Functions

8.F.A and 8.F.B Conceptual Understanding & Application Mini-Assessment by Student Achievement Partners

OVERVIEW

This mini-assessment is designed to illustrate the clusters 8.F.A and 8.F.B, which set an expectation for understanding functions and using functions to model relationships between quantities. This mini-assessment is designed for teachers to use either in the classroom, for self-learning, or in professional development settings to:

- **Evaluate** students' understanding of 8.F.A and 8.F.B in order to prepare to teach this material or to check for student ability to demonstrate understanding and application of these concepts;
- **Gain knowledge about** assessing function application problems;
- **Illustrate CCR-aligned** assessment problems;
- **Illustrate best practices** for writing tasks that allow access for all learners; and
- **Support mathematical language acquisition** by offering specific guidance.

MAKING THE SHIFTS

This mini-assessment attends to **focus** as it addresses understanding of functions and using functions to model relationships between quantities, which is at the heart of the grade 8 standards and a key component of the Major Work of the Grade.[1] It addresses **coherence** across grades as students formally work with functions and function notation in grade 8, but build from previous work on algebraic patterns, input/output rules, and ratios and proportional relationships, before ever learning the term "function." Function learning in grade 8 lays the foundation for a topic that is a widely applicable prerequisite for college and careers. The Functions domain in grade 8 and this mini-assessment target *conceptual understanding* and *application*, two of the three elements of **rigor**.

A CLOSER LOOK

8.F.A. Define, evaluate, and compare functions.

At the heart of this mini-assessment is the concept of a function as a rule that assigns to each input exactly one output. Students spend extensive time in grade 8 developing this understanding using functions to model real-world situations. This mini-assessment requires students to show both a strong understanding of functions and the ability to apply function concepts in context-free and context-rich questions. In grade 8, students synthesize learnings around patterns and ratios and proportional relationships to work with functions expressed as equations, tables, and graphs. Through this work, students see different contexts and representations of functions.

The distinction between function and equation is an important one as the two terms are often conflated by students and teachers. Equations answer a question: "For what values of the variable(s) is the equation true?" Functions, on the other hand, are neither true nor false. Confusion may arise when graphing: the graph of the solutions to an equation in two variables (an infinitude of (u, v) pairs that satisfy the equation) can often look the same as the graph of a function (or picture of how the output changes as the input changes).

8.F.B. Use functions to model relationships between quantities.

Real-world examples are concrete ways for students to understand linear and non-linear functions. For example, the distance a car drives as a function of time is non-linear because cars do not move at the

[1] For more on the Major Work of the Grade, see achievethecore.org/focus.

same speed continuously; they accelerate, decelerate, and have periods of no movement (e.g., at a red light). Asking students to graph this function will help them to better understand how to interpret the graph's features. This is not only important for linear functions in grade 8, but will be a critical skill in high school as well.

The situations in grade 8 should be more robust than those in previous grades, including more real-world data, using rational numbers, and students needing to construct their own functions and define their own variables. Note that concepts like domain, range, and zeroes, as well as formal function notation, are all saved for high school.

This mini-assessment should follow work with radicals, the Pythagorean Theorem, and solving linear equations. It is designed to take one class period or less. Whether or not to use calculators on this mini-assessment is at the discretion of the teacher. Not allowing a calculator gives students the opportunity to practice work with rational numbers and ensure students sketch their own graphs. Allowing a calculator would minimize the computational demand and allow students to use technology strategically.

SUPPORT FOR ENGLISH LANGUAGE LEARNERS

This lesson was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction and assessment. Go here to learn more about the research behind these supports. Features that support access in this mini-assessment include:

- Tasks that allow for multi-modal representations, which can deepen understanding of the mathematics and make it easier for students, especially ELLs, to give mathematical explanations.
- Tasks that avoid unnecessarily complex language to allow students, especially ELLs, to access and demonstrate what they know about the mathematics of the assessment.

Prior to this mini-assessment, ensure students have had ample opportunities in instruction to read, write, speak, listen for, and understand the mathematical concepts that are represented by the following terms and concepts:

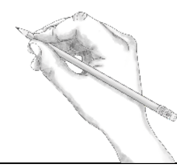
- function
- set of positive integers
- input
- output
- square (exponent)
- value
- coordinate
- linear
- accurately

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work (for example, through engagement in mathematical routines).

ELLs may need support with the following words found in this mini-assessment:

- rule
- describe
- produce
- shift
- commute
- same
- cheaper

In preparation for giving this mini-assessment, teachers should strive to use these words in context so they become familiar to students. It will be important to offer synonyms, rephrasing, visual cues, and modeling of what these words mean in the specific contexts represented in the items in this mini-assessment. Additionally, teachers may offer students the use of a student-friendly dictionary, or visual glossary to ensure they understand what is being asked of them in each item.

Sketch	
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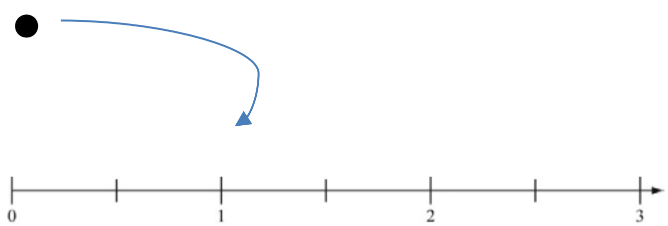
Locate	

An example of a visual glossary for student use.

Name:____Date:___________________

1. The following rules define two functions. The inputs for the functions are the set of positive integers.

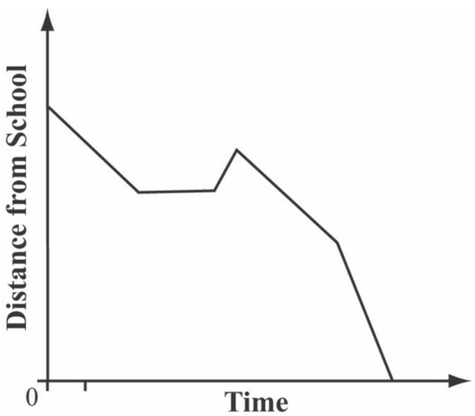
Rule 1: The output is the square of the input.
Rule 2: The output is 90 greater than the input.

(a) What is a positive integer input that gives the same output for both functions?

(b) Do Rule 1 and Rule 2 define the same function? Explain your reasoning.

2. The graph below shows Jonas' commute to school this morning.

Based on the graph, describe what is happening as carefully as you can. You do not need to measure anything accurately.



3. A machine produces granola bars from long strips of granola. The function below shows that L meters of granola produces B granola bars.

$$B = 25\left(\frac{L}{10} + \frac{3}{4}\right)$$

How many granola bars are produced from 2.5 meters of granola?

4. Mrs. Garner opens a factory that produces cans of soda 24 hours a day. There are three work shifts. Each shift produces cans of soda at different rates.

Shift	Rate of Soda Production
8am – 4pm	Normal production rate
4pm – 12am	$\frac{1}{2}$ of normal production rate
12am – 8am	$\frac{1}{4}$ of normal production rate

She needs to determine what the normal production rate must be to produce 200,000 cans of soda each day.

How many cans of soda must the factory produce from 8am to 9am to produce 200,000 cans each day? Explain your answer using numbers, words, and/or pictures.

5. Maureen and Shannon decide to rent paddleboards while on vacation. Shop A rents paddleboards for \$7.75 per hour. Shop B's prices are shown on the poster to the right.

Which shop offers a cheaper hourly rental rate? Explain your answer using numbers, words and/or pictures.

Shop B

Rental Rates

Hours	Price
0.5	\$3.80
4	\$30.40
7	\$53.20
8	\$60.80

6. The graph of a linear function includes the points (6, 3.5) and (1, –2.5).

a. The x-coordinate is the input and the y-coordinate is the output. Write an equation that gives y in terms of x.

b. Find another point on the graph with an x-coordinate that has a negative value.

1. **(8.F.A)** The following rules define two functions. The inputs for the functions are the set of positive integers.

   Rule 1: The output is the square of the input.
   Rule 2: The output is 90 greater than the input.

   (a) What is a positive integer input that gives the same output for both functions?

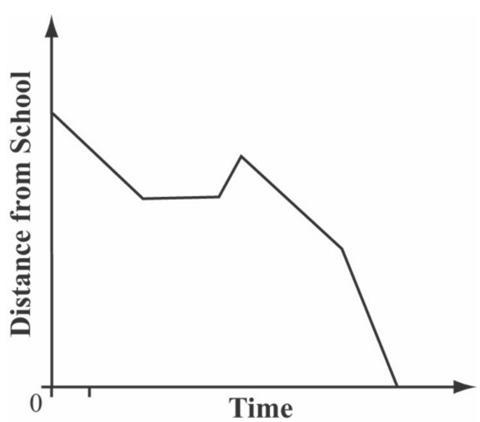
   10

   (b) Do Rule 1 and Rule 2 define the same function? Explain your reasoning.

Student states that the two rules do not define the same function and reasons that since positive integers other than 10 produce different outputs for the given rules and provide an example to support their statement. Some students may choose to include a counterexample such as: the input value of 2 gives an output of 4 under Rule 1 and an output of 92 under Rule 2.

2. **(8.F.B.5)** The graph below shows Jonas' commute to school this morning.

   Based on the graph, describe what is happening as carefully as you can. You do not need to measure anything accurately.



Example description: Jonas began walking to school quickly at a constant rate. He stopped for a while before going backwards for a short time. Then, he headed back toward school at the same rate as when he first left. Jonas then finished his commute at a faster rate.

Rubric: Correct answers should include:

1. Noting that Jonas walked toward school first

2. Noting that Jonas has a period with no movement toward school and a period moving away from the school
3. Noting that the last leg of the commute to school is at the fastest rate

NOTE: The question asks for careful description to encourage precise language but students do not necessarily have to add contextual information (e.g., "Jonas stopped to pet a dog."), but they should include information about the varying rates during the commute.

3. **(8.F.A)** A machine produces granola bars from long strips of granola. The function below shows that L meters of granola produces B granola bars.

$$B = 25(\frac{L}{10} + \frac{3}{4})$$

How many granola bars are produced from 2.5 meters of granola?

Key: $B = 25$ granola bars

Note: The variables M and B are defined for the student in the question stem. Students' ability to correctly answer questions like this relies on their ability to understand what these variables actually represent.

4. **(8.F)** Mrs. Garner opens a factory that produces cans of soda 24 hours a day. There are three work shifts. Each shift produces cans of sodas at different rates.

Shift	Rate of Soda Production
8am – 4pm	Normal production rate
4pm – 12am	$\frac{1}{2}$ of normal production rate
12am – 8am	$\frac{1}{4}$ of normal production rate

She needs to determine what the normal production rate must be to produce 200,000 cans of soda each day.

How many cans of soda must the factory produce from 8am to 9am to produce 200,000 cans each day? Explain your answer using numbers, words, and/or pictures

> *Note: Defining variables is a key part of building functions. Both functions given as examples are correct based on differently defined variables.*

Key: approximately 14,286 cans of soda

Sample answer (where *a* is the number of cans produced in a normal 8-hour shift):

$$a + \frac{a}{2} + \frac{a}{4} = 200{,}000$$

So, that means $\frac{7}{4}a = 200{,}000$ and $a = 114{,}285.7$. So, in one hour they must produce 114,285.7 ÷ 8 or about 14,286 cans of soda.

Sample answer (where *a* is the number of cans produced in 1 hour at the normal rate):

$$8a + \frac{8a}{2} + \frac{8a}{4} = 200{,}000, \text{ or } 14a = 200{,}000$$

So that means the number of cans produced from 8am to 9am is 200,000 ÷ 14 or about 14,286 cans of soda.

5. **(8.F.A.2)** Maureen and Shannon decide to rent paddleboards while on vacation. Shop A rents paddleboards for \$7.75 per hour. Shop B's prices are shown on the poster to the right.

Which shop offers a cheaper hourly rental rate? Explain your answer using numbers, words and/or pictures.

Shop B
Rental Rates

Hours	Price
0.5	\$3.80
4	\$30.40
7	\$53.20
8	\$60.80

Key: Shop B (with appropriate justification)

Sample answer: Shop B rents paddleboards at a cheaper hourly rate, at \$7.60 per hour. I found the amounts in the table to be in a proportional relationship: for every 1 hour, there is a cost of \$7.60.

6. (8.F.B.4) The graph of a linear function includes the points (6, 3.5) and (1, –2.5).

a. The x-coordinate is the input and the y-coordinate is the output. Write an equation that gives y in terms of x.

Note: As a function, the two points can be thought of pairs of inputs and outputs. Finding the equation for the function amounts to calculating the slope between the two points and using that to find the y-intercept.

Key: $y = 1.2x - 3.7$

b. Find another point on the graph with an x-coordinate that has a negative value.

Key: Any point that makes the equation true is acceptable. Possible points include: (–1.5, –5.5); (–2, –6.1); (–2.75, –7); (–4, –8.5)