Identifying Proportional and Non-Proportional Relationships in Graphs

This lesson was adapted by Student Achievement Partners from Great Minds (as featured on EngageNY) and includes annotation by Student Achievement Partners

GRADE LEVEL Seventh

IN THE STANDARDS 7.RP.A.2A

WHAT WE LIKE ABOUT THIS SET OF LESSONS

Mathematically:

- Requires students to make connections between the different representations of a situation, and requires students to create tables from ratios in various formats.
- Requires students to use multiple methods and reasoning to determine whether a relationship is proportional; provides examples and non-examples.
- Allows students to draw conclusions based on mathematical evidence.

In the classroom:

- Offers an engaging exploration that connects students' procedural skill and conceptual understanding to real-life situations.
- Gives students the opportunity to work collaboratively in groups.
- Provides students with an opportunity to critique each other's work.
- Gives formal and informal opportunities for teachers to check for understanding.
- Includes a problem set that can be used for homework or for additional practice, as well as an exit ticket that summarizes the mathematics of the lesson.

This lesson was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practices in the following ways:

- Instruction and tasks allow for multimodal representations.
- Lesson provides opportunities for students to practice and refine their use of mathematical language.
- Lesson provides students with support in negotiating written word problems.
- Lesson provides opportunities for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Embedded sentence starters help students communicate their mathematical thinking.
- Debrief allows for the opportunity to point out metacognitive strategies.

MAKING THE SHIFTS[1]

Focus	Belongs to the Major Work[2] of seventh grade
Coherence	Builds on key understandings of ratios, rates, and unit rates (6.RP.A), and prior understanding of proportional relationships in grade 7
Rigor[3]	This lesson touches on all three aspects of rigor: conceptual understanding, procedural skill and fluency, and application.

[1] For more information read Shifts for Mathematics.
[2] For more information, see Focus in Grade Seven.
[3] Lessons may target one or more aspect(s) of rigor.

ADDITIONAL THOUGHTS

It's important to note that this sample lesson is the last of a 6-lesson series on "Proportional Relationships," which is part of a 22-lesson unit on Ratios and Proportional Relationships. This sample lesson lays a strong foundation for the work that is to come in the unit, but it is not intended for students to meet the full expectations of the standards through only this lesson. In subsequent lessons, students explore ratios and rates involving fractions, as well as ratios of scale drawings.

In this particular lesson, students work in groups to demonstrate their understanding of proportional relationships. The "art gallery" provides an opportunity for students to showcase their work and thinking, as well as to reflect on each other's representations and reasoning. This lesson could be strengthened by concluding the lesson with questions that ask students to discuss and compare various strategies for determining the proportionality of relationships and having them make connections between the different representations of the situations.

This activity has students think about the structure of the situations to determine whether a proportional relationship exists, as opposed to using the traditional method of "cross-multiplying" for solving proportions ($a/b = c/d$). For more insight on the grade-level concepts addressed in this lesson, read page 8 of the progression document, *Grade 6–7, Ratios and Proportional Relationships* available at http://www.achievethecore.org/progressions.

The structure of these lessons and the unit overall have some interesting aspects to highlight. Each unit is divided into topics (a set of lessons) that are connected to prior learning and also point to the lesson that follows in the learning progression. Within individual lessons, there are a number of components that add to their strength including variety in questioning techniques and frequent opportunities for students to debrief about their learning. Through the series of lessons, students have the opportunity to engage in all three aspects of rigor.

LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Ratio
- Proportional
- Origin
- Graph
- Line
- Quantities

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work.

For a direct link, go to: http://achievethecore.org/page/902/grade-7-engageny-lesson-rp-proportional-and-nonproportional-relationships

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Student Outcomes

- Students examine situations carefully to decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether all the points would fall on a line that passes through the origin.
- Students study examples of relationships that are not proportional as well as those that are.

Classwork

Today's Exploratory Challenge is an extension of Lesson 5. You will be working in groups to create a table and graph, and identify whether the two quantities are proportional to each other.

Preparation (5 minutes)

Place students in groups of two. Give each pair a worksheet and an envelope with 5 ratios. Have pairs assign one member the reader and one member the recorder.

- Instruct the reader to take out the contents of the envelope (located at the end of the lesson), and read the problem out loud.
- Instruct students to work together to arrange data into a table and on a graph. The recorder will be responsible for capturing the pairs thinking on the worksheet. Students will then determine if the ratios in the envelope are proportional to each other and record how they know on the worksheet.
- Be sure students work in pencil, so they can revise their work.

For English Language Learners: Support students in understanding the context of the problem their group is assigned. This could involve sharing a picture (e.g., a visual of yogurt sundae with toppings), acting out the situation (e.g., collecting money for selling books), showing a video (e.g., mold growing on bread over time), and/or having the problem translated into their native language.

Exploratory Challenge (30 minutes)

Give students 15 minutes to discuss the problem and record their responses. When completed, have students find another pair that had the same ratios. The two pairs with the same ratios will identify and discuss the differences of their posters.

For English Language Learners: Supply some sentence starters to support this discussion.

"I notice that we both _____"

"A difference in our work is _____"

"Can you explain why you _____?"

"How did you know _____?"

Allow pairs to go back and revise their original work.

If time allows, have students find another pair that had the same ratios and repeat the same process of discussing similarities and differences and returning to revise their final answer.

Closing (10 minutes)

Choose one student (from each of the four problems) to share their work (ideally with an overhead projector) with the whole class. Emphasize the connection between the graph and whether the ratios were proportional to each other. Encourage students to ask questions of the presenter.

Group 1

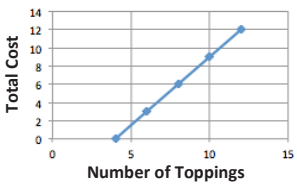
Problem:

A local frozen yogurt shop is known for their monster sundaes. Create a table, and then graph and explain if the quantities are proportional to each other.

Table:

Number of Toppings	Total Cost of Toppings ($)
4	0
6	3
8	6
10	9
12	12

Graph:



Explanation:

Although the points appear on a line, the quantities are not proportional to each other because the line does not go through the origin. Each topping does not have the same unit cost.

Group 2

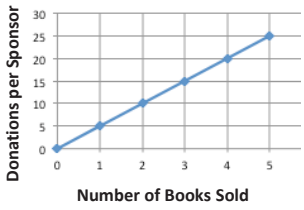
Problem:

The school library receives money for every book sold at the school's book fair. Create a table, and then graph and explain if the quantities are proportional to each other.

Table:

Number of Books Sold	Donations per Sponsor ($)
1	5
2	10
3	15
4	20
5	25

Graph:



Explanation:

The quantities are proportional to each other because the points appear on a line that goes through the origin. Each book sold brings in $5.00 no matter how many books are sold.

Group 3

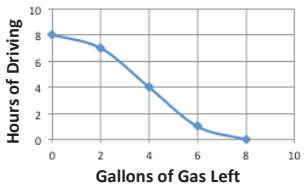
Problem:

Your uncle just bought a hybrid car and wants to take you and your siblings camping. Create a table, and then graph and explain if the quantities are proportional to each other.

Table:

Gallons of Gas Left in Tank	Hours of Driving
8	0
6	1
4	4
2	7
0	8

Graph:



Explanation:

The graph is not represented by a line passing through the origin, so the quantities are not proportional to each other. The number of gallons of gas varies depending on how fast or slow the car is driven.

Group 4

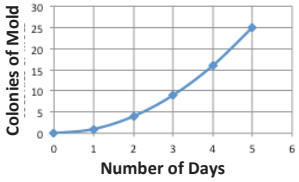
Problem:

For a science project, Eli decided to study colonies of mold. He observed a piece of bread that was molding. Create a table, and then graph and explain if the quantities are proportional to each other.

Table:

Number of Days	Colonies of Mold
1	1
2	4
3	9
4	16
5	25

Graph:



Explanation:

Although the graph looks as though it goes through the origin, the quantities are not proportional to each other because the points do not appear on a line. Each day does not produce the same amount of colonies as the other days.

Lesson Summary

Graphs of Proportional Relationships: The graph of two quantities that are proportional appear on a line that passes through the origin.

Exit Ticket (5 minutes)

Name _____ Date_____

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Exit Ticket

1. What are the characteristics of graphs that represent proportional relationships?

Exit Ticket Sample Solutions

1. What are the characteristics of the graphs that represent proportional relationships?

   Graphs appear on a line and go through the origin.

Problem Set Sample Solutions

Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios below represent the number of years to the amount of money in the savings account.

- After one year, the interest accumulated, and the total in Sally's account was $\$312$.
- After three years, the total was $\$340$. After six years, the total was $\$380$.
- After nine years, the total was $\$430$. After 12 years, the total amount in Sally's savings account was $\$480$.

Similar to today's lesson, create a table and a graph, and decide whether the amount of money accumulated and time elapsed are proportional to each other. Use your table and graph to support your reasoning.

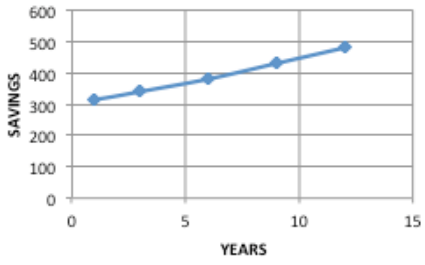
Problem:

Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping the money in the bank. The ratios below represent the number of years to the amount of money in the savings account. Create a table and a graph, and explain whether the quantities are proportional to each other.

Table:

Years	Savings ($)
1	312
3	340
6	380
9	430
12	480

Graph:



Explanation:

The graph is not proportional because although the data appears to be a line, it is not a line that goes through the origin. The amount of interest collected is not the same every year.

Ratios for Exploratory Challenge

Cut and place in labeled envelopes prior to instructional time.

Group 1	Group 2	Group 3	Group 4
A local frozen yogurt shop is known for its monster sundaes to be shared by a group. The ratios represent the number of toppings to the total cost of the toppings. Create a table, and then graph and explain if the quantities are proportional to each other.	The school library receives money for every book sold at the school's book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table, and then graph and explain if the quantities are proportional to each other.	Your uncle just bought a hybrid car and wants to take you and your siblings camping. The ratios represent the number of gallons of gas remaining to the number of hours of driving. Create a table, and then graph and explain if the quantities are proportional to each other.	For a science project, Eli decided to study colonies of mold. He observed a piece of bread that was molding. The ratios represent the number of days passed to the number of colonies of mold on the bread. Create a table, and then graph and explain if the quantities are proportional to each other.
4 to 0	1 to 5	8 to 0	1 to 1
$6 : 3$	2 to 10	After 1 hour of driving, there are 6 gallons of gas left in the tank.	2 to 4
$8 : 6$	The library received $\$15$ for selling 3 books.	$4 : 4$	$3 : 9$
The total cost of a 10-topping sundae is $\$9$.	$4 : 20$	2 to 7	$4 : 16$
12 to 12	$5 : 25$	$0 : 8$	Twenty-five colonies were found on the 5^{th} day.

Problem: A local frozen yogurt shop is known for their monster sundaes to be shared by a group. The ratios represent the number of toppings to total cost. Create a table then graph and explain if the quantities are proportional to each other or not.

Table:

Graph:

Proportional or Not? Explain:

Problem: The school library receives money for every book sold at the school's book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table then graph and explain if the quantities are proportional to each other or not.

Table:

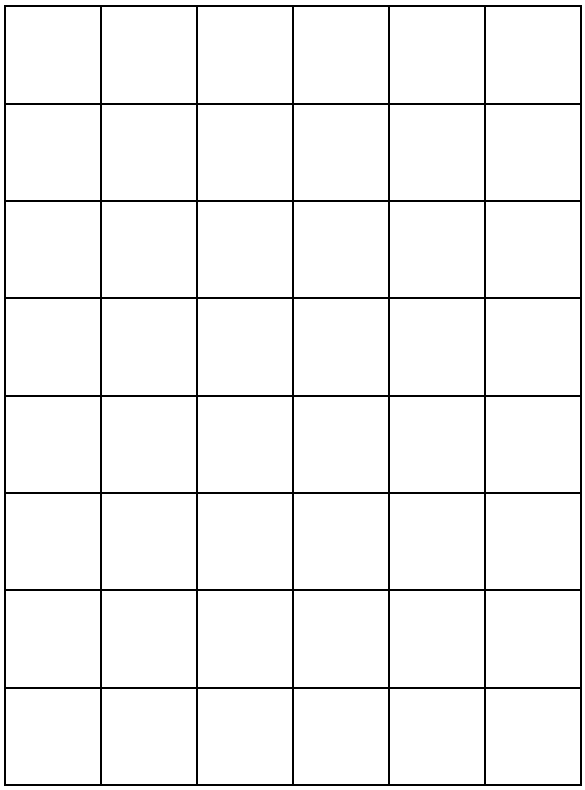
Graph:

Proportional or Not? Explain:

Problem: Your uncle just bought a hybrid car and wants to take you and your sibling camping. The ratios represent the number of gallons remaining to hours of driving. Create a table then graph and explain if the quantities are proportional to each other or not.

Table:

Graph:



Proportional or Not? Explain:

Problem: For a science project Eli decided to study colonies of mold. He observes a piece of bread that was molding. The ratios represent the number of days passed to colonies of mold on the bread. Create a table then graph and explain if the quantities are proportional to each other or not.

Table:

Graph:

Proportional or Not? Explain:

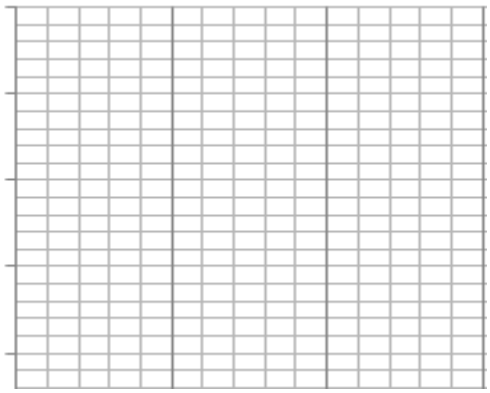
Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Today's activity is an extension of Lesson 5. You will be working in groups to table, graph and identify whether or not the two quantities are proportional to each other.

Classwork

Poster Layout

Use for notes

Problem	Table
Graph 	Proportional or not? Explain.

Lesson Summary:

Graphs of Proportional Relationships: The graph of two quantities that are proportional fall on a straight line that passes through the origin.

Note about Lesson Summary

Problem Set

1. Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios below represent years to amount of money in her savings account.
   - After one year, the interest had accumulated and the total was $312 in Sally's account.
   - After three years, the total was $340. After six years, the total was $380.
   - After nine years, the total was $430. After 12 years, the total amount in Sally's savings account was $480.

   Using the same four-fold method from class, create a table then graph and determine whether the amount of money accumulated and time elapsed are proportional to each other or not. Use your table and graph to support your reasoning.