# Modeling Equivalent Expressions

Lesson written by Shelbi Cole

GRADE LEVEL Sixth

IN THE STANDARDS 6.EE.A.2, 6.EEA.4

WHAT WE LIKE ABOUT THIS LESSON

Mathematically:

- Allows students to develop an understanding of the meaning of equivalent expressions
- Promotes coherence by highlighting prior knowledge and pointing to the mathematics that will be built from these ideas
- Requires students' use of precise course-appropriate mathematical language (MP.6)
- Relates concrete quantities and abstract symbols (MP.2)
- Encourages students to make use of structure when creating expressions from the models (MP.7)

In the classroom:

- Uses multiple concrete representations and visual models to make the mathematics explicit
- Gives formal and informal opportunities for teachers to check for understanding
- Provides opportunities for students to discuss mathematical concepts; includes guiding questions for teachers to use to facilitate discussion

MAKING THE SHIFTS[1]

| | |
|---|---|
| Focus | Belongs to the major work[2] of sixth grade |
| Coherence | Builds on grade 5 work with numerical expressions. |
| Rigor[3] | Conceptual Understanding: Primary in this lesson<br>Procedural Skill and Fluency: Secondary in this lesson<br>Application: Not addressed in this lesson |

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Six in the Supplemental Resources below.
[3]Lessons may target one or more aspect(s) of rigor.

## ADDITIONAL THOUGHTS

It's important to note that this lesson would fit best early in the Expressions and Equations unit of sixth grade. It is not intended for students to meet the full expectations of the grade-level standards addressed through only this selected lesson. Within the individual lesson, the selection of representations is purposeful and connects to the student's familiarity with arrays. This lesson builds on grade 5 work of interpreting numerical expressions to describe calculations and transitions students to grade 6 work of viewing expressions as objects.

The format of this lesson has some interesting aspects to highlight. This engaging and active lesson is meant to spark conversation between teacher and students as well as students with one another. The teacher checks for understanding of algebraic notation and interpretation of visual models. This ensures that the students' understanding of notation, properties of operations, and variables is sufficient to move forward.

For more insight on the grade-level concepts addressed in this lesson, read pages 4 - 7 of the progression document, Grades 6-8, Expressions and Equations, https://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

# Grade 6: Modeling Equivalent Expressions

## Big Mathematical Ideas

In grade 6, students build on grade 5 work with numerical expressions and apply those understandings to work with algebraic expressions. The standards in grade 6 require that students develop a deep understanding of the meaning of equivalent expressions by both examining the value of two expressions for different values of the variable and by demonstrating the equivalence of two expressions using properties of operations.

## Lesson Objectives

- Students will write expressions to model a given scenario.
- Students will understand equivalent expressions by connecting written statements to expressions to physical models.

## Standards Addressed

6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number $y$ stands for.*

6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as $5 – y$.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*

## Materials

- Student Page: "Exploring Equivalent Expressions"
- Student Page: "Investigating Equivalent Expressions"
- Toothpicks (optional)

## Lesson Overview

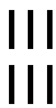
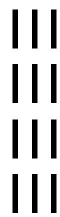
This lesson is written to precede formal work with using properties of operations to generate equivalent algebraic expressions, and may serve as a good introduction to that work. In this lesson, students will relate verbal statements to algebraic expressions to physical models to develop an understanding of equivalent expressions.

## Initiate

Present students with the student page "Exploring Equivalent Expressions." Each line segment represents one toothpick. If desired, students may create the figures using real toothpicks. Give students some time to work through the questions, and then facilitate a discussion using the "Answers & Teaching Notes" section for this part of the lesson.

Student-Facing Questions

1. Each line segment represents a toothpick. Based on the observed pattern, draw Figure 5.

| Figure 1 | Figure 2 | Figure 3 | Figure 4 | Figure 5 |
| --- | --- | --- | --- | --- |
|  |  |  |  | |

2. What is the relationship between the number of toothpicks in any one figure and the figure before it?

3. What is the relationship between the number of toothpicks needed to create the figure and the figure number (e.g., Figure 1)? Write your answer as a sentence.

4. Write an expression that gives the number of toothpicks needed to create Figure $n$, where $n$ is the number assigned to the figure.

5. Is there another way to write the expression you created in #4?

Answers & Teaching notes

#1 – Students should create figure 5 by adding another row of 3 toothpicks to the previous figure (i.e., draw 5 rows of 3 toothpicks per row.)

#2 Each figure has 3 more toothpicks than the figure before it.

#3 The number of toothpicks needed to create each figure is 3 times the figure number.

Teaching Notes for #3: Here students might note that the number of rows is the same as the figure number. The concept of the array was first introduced in grade 3. When the number of objects per row is 3, the total number of objects in the array will be 3 times the number of rows (in this case, the number of rows is also equal to the number assigned to the figure). Students may benefit from seeing a set of numerical expressions assigned to each figure before moving on to writing the algebraic expression in question #4.

Especially for students who may struggle with this question, ask them to write the numerical expression for each figure (the expression generated by multiplying the number of rows times the number of objects per row:1x3; 2x3; 3x3; 4x3; 5x3). During the whole class discussion, check that students understand the meaning of the factors in these numerical expressions to deepen their understanding of why we will eventually use a variable to represent the number in the expression that changes with each figure.

#4 Possible answers: $3n$; $n + n + n$

Teaching Notes: Students should connect the response to #3 to the response to #4, recognizing that the expression $3n$ (or equivalent) gives the total number of toothpicks needed to create Figure *n.* Students may be inclined to define the variable as the "number of rows" which helps them relate it to the concrete model which is the figure. To support students' ability to generalize their learning to non-array patterns later in the lesson, it is important to define the variable as the figure number noting that the expression defines the number of toothpicks needed to create Figure *n.*

#5 $3n$; $n + n + n$; $2n + n$; $n + 2n$; or other equivalent expression

Teaching Notes: Where necessary, continue to go back to the numerical expressions to help students generate an equivalent expression. For example, ask, "What would be another way to find the number of toothpicks in Figure 5? Does that same approach work for Figure 4? How can we write that using a variable like we did in #4?" If time allows, allow students to demonstrate using substitution that the expressions they created in #4 and #5 "work" to find the total number of toothpicks for any figure in the pattern. During the classroom discussion, record all of the different expressions that students come up with and challenge them to provide evidence that the expressions all yield the correct total number of toothpicks or to provide evidence that one or more of the expressions must not be equivalent in the case of an incorrect expression.

## Investigate

Present students with the student page "Investigating Equivalent Expressions." Each line segment represents one toothpick. If desired, students may create the figures using real toothpicks. Give students some time to work through the questions, and then facilitate a discussion using the "Answers & Teaching Notes" section for this part of the lesson.

Student-Facing Questions

1. Each line segment represents a toothpick. Based on the observed pattern, draw Figure 5.

| Figure 1 | Figure 2 | Figure 3 | Figure 4 | Figure 5 |
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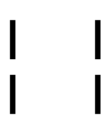
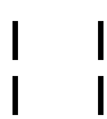
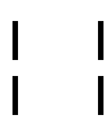
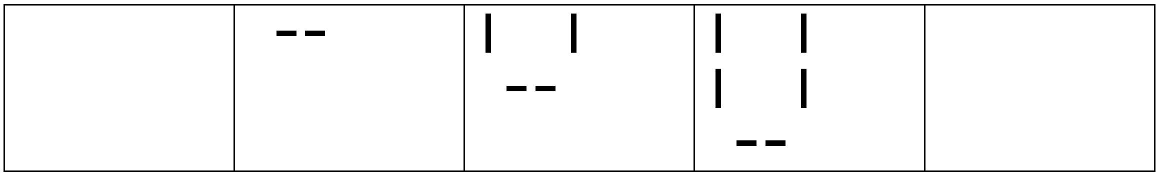
|  |  |  |  | |



1. What is the relationship between the number of toothpicks in any one figure and the figure before it?

2. What is the relationship between the number of toothpicks needed to create the figure and the figure number (e.g., Figure 1)? Write your answer as a sentence.

3. Write an expression that gives the number of toothpicks needed to create Figure *n*, where *n* is the number assigned to the figure.

4. Several students wrote different expressions to show the number of toothpicks needed to create Figure *n.* Write YES or NO to tell whether the given expression can be used to find the total number of toothpicks needed to create Figure *n* for all values of *n.*

| Expression | Correctly assigns the number of toothpicks needed to each figure? | How do you know it is correct/incorrect for all figures? |
| --- | --- | --- |
| $2n + 2$ | | |
| $n + n + 2$ | | |
| $n + n + n + n$ | | |
| $2(n +1)$ | | |
| $1 + 1 + n + n$ | | |
| $2n + 1$ | | |

Answers & Teaching notes

#1 – Students should create figure 5 by replicating figure 4 and adding 1 toothpick to each of the vertical portions of the figure for a total of 12 toothpicks.

#2 Each figure has 2 more toothpicks than the figure before it.

#3 One possible answer: The number of toothpicks needed to create each figure is 2 times the figure number plus 2.

Teaching Notes for #3: Here students might note that they noticed that the number of toothpicks on each side of a figure was equal to the figure number. To get the total, they needed to account for the 2 sides plus the 2 additional toothpicks at the bottom. Other students may respond differently depending on their view of the figures. See section "Connecting to the visual models" for suggestions on handling any variations to the expression that students might give.

#4 Possible answers: $2n+2$

Teaching Notes: Students should connect the response to #3 to the response to #4, recognizing that the expression $2n + 2$ (or equivalent) gives the total number of toothpicks needed to create Figure *n.* Students may be able to describe how they used the shape of the figure to generate their expressions, while others may have generated an expression simply by examining the relationship between the numbers themselves (the figure number and the total number of toothpicks).

#5

| | | |
|---|---|---|
| $2n + 2$ | YES | Example answer: Substituting the figure number for *n* gives the correct number of total toothpicks for that figure for all figures 1-5. |
| $n + n + 2$ | YES | Example answer: Substituting the figure number for *n* gives the correct number of total toothpicks for that figure for all figures 1-5. |
| $n + n + n + n$ | NO | Example answer: This expression works for figure 1 where it correctly gives 4 total toothpicks. However, for figure 2, it says there should be 2+2+2+2, or 8, toothpicks and there are only 6 toothpicks for figure 2. |
| $2(n +1)$ | YES | Example answer: Substituting the figure number for *n* gives the correct number of total toothpicks for that figure for all figures 1-5. |
| $1 + 1 + n + n$ | YES | Example answer: Substituting the figure number for *n* gives the correct number of total toothpicks for that figure for all figures 1-5. |
| $2n + 1$ | NO | Example answer: Since 2 times the figure number plus 2 gives a correct value, it is not possible for 2 times the figure number plus 1 to ever give the same result. |

Teaching notes: Depending on where this lesson is incorporated into the teaching sequence, students may be in a position to use other explanations in the table to note why a given expression gives the correct number of toothpicks. For example, a student might recognize $n + n$ as equal to $2n$ and reason that $2n + 2 = n + n + 2$. Although 6.EE.A.3 is not a direct target of this lesson, students may use properties of operations as a way of showing equivalence of two expressions and concluding that since one gives the correct number of toothpicks, any expression equivalent to that one will also give the correct number of toothpicks. This reasoning should be valued and used as a way to tie the work of using properties of operations to the understanding that equivalent expressions name the same number regardless of which value is substituted (articulated in 6.EE.A.4).

Other students may use reasoning that relates the expression to the structure of the figures themselves. For more information on this perspective, see the next section on "Connecting Expressions to the Visual Models."

**Lesson Discussion Connecting Expressions to the Visual Models**

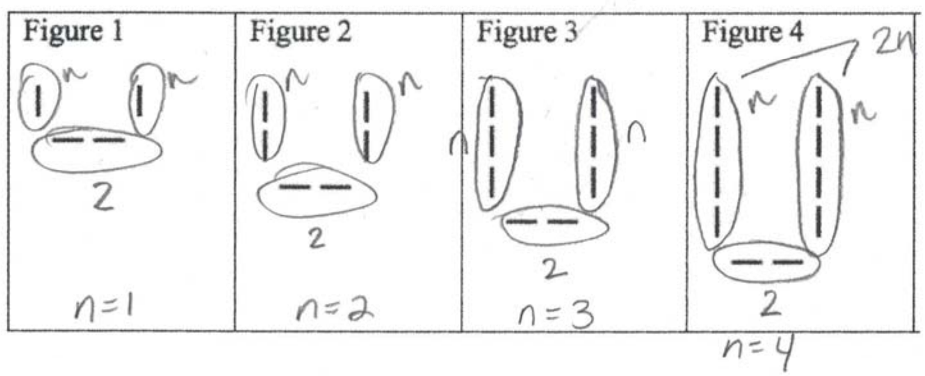
Refer to the page "Variations in how students relate expressions to the physical models." This page shows how students might relate the figure number, $n$, to the figure shown and use that to generate an expression. If this idea doesn't come up during the natural flow of the lesson, take the opportunity to help students understand why other students might come up with different but equivalent expressions depending on how they view the figures. You might ask a question like the following: *"How does the expression n + n + 1 + 1 relate to the figures shown? What might a student who wrote that as his expression for the total number of toothpicks been thinking as he looked at the figures?"* Another question might be, *"What might a student who wrote 4n have been thinking when looking at the figures? How could you explain to the student why 4n doesn't work for all of the figures shown?"*

**Extension/Homework**

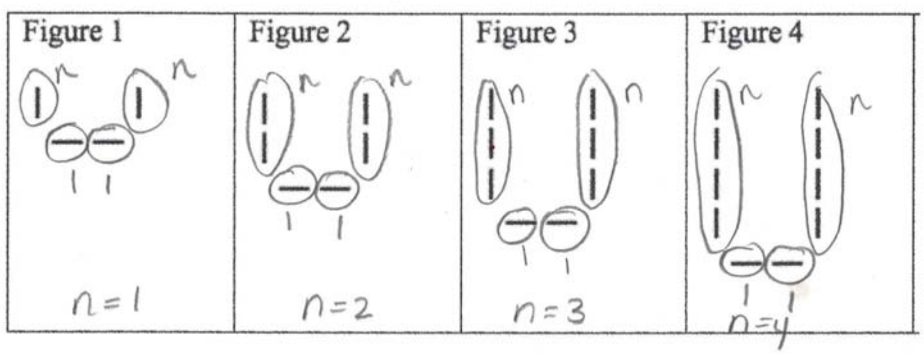
Can you write any other expressions that are equivalent to $2n + 2$ that we haven't discussed today? How do you know they are equivalent?

## Variations in how students relate expressions to the physical models

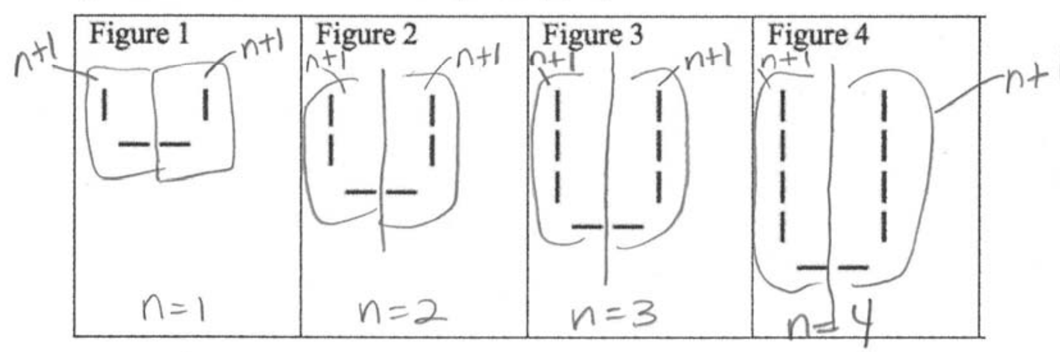
Expression for total number of toothpicks: $2n+2$




Expression for total number of toothpicks: $n+n+1+1$




Expression for total number of toothpicks: $2(n + 1)$



Name: ________________________________ Date: ________________________

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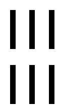
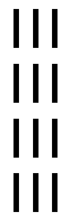
|  |  |  |  | |

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Investigating Equivalent Expressions

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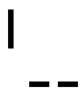
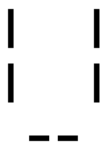
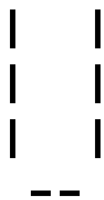
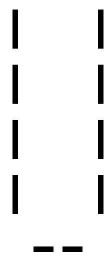
|  |  |  |  | |

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