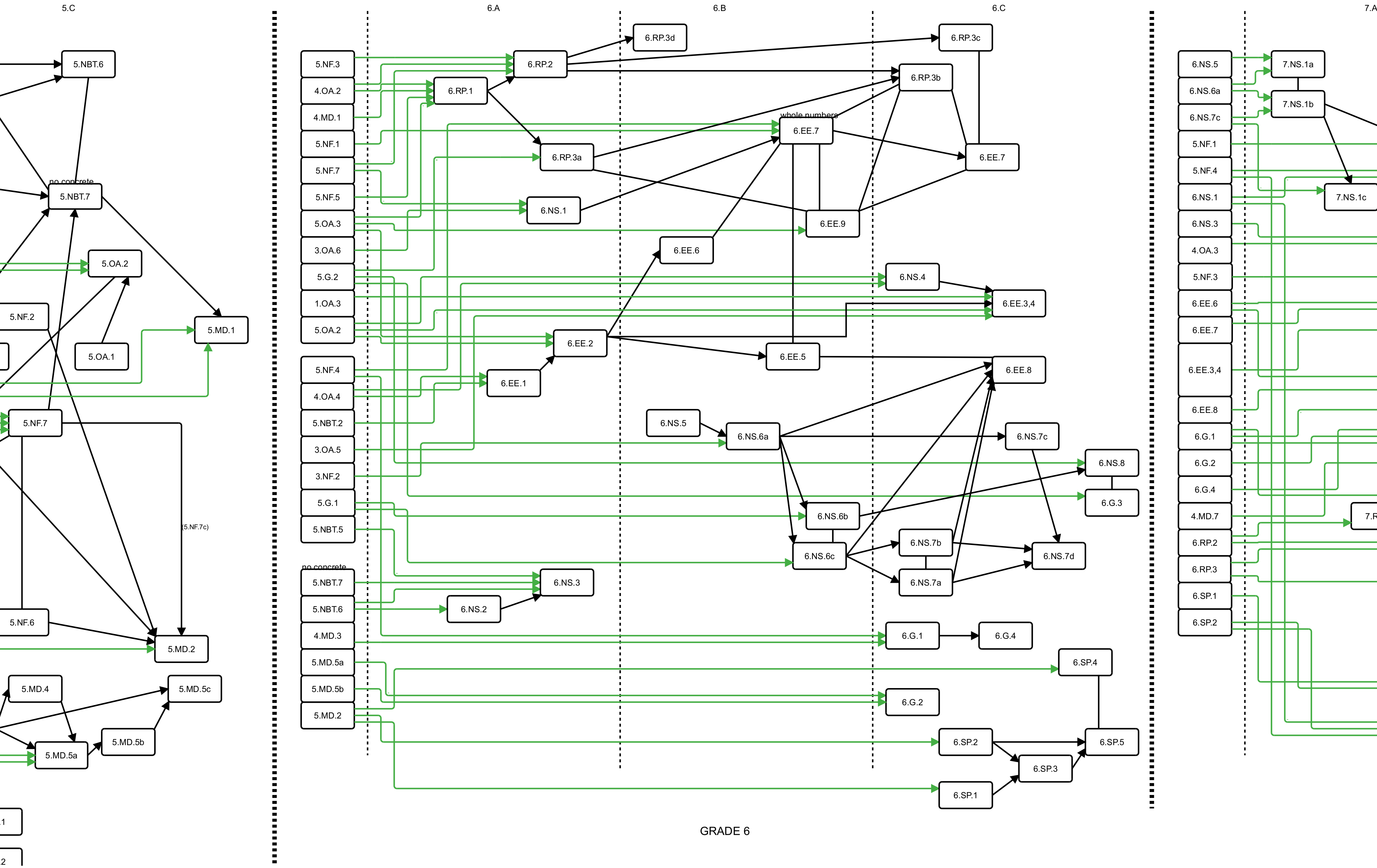

2.A
2.B
2.C
3.A
3.B
3.C
fluency to 15; no memory
no memory
no fluency
no two-step
no two-step; harder types
two-step and harder one-step
2.OA.2
2.NBT.5
2.OA.1
2.MD.10
2.MD.1
2.MD.2
2.MD.3
2.MD.4
2.MD.5
2.MD.8
2.MD.6
2.MD.9
2.NBT.4
2.NBT.1
2.NBT.6
2.NBT.2
2.NBT.3
2.NBT.7
no concrete or drawings
2.NBT.9
2.OA.3
2.NBT.8
2.OA.4
2.G.2
2.G.3
2.MD.7
2.G.1
GRADE 2
no concrete or drawings
2.NBT.7
2.NBT.8
3.NBT.2
2.NBT.1
3.NBT.1
3.NBT.3
2.OA.4
3.OA.1
3.OA.6
2.OA.3
no measurement
3.OA.2
3.OA.3
3.OA.4
no memory
3.OA.5
3.OA.7
two-step and harder one-step
2.OA.1
no judging reasonableness
3.OA.8
3.OA.9
3.MD.7c
3.MD.3
3.MD.7d
2.G.2
3.MD.6
3.MD.7a
3.MD.7b
1.G.2
3.MD.5
3.MD.8
2.MD.1
measure, estimate only
3.MD.2
3.G.2
2.MD.2
2.G.3
3.NF.1
3.NF.3
2.MD.6
3.NF.2
3.MD.4
2.G.1
3.G.1
Not shown: 3.MD:1
GRADE 3
3.NBT.1
3.NBT.2
2.NBT.1
3.OA.5
3.NBT.3
3.OA.8
3.OA.9
3.OA.7
3.OA.1
3.OA.3
3.MD.2
3.OA.4
3.MD.7b
3.MD.8
3.NF.1
3.NF.3
1.OA.3
1.OA.4
2.OA.1
3.NF.2
3.MD.4
3.G.1
1.OA.8

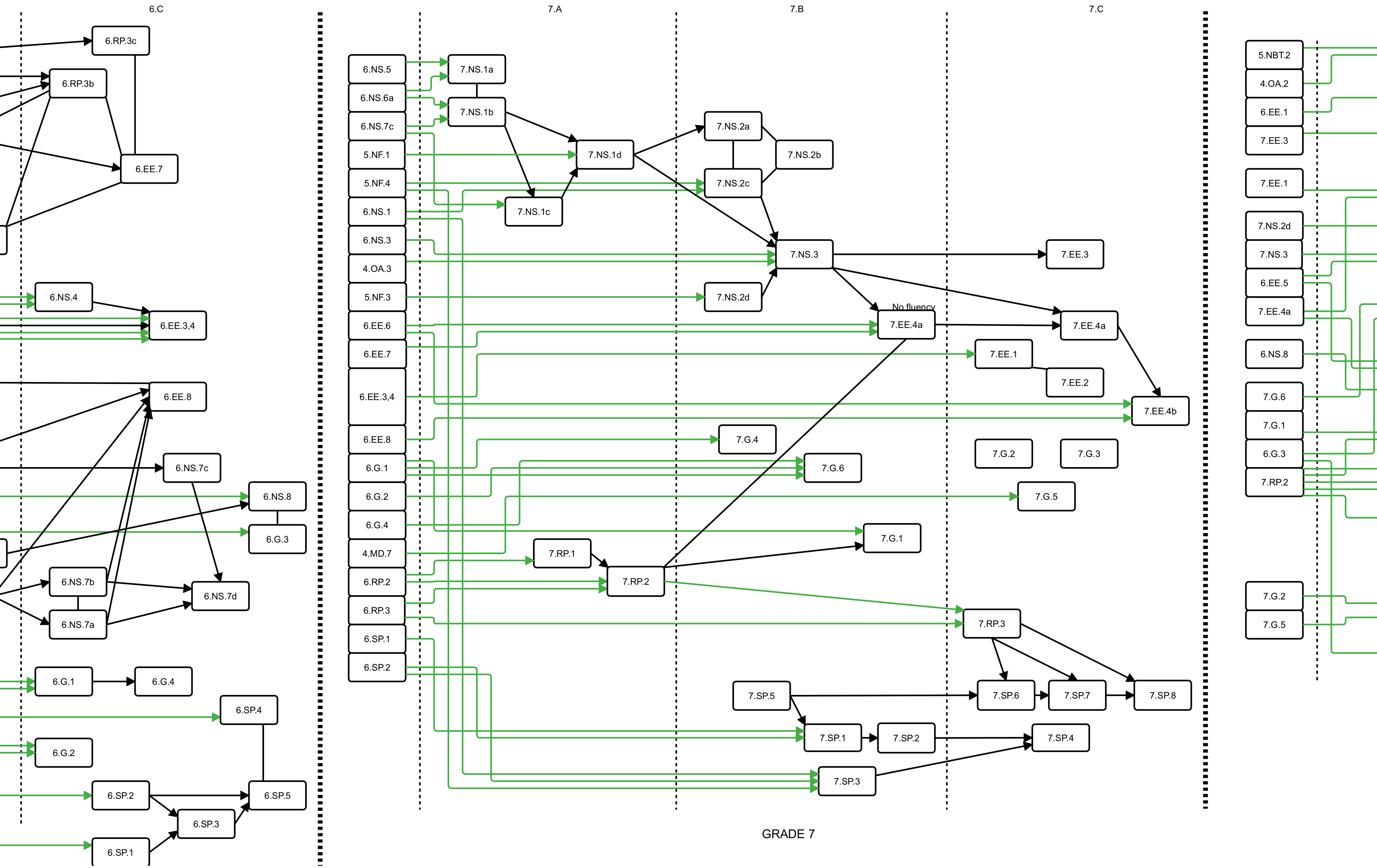

3.A
3.B
3.C
no concrete or drawings
2.NBT.7
2.NBT.8
3.NBT.2
2.NBT.1
3.NBT.1
3.NBT.3
2.OA.4
2.OA.3
3.OA.1
3.OA.6
3.OA.2
no measurement
3.OA.3
3.OA.4
3.OA.5
no memory
3.OA.7
two-step and harder one-step
2.OA.1
no judging reasonableness
3.OA.8
3.OA.9
3.MD.7c
3.MD.3
3.MD.7d
2.G.2
3.MD.6
3.MD.7a
3.MD.7b
1.G.2
3.MD.5
3.MD.8
2.MD.1
measure,
estimate only
3.MD.2
3.G.2
2.MD.2
2.G.3
3.NF.1
3.NF.3
2.MD.6
3.NF.2
3.MD.4
2.G.1
3.G.1
Not shown: 3.MD:1
GRADE 3
4.A
4.B
4.C
3.NBT.1
4.NBT.3
4.NBT.2
3.NBT.2
4.NBT.4
2.NBT.1
4.NBT.1
No four-digit dividends
4.NBT.6
3.OA.5
3.NBT.3
No two-digit by two-digit
4.NBT.5
3.OA.8
No judging reasonableness
4.OA.3
3.OA.9
4.OA.5
3.OA.7
4.OA.4
4.NF.4b
3.OA.1
4.NF.4a
4.OA.2
3.OA.3
4.NF.4c
4.MD.2
4.OA.1
3.MD.2
4.MD.1
3.OA.4
4.MD.3
3.MD.7b
4.NF.5
3.MD.8
3.NF.1
4.NF.6
3.NF.3
4.NF.1
4.NF.2
4.NF.7
1.OA.3
(4.NF.3c)
4.NF.3a-c
1.OA.4
4.NF.3d
two-step and harder one-step
2.OA.1
3.NF.2
3.MD.4
4.MD.4
3.G.1
4.G.1
4.G.2
1.OA.8
4.MD.5
4.MD.6
4.MD.7
1.G.2
4.G.3
4.NBT.1
4.NF.5
4.NF.6
4.NF.7
4.NBT.6
4.NBT.5
4.NBT.4
4.NBT.3
4.NBT.2
(K-2).OA
4.NF.3a-c
4.NF.3d
4.NF.2
4.OA.5
4.MD.1
3,4.OA.1,2
4.NF.1
3.OA.6
3.NF.1
4.NF.4
4.MD.2
3.MD.7b
4.MD.4
3.MD.5
3.OA.5
4.MD.3
3.G.1
4.G.2
3.NF.2

GRADE 4

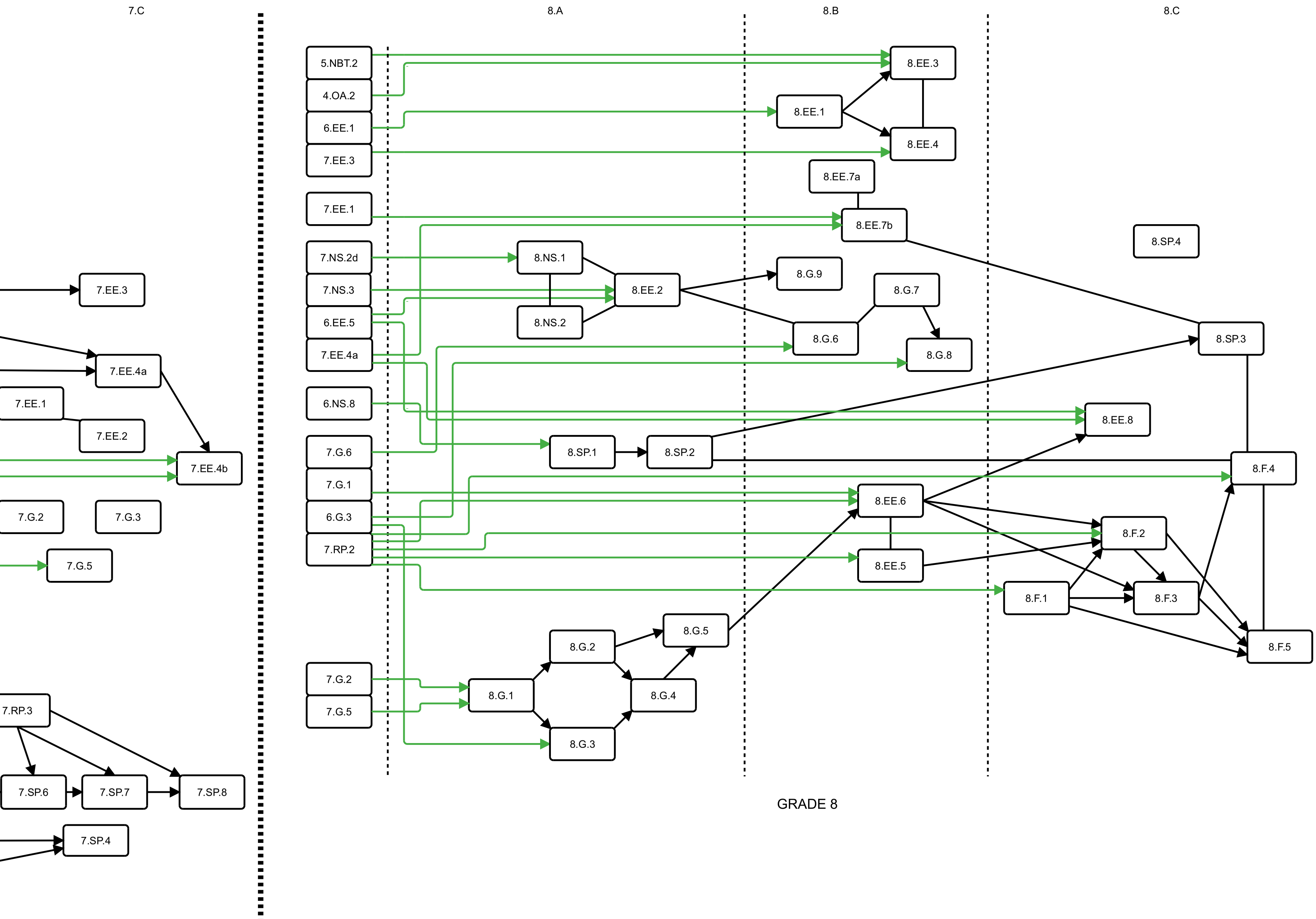


4.C
5.A
5.B
5.C
6.A
No four-digit dividends
No judging reasonableness
3-digit dividends
no fluency
no concrete
no estimation
a/b x n
no mixed
(5.NF.7c)
GRADE 5


5.C
6.A
6.B
6.C
7.A
GRADE 6
5.NBT.6
5.NBT.7
no concrete
5.OA.2
5.NF.2
5.MD.1
5.OA.1
5.NF.7
(5.NF.7c)
5.NF.6
5.MD.2
5.MD.4
5.MD.5c
5.MD.5a
5.MD.5b
5.NF.3
4.OA.2
4.MD.1
5.NF.1
5.NF.7
5.NF.5
5.OA.3
3.OA.6
5.G.2
1.OA.3
5.OA.2
5.NF.4
4.OA.4
5.NBT.2
3.OA.5
3.NF.2
5.G.1
5.NBT.5
no concrete
5.NBT.7
5.NBT.6
4.MD.3
5.MD.5a
5.MD.5b
5.MD.2
6.RP.1
6.RP.2
6.RP.3a
6.NS.1
6.EE.2
6.EE.1
6.NS.3
6.NS.2
6.RP.3d
6.EE.6
6.NS.5
6.NS.6a
6.EE.5
6.RP.3c
6.RP.3b
whole numbers
6.EE.7
6.EE.9
6.EE.7
6.NS.4
6.EE.3,4
6.EE.8
6.NS.6b
6.NS.6c
6.NS.7b
6.NS.7a
6.NS.7c
6.NS.7d
6.NS.8
6.G.3
6.G.1
6.G.4
6.SP.4
6.G.2
6.SP.2
6.SP.3
6.SP.5
6.SP.1
6.NS.5
6.NS.6a
6.NS.7c
5.NF.1
5.NF.4
6.NS.1
6.NS.3
4.OA.3
5.NF.3
6.EE.6
6.EE.7
6.EE.3,4
6.EE.8
6.G.1
6.G.2
6.G.4
4.MD.7
6.RP.2
6.RP.3
6.SP.1
6.SP.2
7.NS.1a
7.NS.1b
7.NS.1c


6.C
6.RP.3c
6.RP.3b
6.EE.7
6.NS.4
6.EE.3,4
6.EE.8
6.NS.7c
6.NS.8
6.G.3
6.NS.7b
6.NS.7d
6.NS.7a
6.G.1
6.G.4
6.SP.4
6.G.2
6.SP.2
6.SP.5
6.SP.3
6.SP.1
7.A
7.B
7.C
6.NS.5
6.NS.6a
6.NS.7c
5.NF.1
5.NF.4
6.NS.1
6.NS.3
4.OA.3
5.NF.3
6.EE.6
6.EE.7
6.EE.3,4
6.EE.8
6.G.1
6.G.2
6.G.4
4.MD.7
6.RP.2
6.RP.3
6.SP.1
6.SP.2
7.NS.1a
7.NS.1b
7.NS.1d
7.NS.1c
7.NS.2a
7.NS.2b
7.NS.2c
7.NS.3
7.NS.2d
No fluency
7.EE.4a
7.EE.3
7.EE.4a
7.EE.1
7.EE.2
7.EE.4b
7.G.4
7.G.6
7.G.2
7.G.3
7.G.5
7.G.1
7.RP.1
7.RP.2
7.RP.3
7.SP.5
7.SP.6
7.SP.7
7.SP.8
7.SP.1
7.SP.2
7.SP.4
7.SP.3
5.NBT.2
4.OA.2
6.EE.1
7.EE.3
7.EE.1
7.NS.2d
7.NS.3
6.EE.5
7.EE.4a
6.NS.8
7.G.6
7.G.1
6.G.3
7.RP.2
7.G.2
7.G.5

GRADE 7


7.C
8.A
8.B
8.C
7.EE.3
7.EE.4a
7.EE.1
7.EE.2
7.EE.4b
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7.G.3
7.G.5
7.RP.3
7.SP.6
7.SP.7
7.SP.8
7.SP.4
5.NBT.2
4.OA.2
6.EE.1
7.EE.3
7.EE.1
7.NS.2d
7.NS.3
6.EE.5
7.EE.4a
6.NS.8
7.G.6
7.G.1
6.G.3
7.RP.2
7.G.2
7.G.5
8.NS.1
8.NS.2
8.EE.2
8.SP.1
8.SP.2
8.G.1
8.G.2
8.G.3
8.G.4
8.G.5
8.EE.1
8.EE.3
8.EE.4
8.EE.7a
8.EE.7b
8.G.9
8.G.7
8.G.6
8.G.8
8.EE.6
8.EE.5
8.SP.4
8.SP.3
8.EE.8
8.F.4
8.F.2
8.F.1
8.F.3
8.F.5
GRADE 8