Common Core Knowledge and Practice Survey Report Companion

Math Grades 3-5

About

The Common Core Knowledge and Practice Survey (Survey) is a tool for educators to use to reflect on their instructional practice and understanding of the Common Core State Standards (CCSS). Designed for use in a professional learning community (PLC) setting within a school, the Survey is meant to spark conversation, identify areas for growth, and offer concrete ways for teams of teachers to continue to align their practice to the Shifts. Following the administration of the Survey in a PLC, this resource can be used by coaches and teachers to better understand the Survey questions and their relationship to the Shifts.

The Survey was intentionally designed to take no more than 30 minutes. As a result, there are a limited number of questions pertaining to each aspect of the Shifts. When considering the performance of an individual or group on a Shift, it is important to look closely at the questions to tease out which aspect(s) of the Shift may be creating confusion. This resource is broken into two parts.

PART 1 : SURVEY

The Survey uses a variety of question types and formats to highlight specific features of the Standards and Shifts. It is intended to capture information from educators about their understanding of different elements of the Standards and how the Shifts manifest in planning and classroom instruction. Each Survey question will be followed by commentary. The "Purpose" section will offer information about why the question was included in the Survey. The "Rationale" section will explain the different response options.

Some of the Survey questions have correct answers because they ask about items that can be objectively verified. Other questions ask participants to apply their knowledge of the Shifts and Standards to a specific example or classroom scenario. For these questions, there may be circumstances in the course of instruction where any number of answer choices could be correctly applied; however, for the purpose of this Survey the correct answers are those actions that most closely tie instruction practice to the expectations of the Standards and Shifts. Finally, a subset of the questions about practice are designed to help illuminate what's currently happening in classrooms and schools, in order to facilitate conversation about what training or structures might support future work.

Each question will have a label to indicate which category the question relates to – one of the Shifts (Focus, Coherence, Rigor) or "Practice-Content Connections." Questions that have one or more correct answers will have the correct response(s) indicated.

Shift of Focus

Focus strongly where the Standards focus

Shift of Coherence

Think across grades, and link to major topics within grades

Shift of Rigor

In major topics pursue: conceptual understanding, procedural skill and fluency, and application with equal intensity.

Practice-Content Connections

Knowledge of the Mathematical Practice Standards and strategic connection of the Practice and Content standards in materials and instruction

PART 2 : SHIFTS SUMMARY

Part two shows which questions relate to each category. Part two also includes discussion questions and resource recommendations intended to drive conversation and provide a starting point for next steps and long-term professional learning.

Part 1 : Survey

Q. 4
Shift of Focus

Question
Which of the following belongs to the Major Work of the indicated grade or course? Please select all that apply.

2nd grade
- ❒ a) Identify line of symmetry in two dimensional figures
- ❒ b) Understand the place value
- ❒ c) Apply and extend previous understandings of multiplication and division to multiply and divide fractions
- ❒ d) Represent and solve problems involving addition and identify line of symmetry in two dimensional figures
- ❒ e) I don't know

3rd grade
- ❒ a) Multiply and divide within 100
- ❒ b) Identify the measures of central tendency and distribution
- ❒ c) Develop understanding of fractions as numbers
- ❒ d) Understand meaning of addition and subtraction
- ❒ e) I don't know

4th grade
- ❒ a) Examine transformations on the coordinate plane
- ❒ b) Generalize place value understanding for multi-digit whole numbers
- ❒ c) Extend understanding of fraction equivalence and ordering
- ❒ d) Create and extend patterns and sequences
- ❒ e) I don't know

5th grade
- ❒ a) Apply and extend previous understandings of multiplication and division to multiply and divide fractions
- ❒ b) Understand the place value system
- ❒ c) Understand and calculate probability of single events
- ❒ d) Identify line of symmetry in two dimensional figures
- ❒ e) I don't know

6th grade
- ❒ a) Understand ratio concepts and use ratio reasoning to solve problems
- ❒ b) Identify the measures of central tendency and distribution
- ❒ c) Identify and utilize rules of divisibility
- ❒ d) Apply and extend previous understandings of arithmetic to algebraic expressions
- ❒ e) I don't know

Correct Answer - 2nd Grade: b, d; 3rd Grade: a, c; 4th Grade: b, c; 5th Grade: a, b; 6th Grade: a, d

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PURPOSE

Rather than racing to cover topics in a mile-wide, inch-deep curriculum, the Standards require us to significantly narrow and deepen the way time and energy is spent in the math classroom. By focusing deeply on the Major Work of each grade, students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom. This question addresses teachers' knowledge of the Major Work of the grade. More information can be found at www.achievethecore.org/focus.

RATIONALE

This question showed different grade levels depending on the grade the teacher selected at the beginning of the Survey. The selected grade, one grade level below, and one grade level above were displayed for this question.

2nd grade
a) Identify line of symmetry in two dimensional figures - 4th grade expectation
b) Understanding the place value system - Major Work of 2nd grade
c) Multiplication and division of fractions – begins in 5th grade
d) Solving addition and subtraction word problems - Major Work of 2nd grade.
e) I don't know.

3rd grade
a) Multiplication and division within 100 - Major Work of 3rd grade.
b) Measures of central tendency – begins in 6th grade.
c) Understanding fractions as numbers - Major Work of 3rd grade.
d) Understanding the meaning of addition and subtraction – Kindergarten expectation
e) I don't know.

4th grade
a) The coordinate plane – begins in 5th grade.
b) Generalize place value understanding for multi-digit whole numbers - Major Work of 4th grade
c) Extend understanding of fraction equivalence and ordering - Major Work of 4th grade
d) Create and extend patterns and sequences - not in the Standards
e) I don't know.

5th grade
a) Applying and extend previous understandings of multiplication and division to multiply and divide fractions - Major Work of 5th grade
b) Understand the place value system - Major Work of 5th grade
c) Understand and calculate probability of single events - introduced in 7th grade
d) Identify line of symmetry in two dimensional figures - 4th grade standard
e) I don't know.

6th grade
a) Understand ratio concepts and use ratio reasoning to solve problems - Major Work of 6th grade
b) Identify the measures of central tendency and distribution - not in the Standards
c) Identify and utilize rules of divisibility - not in the Standards
d) Apply and extend previous understandings of arithmetic to algebraic expressions - Major Work of 6th grade
e) I don't know.

Q. 5

Shift of Coherence

Question

Over the past school year, how frequently have you done the following?

	Almost never/Never	About once a month	Several times a month	About weekly	Several times a week
a) Discussed Common Core State Standards for Mathematics with teachers in other grades					
b) Discussed Common Core State Standards for Mathematics with teachers in your own grade					
c) Looked at student mathematical work with other teachers for the purposes of professional development					
d) Received suggestions from colleagues for curricular materials aligned to the Common Core State Standards for Mathematics					

Correct Answers - N/A

PURPOSE

Supportive environments in which teachers are able to discuss, question, and work collectively to study student expectations support a deeper understanding of the coherence of the Standards. Teacher collaboration can facilitate more consistency in both instructional materials and instruction, and allows teachers to see connections across grades more clearly and make those connections in their classrooms. This question addresses the frequency of teacher collaboration around the Standards.

RATIONALE

There is no correct amount of time required for teachers to spend working together; however, teacher collaboration can support high quality CCSS-aligned instruction.

a) Vertical collaboration helps teachers develop a deeper understanding of the expectations of their own grade and provides an understanding of the progression of the standards in the surrounding grades.
b) Teachers collaborating within a grade allows for a collective reflection on specific content standards, lesson, and activities that are applicable to the content of the grade.
c) Examining student work with other teachers is useful in identifying key misconceptions and discussing how to make classroom adjustments.
d) Finding and sharing aligned resources with colleagues is useful for many schools/districts in which instructional materials are not aligned to the Standards.

Q. 6

Shift of Rigor

Question

Carefully examine each standard below and select which aspect of rigor is being targeted.

	Conceptual Understanding	Procedural Skill and Fluency	Application	I don't know
a) Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.				
b) Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.				
c) Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.				
d) Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.				
e) Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.				

Correct Answers -a) Application, b) Conceptual Understanding, c) Procedural Skill and Fluency, d) Procedural Skill and Fluency, e) Conceptual Understanding

PURPOSE

In order to reach the depth of the Standards, a balance of conceptual understanding, procedural skill/fluency, and application is required in the classroom. Each standard embodies at least one aspect of rigor; often the language in the standard indicates which aspect is being targeted (e.g., "fluently" identifies a procedural standard; "understand" identifies a conceptual understanding standard, and "real world problems" identifies an application standard). This question asks teachers to identify the aspect of rigor in the standard; understanding the intended aspect of rigor in a particular standard is critical for teachers to plan and execute lessons that address the expected learning of the Standards.

RATIONALE

a) The standard includes the phrase "solve real world problems" which indicates students are using their ability to perform arithmetic with fractions within the context of a real-world problem; this standard targets application.
b) "Understand the quantity 1/b as the quantity…" indicates students are learning about the concept of a fraction; this standard targets conceptual understanding.
c) "Read and write multi-digit numbers…compare two multi-digit numbers" indicates students are practicing the skill of writing and reading numbers and comparing their size; this standard targets procedural skill/fluency.
d) "Fluently multiply and divide within 100" targets procedural skill/fluency.
e) "Recognize that in a multi-digit number a digit in one place represents 10 times as much as it represents in the place to its right…" indicates students are reasoning about the size of numbers and how that relates to place value; this standard targets conceptual understanding.

Q. 7

Shifts of Focus, Coherence and Rigor

Question

Please indicate the extent to which you agree or disagree with the following statements as they relate to your mathematics teaching this school year.

	Strongly Disagree	Somewhat Disagree	Somewhat Agree	Strongly Agree	Not Applicable
a) I choose which standards to teach based on the major work of the grade.					
b) I try to incorporate conceptual understanding into every lesson that I teach.					
c) I try to incorporate real-world applications into every lesson that I teach.					
d) I use the textbook to determine the order of the standards that I teach.					
e) I use the wording of the standards to determine if procedural skills, conceptual understanding, and/or real-world applications are emphasized in my lessons.					
f) I order lessons based on the order of the standards at my grade.					
g) I spend less time in the classroom on additional/supporting standards.					
h) I organize which standards to teach based on how they connect to one another within and across units.					
i) I seek to balance my unit across procedural skills, conceptual knowledge and real-world applications.					
j) I consider students' prior knowledge when writing my lesson and learning objective.					
k) I try to give equal importance to all topics throughout the year.					

Correct Answers - N/A

PURPOSE

There are many pedagogical choices a teacher can make when supporting students in mathematics. This question details an array of instructional practices and asks teachers to identify which behaviors they exhibit in the classroom. Given the variety of decisions that can be made in different educational contexts, many of the statements above could be valid. However, some instructional practices better support Standards-based instruction and a teacher fully implementing the Standards would be more likely to emphasize those practices over others. To learn more about the Shifts, go to **www.achievethecore.org/shifts-mathematics**.

RATIONALE

The practices associated with the italicized sections below are those that a teacher fully implementing the Standards is likely to emphasize.

a) *Major Work of the grade is prioritized content within the Standards and requires more class time. Making instructional decisions based on the Major Work of the grade is important for ensuring that students spend the time necessary to master the concepts and skills that lead to algebra.*
b) Conceptual understanding is one of the three aspects of rigor and should be incorporated into lessons appropriately – based on the expectation of the standard. Not all standards focus on the development of mathematical concepts; lessons based on standards that set expectations of fluencies or the application of math should not target conceptual understanding.
c) Application is one of the three aspects of rigor and should be incorporated to lessons appropriately – based on the expectation of the standard. Not all standards focus on applying mathematical concepts; lessons based on standards that set expectations of fluencies or building mathematical concepts should not target application.
d) While the textbook might present a sound mathematical sequence of topics, the curriculum should be carefully reviewed for alignment to the Shifts. Tools such as the Instructional Materials Evaluation Tool (IMET) can help determine whether adjustments should be made. For more information go to **www.achievethecore.org/imet**.
e) *The Standards require a balance of conceptual understanding, procedural skill/fluency, and real-world application across the year, but the realization of these in the classroom should be specific to the intent of the standard being addressed. As noted in the rationale for Q6, the language of a standard indicates the aspect(s) of rigor targeted by the standard. Each of the aspects of rigor does not need to be attended to every day or in every lesson. For more information go to* ***www.achievethecore.org/rigor***.
f) The standards within each grade are organized by domain, not in a suggested instructional sequence. Decisions around how to sequence topics should be made thoughtfully based on the coherence of the content of the grade and the progressions of mathematical ideas.
g) *Teachers should spend the majority of instructional time on the Major Work of the grade, and strategically connect Supporting Work to strengthen an understanding of the Major Work. For more information go to* ***www.achievethecore.org/focus***.
h) *The Standards are built on mathematical progressions that support the development of concepts and skills across grades.*
i) The three aspects of rigor should be balanced across a full year in order to ensure students master the full depth of the Shifts. Units might have a balance of the aspects of rigor, but the relative time on each aspect will depend on which standards are being addressed in that unit.
j) *Building upon students' prior knowledge helps to make connections across grades clear to students, which, in turn, strengthens student understanding. For more information go to* ***www.achievethecore.org/coherence***.
k) *The Shift of Focus highlights that some content is more important for students to succeed and progress towards algebra. Teachers should spend the majority of instructional time on the Major Work of the grade, and strategically connect Supporting Work to strengthen an understanding of the Major Work. For more information go to* ***www.achievethecore.org/focus***.

Q. 8

Shift of Coherence

Question

Consider this standard.

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

This standard is preceded by which of the following standards? *(Select one.)*

○ a) Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

○ b) Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

i. Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent 5/4 as the product of 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4).

ii. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

○ c) Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

○ d) Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share ½ lb. of chocolate equally?*

How many ¾ cup servings are in 2/3 cup of yogurt? How wide is a rectangular strip of land with length ¾ mi. and area ½ square mi.?

○ e) I don't know.

Correct Answers - a

PURPOSE

The Standards are designed around coherent progressions that develop across grades. Understanding how content builds and connects to grades before and after is critical for teachers to properly support all students. This question asks teachers to choose the closest related standard that would precede the standard in the box; the standard comes from a prior grade in a related domain.

RATIONALE

a) In second grade, students are partitioning a number line into equal lengths which is conceptually a precursor to the standard in the box, in which students develop understanding of a fraction representing equal parts of one whole.

b) Multiplying fractions by a whole number would be taught after students are introduced to the concept of a

fraction.
c) Multiplication of fractions and mixed numbers would also be taught after students are introduced to the concept of a fraction.
d) Interpreting and computing quotients of fractions appears in sixth grade, after students have mastered performing addition and subtraction with fractions.

Q. 9

Shift of Coherence

Question
Consider this standard.

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

This standard prepares students for which of the following standards? *(Select one.)*

- ○ a) Understand a fraction as a number on the number line; represent fractions on a number line diagram.
  - *i.* Represent a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size *1/b* and that the endpoint of the part based at 0 locates the number *1/b* on the number line.
  - *ii.* Represent a fraction *a/b* on a number line diagram by marking off *a* lengths *1/b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.
- ○ b) Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
  - *iii. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product of 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4).*
  - *iv. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5. (In general, n x (a/b) = (n x a)/b.)*
- ○ c) Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share ½ lb. of chocolate equally? How many ¾ cup servings are in 2/3 cup of yogurt? How wide is a rectangular strip of land with length ¾ mi. and area ½ square mi.?*
- ○ d) Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
- ○ e) I don't know.

Correct Answers - c

PURPOSE

This question asks teachers to choose the closest related standard that would follow the standard in the box. The correct answer comes from the next grade in a related domain.

RATIONALE

a) The standards require students to understand of the meaning of a fraction before multiplying fractions.
b) Multiplying a fraction by a whole number precedes multiplying a fraction by a fraction.
c) Multiplying fractions by fractions in fifth grade naturally extends to dividing fractions by fractions in sixth grade.
d) Partitioning a number line into equal whole number lengths is a precursor to developing the concept of a fraction and would precede multiplying fractions.

Q. 10

Shift of Coherence

Question

From the list of five standards below, choose three (in any order) that could be taught together in a coherent unit.

From the list of six standards below, select ***three*** (in any order) that could be taught together in a coherent unit. Please ensure that you select three standards.

☐ a) Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

☐ b) Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

☐ c) Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

☐ d) Use place value understanding to round multi-digit whole numbers to any place.

☐ e) Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

☐ f) Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Correct Answers - b, c, e

PURPOSE

Understanding how standards relate to one another and support each other is a powerful tool in the classroom to ensure each lesson is not treated as a discrete topic and students are given the opportunity to build upon their understanding and prior knowledge. This question asks teachers to identify coherence within a grade as opposed to across grades as seen in questions 8 and 9.

RATIONALE

The standards listed in b, c, and e all relate to fractions and could reasonably be taught together in a coherent unit. The other standards listed in a, d, and f address generating patterns and rules with numbers, rounding, and drawing points, lines, and line segments, which do not directly relate to fractions.

a) Number and shape patterns do not relate to the other standards listed.
b) Making a line plot and solving problems involving addition and subtraction of fractions by using information presented in line plots addresses the early development of understanding and manipulating fractions.
c) Generating equivalent fractions sets the stage for performing arithmetic with fractions which relates to the early development of understanding and manipulating fractions.
d) Rounding multi-digit numbers does not conceptually relate to the other standards listed.
e) Understanding of a fraction as a/b relates to the early development of understanding and manipulating fractions.
f) Drawing points, line segments, rays, and angles and identifying geometric shapes does not relate to the other standards listed.

Q. 11

Shift of Coherence

Question

Please briefly explain your reasoning for your selections in Q. 10

PURPOSE

This question is not scored. It allows teachers to attach a rationale to their responses. We recommend teachers share their ideas and reflections on developing a coherent unit with their PLCs as a collaborative learning opportunity.

Q. 12
Shift of Focus

Question
Yesterday, Mr. Jones taught a math lesson to his 3rd grade class and he would like comments on a few elements of his lesson.

Mr. Jones wants to be sure his lesson plan was Common Core-aligned. This was the objective for his class: *"Understand multiplication as equal groups."*

This objective is:

- ○ a) Supporting work of 3rd grade
- ○ b) Major work of 3rd grade
- ○ c) Not a 3rd grade objective
- ○ d) I don't know.

Correct Answers - b

PURPOSE
A large majority of class time should be devoted to the Major Work of the grade, as it is the content most central to grade-level understanding. The lesson objective is often used to convey the intent of the lesson, and it is frequently where information about the standard(s) in the lesson can be found. This question asks teachers to use the lesson objective to identify the standard in the lesson and identify how that standard relates to the focus of the grade (specifically, whether it is appropriate for the grade and whether it is Major or Supporting Work). For more information, see Criterion #1 of the K–8 Publishers' Criteria for the Common Core at www.achievethecore.org/publisherscriteria-math-k-8.

RATIONALE
The lesson objective refers to students understanding the meaning of multiplication which is Major Work of 1st grade. Understanding and practicing multiplication is critical to students' progress towards algebra.

The Supporting Work of 3rd grade—"Use place value understanding and properties of operations to perform multi-digit arithmetic" and "geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures"—could be connected to the Major Work by posing problems in which place value understanding would be beneficial, or geometric measurement problems that require students to use multiplication.

Q. 13

Practice-Content Connections

Question

Which activity would be appropriate for this lesson objective? Select one.

- ○ a) Students work with counters to create equal groups and writing number sentences for their models
- ○ b) Students play a game for multiplication fact fluency
- ○ c) Students work with partners to write word problems for multi-digit multiplication
- ○ d) Students work with 3 digit numbers and base-10 blocks
- ○ e) I don't know.

Correct Answers - a

PURPOSE

Different standards require different types of activities or tasks depending upon what understanding or application a student needs to demonstate in order to exhibit mastery. It is important that any activity students are asked to do supports the learning and practice of the lesson's target. This question asks teachers to choose the appropriate lesson activities for a targeted standard.

RATIONALE

a) Working with counters to create equal groups supports students' conceptual understanding of multiplication and allows them to use physical models to reinforce their understanding.
b) Playing a game for multiplication fact fluency is an activity that would support the practice of procedural skill/fluency, which is not targeted by this standard.
c) This activity addresses multi-digit multiplication which is far more advanced understanding of multiplication.
d) This activity supports base-ten place value understanding and is not related to multiplication or equal groups.

Q. 14

Shift of Rigor

Question

In the lesson plan below, which type(s) of student learning is/are addressed? Please select all that apply.

Objective:
Understand multiplication as equal groups.

Lesson plan:

- Seat students at tables with personal white boards and 12 counters each.
- Ask students to use their counters and make equal groups of two. Afterwards, ask students how many sets of two they have.
- Ask students to write a number sentence to show these groups on their personal white boards. Students may write an addition sentence (2 + 2 + 2 + 2 + 2 + 2 = 12) or a multiplication sentence (6 × 2 = 12).
  - Write both equations on the board and have students investigate how the addition sentence is similar to the multiplication sentence. In the discussion, be sure to emphasize the meaning of the two factors in the multiplication sentence; 6 is the number of groups and 2 is the number of counters in each group.
- Ask students to turn and talk to their partner. Students should ask each other, "What part of the addition sentence represents the number of groups? What part represents how many are in each group?"
- Repeat the process with groups of three, four, and six, writing equations for each arrangement.

In the lesson plan above, which type(s) of student learning is/are addressed? *(Please check ALL that apply.)*

- ❒ a) Conceptual understanding is addressed.
- ❒ b) Procedural skill is addressed.
- ❒ c) Application is addressed.
- ❒ d) None of the above is addressed.

Correct Answers - a

PURPOSE

It is important to identify the aspect(s) of rigor targeted in a standard in order to know what type of activities are most appropriate for the instructional materials. This question asks teachers to analyze the lesson plan and identify which aspect of rigor is addressed in the objective and activities.

RATIONALE

a) In the above lesson plan, the objective is for students to "understand multiplication as equal groups." The word "understand" clues us into the fact that the teacher intends to develop the concept of multiplication. In the activity, students are using manipulatives to create equal groups and then connecting their models to multiplication or addition sentences. They are then deriving meaning from the multiplication sentences by discussing each part of the sentence and what it represents, which underscores the development of the concept of multiplication.
b) While students are writing and solving addition and multiplication sentences in this lesson, which can sometimes be procedural in nature, they are doing so in order to develop the meaning of multiplication.
c) Students are working on pure math problems throughout this lesson and therefore are not applying math to any real-world situations or problems.

Q. 15

Shift of Focus

Question

Which standard is addressed in the lesson plan?

- ❒ a) Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = __ ÷ 3, 6 × 6 = ?*
- ❒ b) Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
- ❒ c) Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
- ❒ d) Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*
- ❒ e) None of the above.
- ❒ f) I don't know.

Correct Answers - b

PURPOSE

Building lesson objectives from the language of the targeted standard(s) is critical to ensuring the lesson will meet the expectations of that standard. This question asks teachers to use the lesson objective to identify the standard addressed.

RATIONALE

a) Students are working with number sentences in the lesson, but this standard is procedural whereas the lesson is mostly conceptual and the lesson does not address division.
b) The lesson objective "understand multiplication as equal groups" most closely relates to "interpret products of whole numbers…" Students are defining and constructing equal groups in this particular lesson to support the lesson objective.
c) Students are just learning the meaning of multiplication in this lesson and are not yet working on fluently multiplying or dividing in any of the lesson activities.
d) The lesson does not require students to identify any arithmetic patterns or to explain them using properties of operations in the lesson activities.

Q. 16

Rigor

Question

Briefly, how would you improve upon the lesson plan to more thoroughly address this standard?

PURPOSE

This lesson was chosen because it offered specific sample material for the questions being asked, not because it is an exemplary lesson and should be used as a model. This question is not scored; it offers teachers an opportunity to submit revisions and reflect on improvements for the lesson. We recommend teachers share the ideas and reflections on the lesson with their PLCs as a collaborative learning opportunity.

Q. 17

Practice-Content Connections

Question

Given the sample lesson plan, which statement(s) below reflect the lesson? Please select all that apply.

- ❒ a) The lesson asks students to attend to precision.
- ❒ b) The lesson asks students to reason abstractly and quantitatively.
- ❒ c) The lesson asks students to construct arguments and critique reasoning in others.
- ❒ d) The lesson requires students to model with mathematics.
- ❒ e) The lesson has students look for and express repeated reasoning.
- ❒ f) The lesson builds on previous knowledge.
- ❒ g) The lesson encourages students to use appropriate tools strategically.
- ❒ h) I don't know.

Correct Answers - b, e, f

PURPOSE

The mathematical practices describe the varieties of expertise math teachers should seek to develop in their students. These student behaviors and actions are elicited by a teacher in the classroom within a lesson and in connection to the classroom norms. The content and/or approach of a particular lesson should support the development of particular practices. This question asks teachers for the practices required based on the description of the activities in the lesson plan.

RATIONALE

a) The lesson activities do not focus specifically on attention to precision.
b) Reasoning abstractly and quantitatively is reflected in students physically creating equal groups and then representing them with multiplication sentences.
c) Though students are reasoning in this lesson, they are not required to explicitly construct arguments or critique the reasoning of others.
d) Modeling with mathematics means using mathematics to represent real-world situations. This lesson uses manipulatives to communicate and facilitate understanding of mathematical ideas, but because the lesson does not involve representing real-world situations with mathematics, modeling with mathematics is not explicitly targeted.
e) Repeated reasoning is reflected when students compare their repeated addition sentences to their multiplication sentences.
f) The lesson builds on previous student knowledge because they are using their understanding of addition in relation to their development of understanding the meaning of multiplication.
g) Using appropriate tools strategically involves an element of strategy: students must choose which tool to use and when to use it. While this lesson does make use of tools, because the tools are chosen for the students, the students are not engaging in this practice.

Q. 18

Shift of Coherence

Question

Which prerequisite content will prepare students for this lesson? Select one.

- ❍ a) Understand that the digits of a three-digit number represent amounts of hundreds, tens, and ones, e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
- ❍ b) Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
- ❍ c) Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100 900.
- ❍ d) Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
- ❍ e) None of the above.
- ❍ f) I don't know.

Correct Answers - d

PURPOSE

Considering how the content of the Standards connects to one another and builds across grades is important for understanding the coherence of the Standards and for supporting differentiation within the classroom. This question asks teachers to reflect on the content addressed in the lesson in order to identify which mathematical concepts students would need to understand prior to the lesson.

RATIONALE

a) This standard describes content students would have previously learned; however, it is content more closely related to place value and developing an understanding of the number system.
b) This standard is from a previous grade dealing with addition and subtraction; however, it does not directly build towards understanding multiplication.
c) This standard is from a previous grade dealing with addition and subtraction and place value; however, it does not directly build towards understanding multiplication.
d) Using arrays to create equations with equal addends is a logical prerequisite to work with multiplication. Students practice making arrays and use addition/subtraction to find the total, which leads naturally into equal groups and multiplication.

Q. 19

Shift of Coherence

Question

This lesson most directly prepares students to learn which of the following standards? (Select one.)

- ○ a) Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
- ○ b) Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplication comparisons as multiplication equations.
- ○ c) Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- ○ d) Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
- ○ e) None of the above.
- ○ f) I don't know.

Correct Answers - b

PURPOSE

This question asks teachers to reflect on the content addressed in the lesson in order to identify which mathematical concepts students would be prepared to learn next.

RATIONALE

a) This standard is related to multiplication, but work with factors pairs, multiples, primes, and composites is considerably more advanced conceptually in comparison to the foundational multiplication work students are doing in the lesson.
b) This standard describes the next stage in students' understanding of multiplication, moving from multiplication as equal groups to multiplication as comparison.
c) This standard is related to multiplication, but is considerably more advanced procedurally in comparison to the work students are doing in the lesson.
d) This standard is about understanding the place value system. Although it is related conceptually to the idea of multiplication, Choice b—interpreting a multiplication as a comparison—is most directly related to the content of the lesson.

Q. 20

Practice-Content Connections

Question

Thinking about the last complete unit that you taught in 3rd-5th grade, how often did you do the following?

	Never	Rarely	Sometimes	Often	In all or most lessons
a) Build on prior skills and knowledge when teaching new content					
b) Ground procedures and formulas in conceptual understanding					
c) Make the mathematics of the lesson explicit by using explanations, representations, and/or examples					
d) Use repeated practice to improve students' computational skills					
e) Have students do work with and practice grade-level problems and exercises. f) Emphasize one solution method to strengthen all students' understanding of the content					
g) Have students choose and use appropriate tools when solving a problem					
h) Check for understanding throughout the lesson using informal, but deliberate methods (such as questioning or assigning short problems)					
i) Summarize the mathematics with references to student work to reinforce the focus of the lesson					
j) Predominantly use questions and problems that are from the textbook					
k) Review standards from previous grades					
l) Ask students to explain and justify their work					
m) Provide feedback to help students revise initial work					

Correct Answers - N/A

PURPOSE

As with question 7, this question details an array of choices a teacher can make when supporting students with mathematics and asks teachers to identify which behaviors they exhibit in the classroom. Given the variety of decisions that can be made in different educational contexts, many of the statements above could be valid. However, some instructional practices better support the content and structure of the Standards, and a teacher fully implementing the Standards would be more likely to emphasize those practices over others. To learn more about Shifts-aligned instructional practice, go to www.achievethecore.org/instructional-practice.

RATIONALE

The practices associated with the italicized sections below are those that a teacher fully implementing the Standards is likely to emphasize.

a) *Building on previous work ensures that students see math as cohesive as opposed to a set of discrete skills.*
b) *Grounding procedures and formulas in conceptual understanding ensures that students understand why a procedure works and will be able to apply it to more complex mathematics.*
c) *Using a variety of explanations, representations, and examples to make the mathematics of the lesson clear helps to reinforce student understanding. Representations display concepts or problems by using drawings or models, whereas examples may show different strategies or methods to solve a particular problem.*

d) *Repeated practice is important for students to build towards the fluency and procedural skill expectations of their grade-level standards.*
e) *All students need consistent practice with grade-level problems and exercises in order to meet the expectations of the Shifts.*
f) A variety of student solution methods should be shared and examined together to support mathematical understanding for all students.
g) *MP.5 explicitly states that students should choose appropriate tools strategically. Students must have the opportunity to select tools, when appropriate.*
h) *Checks for understanding throughout the lesson allow a teacher to assess progress of all students and make adjustments to instruction, as needed.*
i) *A lesson that includes a summary with references to student work and discussion reinforces the mathematics of the lesson and supports student learning.*
j) All materials should be carefully reviewed for alignment to the Shifts. If the text is found to be aligned to the Shifts, using its tasks frequently is appropriate. However, if the text is not aligned to the Shifts, educators should explore supplementing with other resources for questions and problems.
k) While there may be appropriate times to briefly review the standards of a previous grade, the majority of instruction should be on grade-level standards.
l) *Asking students to regularly explain and justify work and providing feedback that helps students revise initial work supports student in developing mathematical proficiency.*
m) *Providing feedback on student work and allowing students to revise and correct/improve their answers supports students in perseverance and developing strong arguments and evidence in their mathematical reasoning.*

Q. 21

Practice-Content Connections

Question
A teacher walking around the classroom overhears the comments below during student group work. Which comment shows students demonstrating the practice standard "construct viable arguments and critique the reasoning of others"? Select one.

- ❍ a) "That could be the answer, or the answer could be 40."
- ❍ b) "No, the answer can't be 27, because when you multiply two even numbers, the answer must be even."
- ❍ c) "Yep, I agree that the answer is 32."
- ❍ d) "I don't think the answer is 32. I'm going to ask the teacher."
- ❍ e) I don't know

Correct Answers - b

PURPOSE

Understanding how the mathematical practices are translated into the classroom and student behavior is important for ensuring CCSS-aligned instruction. This question asks teachers to select the activity that best embodies MP.3.

RATIONALE

a) The student's explanation does not include any reasoning.
b) The student clearly communicates reasoning in her response.
c) The student agrees with no justification or reasoning.
d) The student relies on the teacher for identifying and/or understanding the correct answer.

Q. 22

Practice-Content Connections

Question

Which scenario shows students demonstrating the practice standard "**model with mathematics**"? Select one.

- ❍ a) Students completing a worksheet on adding and subtracting multi-digit numbers.
- ❍ b) Students explaining the relationship between multiplication and division.
- ❍ c) Students using past sales and inventory in order to predict future sales and calculate a budget for the school store.
- ❍ d) Students answering a set of routine word problems on right rectangular prisms.
- ❍ e) I don't know

Correct Answers - c

PURPOSE

Understanding how the mathematical practices are translated into classroom activities and student behavior is important for ensuring aligned instruction. This question asks teachers to select the activity that best embodies MP.4.

PURPOSE

a) Students are not required to apply mathematics in a real-world situation.
b) Students are developing their conceptual understanding around multiplication and do not apply their knowledge of mathematics to a real-world situation
c) Students are using data in a real-world context which is one example of modeling with mathematics. They are identifying important quantities, analyzing them, and interpreting results to draw conclusions in a grade-appropriate context.
d) Students do not apply their knowledge of mathematics to a real-world situation.

Part 2 : Shifts Summary

In the following section, each category will be shown along with the questions from the Survey that comprise that data. These are followed by discussion questions and recommended resources.
The discussion questions are meant to provide guidance for coaches and teachers to collectively reflect on classroom practice in relation to the Shifts. We recommend that these be used in a professional learning community or professional development setting in which educators can have an open and honest dialogue about the current state of aligning instruction and practice to the Standards. These conversations will help educators continue to develop strategies and make progress towards effective implementation of the Shifts.

The recommended resources are meant to provide a starting point to support the discussion between instructional leaders/coaches and teachers, and to help educators continue to learn about the Shifts and how they can be translated into classroom practice. You can find these and many other professional development and classroom resources at www.achievethecore.org.

Category	Topic	Question Number
Focus	Identify Major Work of the Grade	4, 12, 15
	Agreement with the statements about dedicating time to the major work of the grade.	7a, 7j, 7g
Rigor	Identification of rigor targeted by a standard and how rigor manifests in a lesson plan.	6, 14
	Evaluate the quality of lessons based on aspects of rigor	16 (unscored)
	Agreement with the statements about balance teaching the three aspects of rigor throughout the year.	7b, 7c, 7e, 7i
Coherence	Understanding of the progression of standards within and across grades.	8, 9, 18, 19
	Recognize how standards in a grade form a coherent unit.	10, 11 (unscored)
	Frequency at which teachers meet with colleagues to discuss the Common Core State Standards for Mathematics.	5a, 5b
	Agreement with statements about using coherence to organize their curriculum.	7d, 7f, 7h, 7k
Practice-Content Connections	Frequency in which teachers connect the standards for mathematical practices and standards for mathematical content in the classroom.	20
	Identification of mathematical practices elicited in a lesson plan.	17, 13, 21, 22

Shift of Focus

DISCUSSION QUESTIONS

1. What does Major Work of the grade mean? Why is it so important?
2. What is the Major Work of our grade?
3. Approximately how much time are we teaching Major Work? What's our evidence?
4. How have we adapted our curricular materials to spend more time on the Major Work of the grade?
5. What have we stopped teaching since implementing the CCSS for Mathematics?
6. What have we taught less of since implementing the CCSS for Mathematics?

Resources

1. The Shifts (http://achievethecore.org/shifts-mathematics): Webpage containing information and resources of the three Shifts
2. Focus by Grade Level (http://achievethecore.org/focus): A collection of PDFs detailing the mathematical content emphasized in the Standards by grade level. Includes Widely Applicable Prerequisites for High School.

Shift of Coherence

DISCUSSION QUESTIONS

1. When do we work with teachers in surrounding grade-levels to help students make connections across grades/courses?
2. What opportunities have we found to create coherence among the standards for our students, both within and across grades/courses?
3. Have we found ways to connect major/supporting work in our grade/course? When? What are examples?

Resources

1. The Shifts (http://achievethecore.org/shifts-mathematics): Webpage containing information and resources of the three Shifts
2. Progressions Documents (http://achievethecore.org/progressions): A collection of narratives that explain how mathematical content develops coherently across grades

Shift of Rigor

DISCUSSION QUESTIONS

1. What are the fluency or procedural skill standards for our grade?
2. What standards in our grade focus on conceptual understanding (in which students make meaning of the math)?
3. Which standards in our grade ask students to apply their knowledge in real-world settings?
4. Are all students given time for regular practice with the fluency standards?
5. Does our teaching reflect a balance of conceptual understanding, procedural skill/fluency, and application? What is our evidence?

Resources

1. The Shifts (http://achievethecore.org/shifts-mathematics): Webpage containing information and resources of the three Shifts
2. Annotated Tasks (http://achievethecore.org/math-tasks): Math tasks that illustrate the K-12 standards
3. Annotated Mini-Assessments (http://achievethecore.org/math-mini-assessments): A collection of mini-assessments designed for teachers to use

Practice-Content Connections

DISCUSSION QUESTIONS

1. Are we incorporating the mathematical practices in a way that ensures students learn grade-level content?
2. How do we make decisions about which mathematical practices to target within a specific lesson?
3. What evidence do we look for from students to know that they are demonstrating the mathematical practices?

Resources

1. The Standards for Mathematics Practice (http://achievethecore.org/math-practices): excerpted from the Standards, describe the behaviors and skills meant to be elicited by teachers in the math classroom.
2. Planning Tool (http://achievethecore.org/planning-tool): Lesson Planning Tool
3. Coaching Tool (http://achievethecore.org/coaching-tool): Tool to assist teachers, and those who support them, build understanding about CCSS-aligned instruction