Domino Effect

Lesson by Mathalicious;
Annotation by Student Achievement Partners

GRADE LEVEL Eighth

IN THE STANDARDS 8.F.B.4, 8.F.B.5, 8.F.A.3

WHAT WE LIKE ABOUT THIS LESSON

Mathematically:

- Reinforces multiple representations of a function through the use of input/output tables, graphs, and equations
- Requires students to interpret unit rate as slope (rate of change)
- Deepens students' understanding of slope, y-intercept, and domain having them apply it in a real-world situation
- Allows students to explore plausibility of answers and connect the meaning of mathematical models to a situation in the introduction of the lesson

In the classroom:

- Captures student attention by using an engaging context
- Provides robust opportunities for students to discuss mathematical concepts; includes guiding questions for teachers to use to facilitate discussion
- Uses technology to create a graph and illustrate the mathematics of the lesson
- Offers multiple opportunities for student practice by analyzing pizzas of various sizes

MAKING THE SHIFTS[1]

Focus		Belongs to the major work[2] of eighth grade
Coherence		Builds on unit rates from seventh grade (7.RP.A) and students' understanding or representing situations with equations (7.EE.B.4a); lays foundations for work students will do in high school building functions to represent more complex situations (F-BF.A)
Rigor[3]		Conceptual Understanding: secondary in this lesson
		Procedural Skill and Fluency: not addressed in this lesson
		Application: primary in this lesson

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Eight.
[3]Lessons may target one or more aspect(s) of rigor.

For a direct link, go to: http://achievethecore.org/page/919/domino-effect

ADDITIONAL THOUGHTS

This lesson is designed as an introduction to eighth grade work with functions, which is part of the major work of the grade. Students build on major work of seventh grade with proportional relationships on graphs and in equations. The lesson introduces a context that allows students to consider many concepts that they will study in-depth in their ongoing work with linear functions (recognizing equations that determine linear functions, modeling relationships with functions, identifying the input- and output-values for a given function). Students are then given time to connect features of the function to the real-world situation. It is not intended for students to meet the full expectations of the grade-level standards through only this lesson.

In the lesson, students learn to calculate the price of a pizza by interpreting the unit rate (per topping cost) as slope and by considering the initial cost of a pizza based on its size (y-intercept). Scaffolded questions lead students to calculate slope and y-intercept before ultimately finding the corresponding linear equation. There are short answer and open-ended questions, with opportunities for students to justify their answers. The lesson continues by giving students practice creating equations to represent the cost of additional pizzas with an unknown number of toppings. Students then compare the graphs for all three pizza sizes, both qualitatively and quantitatively.

The lesson concludes by presenting the actual costs of various pizzas on a graph. This allows students to consider a situation where the slope is equal to zero. Because the graphs represent continuous functions, they provide an opportunity to explore real-world constraints and discuss the qualitative aspects of graphs (8.F.B.5). Extension activities are included to allow students to apply other mathematical ideas (ratios and proportional reasoning, area of circles and surveying) to the same context.

The work with functions in this lesson lays the groundwork for functions and modeling, which is a significant focus of high school mathematics. For more insight on the grade-level concepts addressed in this lesson and how they relate to later work, read pages 5 and 6 of the progressions document, Grade 8, High School, Functions.

DOMINO EFFECT

How much does Domino's charge for pizza?

lesson guide

mathalicious

Domino's pizza is delicious. The company's success is proof that people enjoy their pies. The company is also tech savvy: you can order online, and they even have a pizza tracker so you can keep tabs on your delivery!

The website is great. But one thing it's *not* is transparent. Domino's does not tell you how much the component pieces cost; they only tell you an item's final price after you build it. In this lesson, students use **linear equations** to find the base price (**y-intercept**) and cost per additional topping (**slope**). Let's find out how much Domino's is *really* charging for pizza.

Primary Objectives

- Understand the ideas of slope and y-intercept within the context of Domino's pizza pricing
- Write and graph a linear equation given two points on the line
- Understand what it means for a function to be linear (constant rate of change)

Content Standards (CCSS)	Mathematical Practices (CCMP)	Materials
Grade 8 EE.5, F.3, F.4, F.5	MP.2, MP.4	• Student handout • LCD projector • Computer speakers

Before Beginning…

Students should be able to calculate a unit rate (e.g. "cost per topping"). As this lesson is meant to be introductory, no prior knowledge of linear functions is assumed.

Preview & Guiding Questions

Students watch a screen recording of someone ordering a two-topping medium pizza from Domino's in Washington, DC, and find out that it costs $13.97. (More specifically, the Domino's that delivers to 1600 Pennsylvania Avenue, or the White House.) The question: *how much is Domino's charging for toppings?*

Of course, with just one pizza, there's no way to determine the cost of a topping. Still, some students may incorrectly reason that since a 2-topping pizza costs $13.97, each topping must cost $13.97 ÷ 2 toppings = $6.99/topping. Other students might realize that this is unreasonable; not only does this seem absurdly expensive, but they may know that the pizza also includes a base price.

At this point, you can ask a series of leading questions to get them thinking about the relationship between the cost per topping and the base price. For instance*, How much might a topping cost?* ($1.) *In this case, what would the base price be?* ($11.97.) *How much else might they cost, and what would the base price be now?* ($2, $9.97.)

The goal of the conversation is for students to realize that they need more information in order to determine the cost of a medium topping at Domino's. In particular, they need to know how much a medium pizza costs with a different number of toppings.

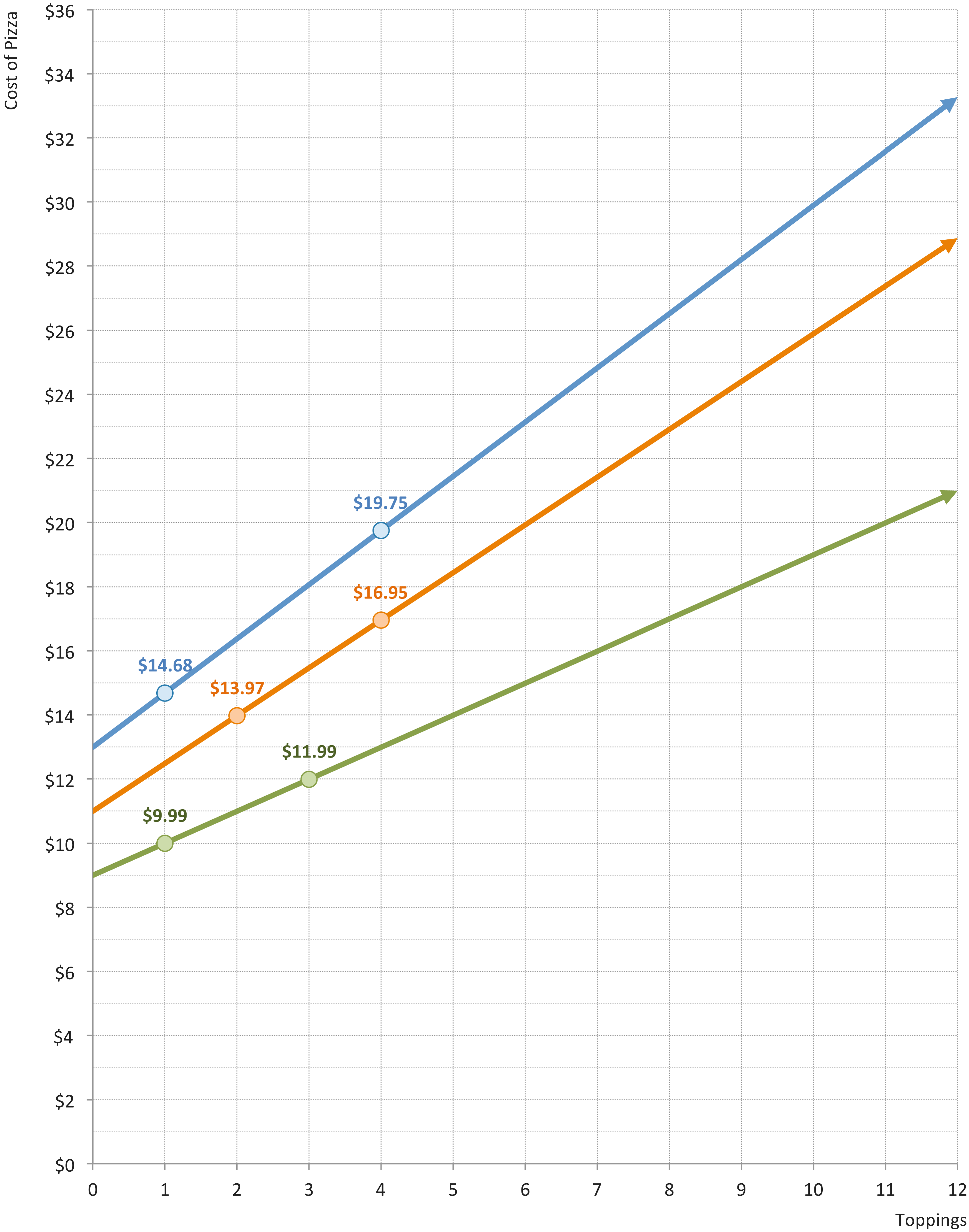
- *How much does a topping cost at Domino's?*
- *How much MIGHT a topping cost? In this case, what would the base price be?*
- *How else might a topping cost, and what would the base price be now?*
- *Could the cost per topping be zero? Could it be negative? (e.g. what if the pepperoni was rancid?!)*
- *Why can't we determine the exact cost per topping?*
- *What other information would you need to determine how much Domino's is charging per topping?*
- *By the way…do you recognize the address 1600 Pennsylvania Avenue??*

Act One

In Act One, students learn that a two-topping medium pizza costs $13.97 and a four-topping medium pizza costs $16.95. Based on this, they'll calculate the cost per topping, assuming that all types of toppings cost the same and that Domino's doesn't charge different amounts for the second topping, the third topping, and so on. After seeing evidence that these assumptions are reasonable, they'll go on to calculate the total amount spent on toppings and the price of a plain cheese pizza. Finally, they'll project out the total cost for pizzas with even more toppings, and they will write and graph an equation describing the price of a pizza in terms of the number of toppings.

Act Two

Act One was all about the medium, but there are other pizza sizes as well! In Act Two, students repeat the analysis in order to write and graph linear equations for the cost of a small and a large pizza. Once again, these equations determine price as a function of the number of toppings on the pizza. Once students have graphed their equations for the small, medium, and large, you can reveal the actual graphs describing how Domino's charges. Students will see that, no matter the size, Domino's stops charging after the fourth topping! They also don't allow you to order more than ten toppings. Students discuss possible reasons for Domino's pricing model.


Cost of Pizza
$36
$34
$32
$30
$28
$26
$24
$22
$20
$18
$16
$14
$12
$10
$8
$6
$4
$2
$0
$19.75
$14.68
$16.95
$13.97
$11.99
$9.99
0
1
2
3
4
5
6
7
8
9
10
11
12
Toppings

Act One: Meaty Yum

1 Below are prices for a medium 2-topping pizza and a medium 4-topping pizza from Domino's in Washington, DC. Plot them on your graph and use the information to answer the following questions.

ITEM		PRICE
	**Medium (12") Hand Tossed Pizza** **Whole:** Pepperoni, Green Peppers	**$13.97**

ITEM		PRICE
	**Medium (12") Hand Tossed Pizza** **Whole:** Bacon, Premium Chicken, Green Peppers, Mushrooms	**$16.95**

a. Based on the information above, how much do you think Domino's is charging for each topping?

For two additional toppings, you pay an additional \$16.95 – \$13.97 = \$2.98. Therefore, each topping costs \$2.98 ÷ 2 = \$1.49.

b. A medium 3-topping pizza costs \$15.46. What would it mean if it cost *more* than this, e.g. \$16?

If a 2-topping pizza costs \$13.97, we'd expect a 3-topping pizza to cost \$13.97 + \$1.49 = \$15.46.

If it cost more than this, though, it would mean that Domino's wasn't charging a constant \$1.49 per additional topping, since the fourth topping would cost less than the third.

c. For the 2-topping pizza, how much in total are you spending *on toppings*? For the 4-topping pizza?

For 2 toppings: \$1.49(2) = \$2.98 on toppings
For 4 toppings: \$1.49(4) = \$5.96 on toppings

d. If you wanted to order a medium cheese pizza, how much would you expect to spend? Explain.

A two topping pizza costs \$13.97. Since \$2.98 of that is spent on toppings, the base price of a plain cheese pizza must be \$13.97 – \$2.98 = \$10.99.

The same result could be calculated by using a 3 or 4-topping pizza.

e. Now write an equation for the price of a medium pizza, and explain what the equation means.

Let C = price of pizza and t = number of toppings

$$C = 10.99 + 1.49t$$

What this equation means is that, for a medium pizza, Domino's is charging a \$10.99 base price plus \$1.49 for each topping.

f. Does a pizza with 12 toppings cost twice as much as a pizza with 6 toppings? Why or why not?

6 topping pizza: \$10.99 + \$1.49(6) = \$19.93
12 topping pizza: \$10.99 + \$1.49(12) = \$28.87

The 12-topping pizza isn't twice as expensive. The reason is because you don't double the base price; you only double the amount that you're spending on the toppings.

Explanation & Guiding Questions (part a)

Since they're dealing with something familiar – pizza and toppings – students shouldn't have too much trouble calculating the cost per topping. You might expect them to say, "When I order two more toppings…the price of the pizza increases by $2.98." One issue you may encounter is students not being able to quickly divide $2.98 ÷ 2 toppings, and getting sidetracked. To remedy this, you might ask them to estimate, e.g. $3 ÷ 2 toppings = $1.50 per topping. Since $2.98 is $0.02 less than $3.00, then each topping must be $0.01 less than $1.50.

Even though the lesson doesn't use the term, the cost per topping is the **slope**: the rate of change, i.e. how the price is changing with each additional topping. As with any rate – miles *per* hour; calories *per* mile – the slope involves units. If students describe the slope as "1.49," you might remind them to specify the units: 1.49 dollars *per* topping.

- *When you increase the number of toppings from two to four, what happens to the price?*
- *If an additional two toppings cost $2.98,* ***approximately*** *how much is each topping? How much, exactly?*
- *(If a student describes the cost per topping as "1.49"), 1.49 WHAT?*

Explanation & Guiding Questions (part b)

This is an important question. The defining characteristic of a linear function is a constant rate of change, i.e. a constant cost per topping. Even though students previously calculated a cost per topping of $1.49, they haven't actually established that Domino's charges this for every number of toppings.

If a 2-topping pizza cost $16, then this would mean that the relationship between the cost of a pizza and the number of toppings was *not* linear. Instead, the third topping would cost around $2 ($16.00 – $13.97), while the fourth topping would only cost around $1 ($16.95 - $16.00). Fortunately, this isn't the case. Domino's *does* charge $15.46, which is exactly what we'd expect: $13.97 + $1.49 = $15.49, and $15.49 + $1.49 = $16.98. Based on this, it seems that Domino's is charging a constant $1.49 for each additional topping, i.e. that the relationship is linear.

- *If a 3-topping pizza cost $16, how much would the third and fourth toppings cost?*
- *Since a 3-topping pizza actually costs $15.46, what does this mean about the cost of each topping?*

Explanation & Guiding Questions (part c)

In part d, students will calculate the base price of a pizza. In part e, they'll write the equation of the line. Having students determine how much a customer is paying *in total* on toppings may help them determine the base price (i.e. the y-intercept), and also recognize what the "mx" term represents in $y = mx + b$.

- *If a medium topping costs* ***around*** *$1.50, approximately how much are you spending in total on toppings?*
- *At $1.49/topping, how much are you actually spending, and how did you get this?*

Explanation & Guiding Questions (part d)

There are a number of ways that students might calculate the base price, i.e. the price of a pizza with zero toppings, i.e. the value of y when $x = 0$, i.e. the **y-intercept**. Since they calculated the total amount that a customer is spending *on toppings*, they can simply subtract this from the price of the pizza. For instance, if a 2-topping pizza costs $13.97, and if $2.98 of this represents what we're paying *for toppings*, then the difference ($13.97) must represent what we're paying on something *other* than toppings, i.e. the base price. Alternatively, students might simply "remove" the toppings to determine the base price: $13.97 – $1.49 – $1.49 = $10.99.

- *What does the base price really represent?*
- *How can you use the total amount spent on toppings to figure out the base price?*
- *Can you think of any other way to determine the base price?*

Explanation & Guiding Questions (part e)

At this point, students know pretty much everything they need to know about how much Domino's charges for a medium pizza: a $10.99 base price, plus $1.49 for each topping. Now they just need to translate this information into an equation, which requires choosing variables. Even though we often introduce linear functions using the slope-intercept form, $y = mx + b$, why not use variables that make sense given the context?

You might see two versions of the equation: $C = 10.99 + 1.49t$, or $C = 1.49t + 10.99$. It doesn't matter which version students use, though the first version may be the most intuitive since Domino's *starts* by charging a base price…then adds toppings.

Once they have their equation, students should be able to explain what each term and coefficient represents:

C	10.99	1.49	t	$1.49t$
Total cost of pizza	Base price	Cost per topping	# of toppings	$ spent on toppings

- *What variables should we use in the equation?*
- *Could we rewrite the equation as C = 1.49t + 10.99? If so, which method do you prefer and why?*
- *In C = 10.99 + 1.49t, what do the following represent: C, 10.99, 1.49, t, 1.49t?*

Explanation & Guiding Questions (part f)

To determine how much 6- and 12-topping pizzas would cost, students might plug the values into the equations (one step), or they might calculate the total spent on toppings, then add this to the base price (two steps). However they choose to do it, it's important that students recognize that each method yields the same result.

When students calculate the prices, they'll find that a medium 6-topping pizza should cost $19.93, while a 12-topping pizza should cost $28.87. Even though we're ordering twice as many toppings, we're not paying twice as much. The reason, of course, is because we don't have to double the base price; we only pay the $10.99 once. That said, we *do* double something: the amount that we're spending in total on toppings. Put another way, the 10.99 amount doesn't change – it's constant – while the 1.49t amount does.

- *Why doesn't the base price double?*
- *Is there anything that* ***does*** *double?*

Deeper Understanding

- *The 4-topping pizza includes "premium chicken." Does Domino's charge more for premium toppings? (No.)*
- *Domino's charges a constant $1.49 per topping. Do all pizza places do this? (Probably not.)*
- *When we graph the price of a medium pizza, should we draw a solid line? (Since we can't order, say, 1.4 toppings, it might make more sense just to plot the points. However, modeling with a line is very useful in visualizing the relationship between the cost of a pizza and the number of toppings, as long as students understand that the line isn't necessarily an accurate representation of reality.)*
- *Actually, should we even call it a "line?" (Probably not. Since we can't order negative toppings, we're restricted to positive values of x. Thus, we're not actually drawing a line but rather a ray.)*
- *We expect a 12-topping pizza to cost $28.87. Would you be willing to spend this much on a pizza? (In Act Two, students will find out that Domino's actually stops charging after four toppings, and that a medium 12-topping pizza would therefore cost the same as a 4-topping pizza: $16.95. However, they'll also learn that Domino's doesn't allow customers to order more than ten toppings.)*
- *Do you expect that Domino's charges $1.49 per topping for every size: small, medium, and large? (In Act Two, students will learn that Domino's charges less for small toppings, and more for large toppings.)*

Act Two: Pizza Tracker

2 Below are the prices for two small pizzas and two large pizzas from Domino's. Write an equation to calculate the cost of each size based on the number of toppings you order.

ITEM		PRICE
	**Small (10") Hand Tossed Pizza** **Whole:** Pepperoni	$9.99
	**Small (10") Hand Tossed Pizza** **Whole:** Premium Chicken, Black Olives, Jalapeno Peppers	$11.99

ITEM		PRICE
	**Large (14") Hand Tossed Pizza** **Whole:** Sliced Italian Sausage, Green Peppers, Roasted Red Peppers, Mushrooms	$19.75
	**Large (14") Hand Tossed Pizza** **Whole:** Philly Steak	$14.68

Cost per Topping	$\frac{\$2.00}{2 \text{ toppings}} = \$1/\text{topping}$	$\frac{\$5.07}{3 \text{ toppings}} = \$1.69/\text{topping}$
Total Spent on Toppings	• *1 × $1 = $1* • *3 × $1 = $3*	• *4 × $1.69 = $6.76* • *1 × $1.69 = $1.69*
Base Price	• *$9.99 – $1 = $8.99* • *$11.99 – $3 = $8.99*	• *$19.75 – $6.76 = $12.99* • *$14.68 – $1.69 = $12.99*
Equation	***C = 8.99 + 1t***	***C = 12.99 + 1.69t***

Explanation & Guiding Questions

Following Act One, students – particularly those on autopilot – might incorrectly assume that Domino's is charging the same amount for a small and large topping as they are for a medium: $1.49. If so, ask them whether $1.49 will get them from $9.99 (small 1-topping) to $11.99 (small 3-topping). It won't. Instead, they must calculate the cost per topping from scratch…pardon the pun! With the small pizza, two more toppings cause the price to increase by $2, which clearly suggests that Domino's isn't charging $1.49 per small topping, but rather $1.

Once they have the cost per topping for the small and large pizzas, students can use the same process as before to determine the total spent on toppings, the base price, and the equation for each size.

- *Does Domino's charge $1.49 per topping for the small and large pizzas? How can we check this?*
- *For each pizza, how much are we spending on toppings? Based on this, what must the base price be?*

Deeper Understanding

- *Which size pizza has the highest cost per topping, and why? (Large, because we get the most topping.)*
- *Which size pizza has the lowest base price, and why? (Small, because we get the least amount of pizza.)*

3 Now graph the equation for each pizza size, and answer the following:

a. Which graph – small, medium, or large – is the steepest, and why do you think this is?	b. Which graph has the lowest starting value, and is this what you'd expect? Explain.
The graph for the large pizza is the steepest. This is because we're paying the most per additional topping, which makes sense: a large pizza can fit more toppings than can a small or medium pizza.	*The small pizza has the lowest starting value. This is because it has the lowest base price, which also makes sense: a small cheese pizza is the smallest of the three sizes, and so it should cost the least.*

Explanation & Guiding Questions

Students already know the highest cost per topping for a large pizza, and the lowest base price for a small pizza. The goal of this question is for them to translate their intuitive (and algebraic) understanding of cost per topping and base price to the graphical representations of slope and *y*-intercept.

Students will likely have a fairly easy time identifying the meaning of the "starting value," or *y*-intercept: it's just the base price. Mathematically, they should be able to describe that it's the cost when the number of toppings is zero.

They may have a harder time making sense of the "steepness," though, as this is a bit more abstract. To help them understand what the steepness really represents, remind them that the cost per topping is really the cost per *additional* topping: *when we order one more topping, by how much does the price increase? When we go from 0 toppings to 1 topping…to 2 toppings…to 3 toppings, etc., by how much does the price* ***rise****?* With this, students should see – literally – why the graph is climbing faster with the large pizza than with the other sizes.

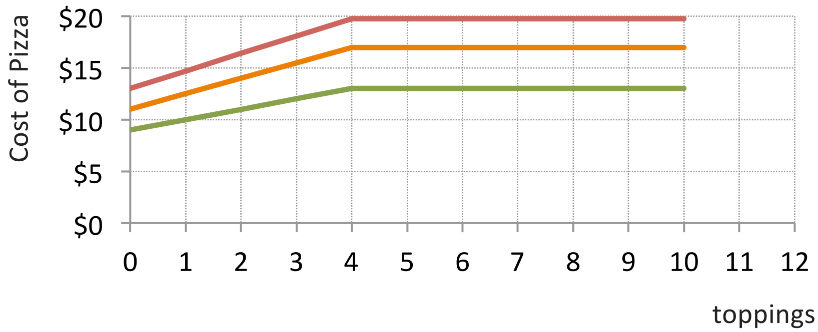
(Note: it's dangerous to equate steepness with slope. If you change the scale of the axes – for instance, if you make the y-axis go from $0 to $1000 – then the lines will appear much less steep even though the slope ($/topping) will remain unchanged. Put another way, steepness is a way to *visualize* slope, but it's not the same thing as slope. The reason the question asks which line is the "steepest" is simply because it will be fairly intuitive for students who are just being introduced to linear functions.)

- *What does the steepness of the graph represent?*
- *What does the starting value represent?*

Deeper Understanding

- *We know how the graphs are different, but how are they alike? (They're all lines, which means that Domino's is charging a constant amount for each pizza size…even though the amount is different.)*
- *Can you think of any pizza chains that charge the same amount per topping regardless of the size? The same base price? (Answers may vary, though this would certainly be a surprising pricing scheme.)*
- *If we were drawing a line rather than a ray – i.e. if the situation allowed for negative values of x – would it make sense to talk about a "starting value?" (No. A line is infinitely long, which means it has no start!)*

4 Look at the graph of how much Domino's *really* charges for pizza in Washington, DC. How is the actual situation different than what you expected…and why do you think Domino's does this?



According to this graph, everything looks normal until the fourth topping. At this point, Domino's appears to stop charging for additional toppings; toppings 5-10 are free, which is why the line is flat (slope = 0).

Also, it looks like Domino's doesn't allow you to order more than ten toppings. They cut you off!

Explanation & Guiding Questions

Throughout the lesson, we assumed that Domino's charged a constant amount per additional topping, and that each line would continue forever. In reality, Domino's – or at least the one that delivers to the White House – has a surprising price twist: it charges what we'd expect…until four toppings. After four toppings, it stops charging for each additional topping. A 4-topping pizza costs the same as a 5-topping pizza, which costs the same as a 10-topping pizza. Mathematically, after four toppings the slope becomes $0/topping.

However, it doesn't cost the same as an 11-topping pizza, since there is none. Domino's doesn't allow customers to order more than ten toppings. According to the website, having too many toppings will prevent the pizza from cooking properly. Maybe this is true…or maybe they're just trying to avoid going bankrupt from all the topping hogs out there!

- *After four toppings, what is the cost per additional topping, i.e. the slope? (Zero, or $0/topping.)*

Deeper Understanding

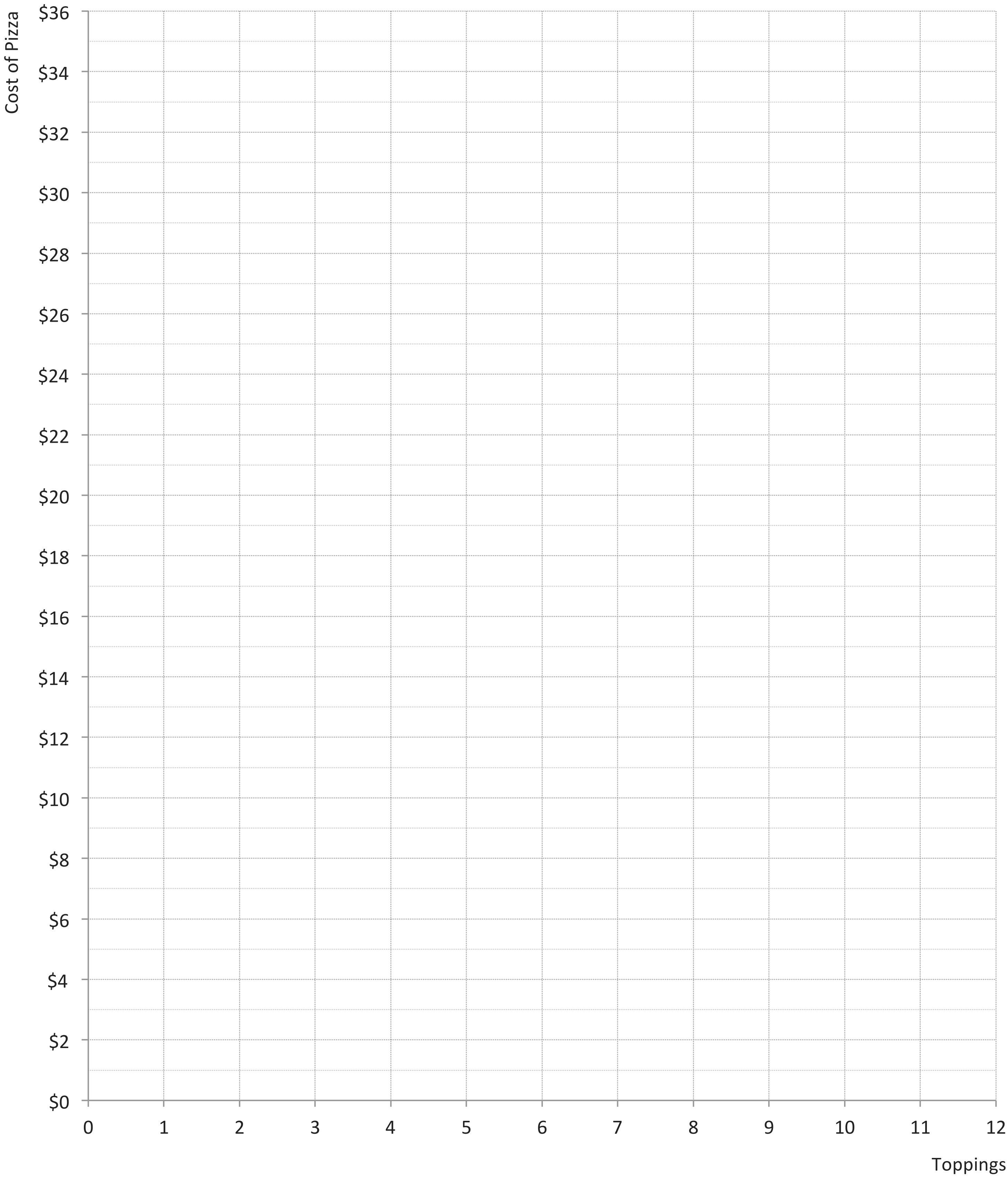
- *Why do you think Domino's stops charging for additional toppings after four toppings? (Maybe there's a maximum price that people are willing to pay. For instance, the large pizza maxes out at $19.75. Perhaps Domino's has done research and found that people are unwilling to spend more than $20 for a pizza. Then again, Papa John's continues to charge for each additional topping, so who knows why Domino's does this?!)*
- *Why do you think Domino's doesn't allow people to order more than ten toppings? (Answers will vary.)*

DOMINO EFFECT

How much does Domino's charge for pizza?

name

date



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Act One: Meaty Yum

1 Below are prices for a medium 2-topping pizza and a medium 4-topping pizza from Domino's in Washington, DC. Plot them on your graph and use the information to answer the following questions.

ITEM		PRICE
	**Medium (12") Hand Tossed Pizza** **Whole:** Pepperoni, Green Peppers	**$13.97**

ITEM		PRICE
	**Medium (12") Hand Tossed Pizza** **Whole:** Bacon, Premium Chicken, Green Peppers, Mushrooms	**$16.95**

a. Based on the information above, how much do you think Domino's is charging for each topping?

b. A medium 3-topping pizza costs $15.46. What would it mean if it cost *more* than this, e.g. $16?

c. For the 2-topping pizza, how much in total are you spending *on toppings*? For the 4-topping pizza?

d. If you wanted to order a medium cheese pizza, how much would you expect to spend? Explain.

e. Now write an equation for the price of a medium pizza, and explain what the equation means.

f. Does a pizza with 12 toppings cost twice as much as a pizza with 6 toppings? Why or why not?

Act Two: Pizza Tracker

2 Below are the prices for two small pizzas and two large pizzas from Domino's. Write an equation to calculate the cost of each size based on the number of toppings you order.

ITEM		PRICE
	**Small (10") Hand Tossed Pizza** **Whole:** Pepperoni	**$9.99**
	**Small (10") Hand Tossed Pizza** **Whole:** Premium Chicken, Black Olives, Jalapeno Peppers	**$11.99**

ITEM		PRICE
	**Large (14") Hand Tossed Pizza** **Whole:** Sliced Italian Sausage, Green Peppers, Roasted Red Peppers, Mushrooms	**$19.75**
	**Large (14") Hand Tossed Pizza** **Whole:** Philly Steak	**$14.68**

3 Now graph the equation for each pizza size, and answer the following:

a. Which graph – small, medium, or large – is the steepest, and why do you think this is?	b. Which graph has the lowest starting value, and is this what you'd expect? Explain.

4 Look at the graph of how much Domino's *really* charges for pizza in Washington, DC. How is the actual situation different than what you expected…and why do you think Domino's does this?