

## High-Quality Mathematics Items Module (Grades 6-8)

The pages that follow contain a selection of items from chapter assessments in grades 6-8. These items are representative of the range available in many textbook series. This activity is designed to help teachers think about how they can revise chapter tests to better align to the Standards. Each item below can be revised to more closely embody the characteristics described in the High-Quality Mathematics Items Modules.

1. Solve all of the items.
2. Take a close look at each item, thinking about what the modules explain about expectations of high-quality mathematics items. Focus on the following features:
  - a. If the item aligns to a Supporting Work standard, does the item connect to Major Work? (Principle 1)
  - b. Does the item align to the aspect of rigor targeted in the Standards? (Principle 2)
  - c. Does the item align to the grade-level expectations? (Principle 3)
  - d. Does the item address the central concern of the identified standard? (Principle 4)
  - e. If the item aligns to a Standard for Mathematical Practice, is the item appropriate to the grade? (Principle 7)
  - f. Does the item type/format of the item match the content? (Principle 8)
3. Using the chart below, record your thoughts about which Alignment Principle can be used to improve each item.
4. With the Alignment Principle in mind, revise the item.
5. After time for individual reflection, discuss your findings and your proposed revision with your colleagues.

**Answer Key:**

*(Note: There may be other principles that could help to strengthen the alignment of the items. The ones listed here are examples.)*

Standard	Item	Assessment Principle?	Sample Revised Item
<p>6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math> and <math>x</math> are all nonnegative rational numbers.</p>	<p>Theresa signed up for a new phone plan. She has to pay \$74.99 a month, which includes all phone calls. Text messages cost an additional 2 cents each. Which equation represents how many text messages she sent if her phone bill was \$76.35, and <math>y</math> represents the number of text messages she sent that month?</p> <p>a) <math>74.99 + 0.02y = 76.35</math>            b) <math>74.99 + .2y = 76.35</math>            c) <math>76.35 + 0.02y = 74.99</math>            d) <math>76.35 = 74.99 - 0.02y</math></p>	<p>3. Items are designed to attend to content limits articulated in the standards.</p>	<p>Theresa's new phone plan charges 2 cents per text, with the first 10 texts free. Which two equations can be used to show how many texts she sent in a month where she paid \$4.52 for text messages? Let <math>y</math> equal the number of texts she sent.</p> <p>a) <math>0.02(y-10) = 4.52</math>            b) <math>.2(y-10) = 4.52</math>            c) <math>0.02y - .2 = 4.52</math>            d) <math>0.02y - 10 = 4.52</math></p> <p>Rationale: The standard limits the work to one-step equations. This problem still engages students with the extent of the rational number system appropriate to the grade but limits to the form of equations named in the standard.</p>

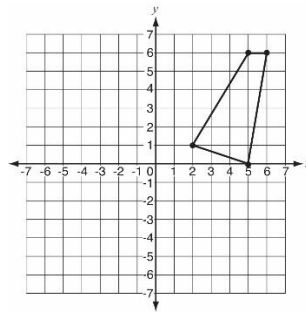
<p>6.NS.A Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</p>	<p>Find the quotient:</p> $100 \div \frac{2}{3} =$ <p>a) <math>\frac{200}{3}</math></p> <p>b) <math>\frac{300}{200}</math></p> <p>c) <math>\frac{50}{3}</math></p> <p>d) <math>\frac{300}{2}</math></p>	<p>2. Items are designed to address the aspect(s) of rigor (conceptual understanding, procedural skill, and application) evident in the language of the content standards.</p>	<p>When you divide 100 by a positive whole number, the result is always less than or equal to 100. This is not always true when you divided by a positive fraction.</p> <p>Give an example of a fraction <math>\frac{a}{b}</math> where <math>100 \div \frac{a}{b} &lt; 100</math>: _____</p> <p>Give an example of a fraction <math>\frac{c}{d}</math> where <math>100 \div \frac{c}{d} &gt; 100</math>: _____</p> <p><i>Adapted from Smarter Balanced Grades 6-8 Claim 3 Item Specifications; accessed July 2016</i></p> <p>Rationale: This item aligns to the cluster level, which emphasizes understanding by expecting students to apply previous understandings of operations to working with fractions. The original item is purely procedural and will not generate information on students' conceptual understanding (although it may be helpful for assessing the procedural skill of 6.NS.A.1.) The revised item assesses conceptual understanding by abstracting the work students are doing.</p>
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6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

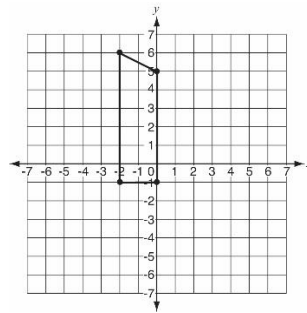
Which coordinate grid shows the trapezoid formed by the following points?

- (-2, 6)
- (-2, -1)
- (5,6)
- (5,0)

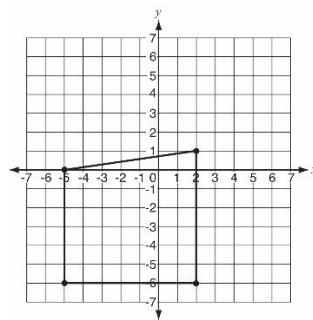
a)



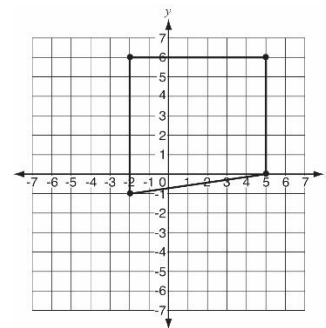
b)



c)



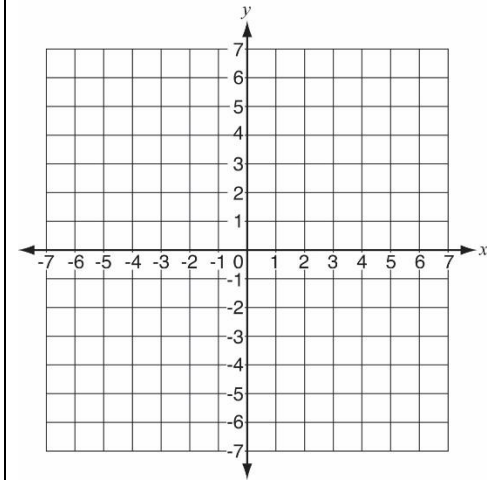
d)



8. Item types are chosen to match the item's purpose and as part of the evidence required by the standards.

Graph the following points on the coordinate grid:

- (-2, 6)
- (-2, -1)
- (5,6)
- (5,0)



Connect them with line segments. What polygon do the points create?

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Rationale: The standard requires that students draw polygons on the coordinate plane. Therefore, the multiple-choice item type is not able to assess students' ability to perform the action of the standard. The revised item matches the verb of the standard (draw).

<p>7. SP.A Use random sampling to draw inferences about a population.</p>	<p>Jean wants to find out information about whether more students at her middle school have brothers or sisters. She knows she can't ask all 700 students in her school. Which would be the most effective way to sample the student population?</p> <ul style="list-style-type: none"> <li>a) Talk to every 10th student that walks out of the building after school</li> <li>b) Survey all students who have math 1st period</li> <li>c) Survey all the students who ride the bus with her</li> <li>d) Talk to 20 students in each grade</li> </ul>	<p>1. Most items aligned to standards in supporting clusters connect to the Major Work of the grade.</p>	<p>Jean wants to find out information about whether more students at her middle school have brothers or sisters. She conducts a survey after school and asks every 10th student who exits the building. These are her survey results:</p> <table border="1" data-bbox="1360 505 1791 821"> <thead> <tr> <th>Siblings</th> <th>Number of students</th> </tr> </thead> <tbody> <tr> <td>Just brothers</td> <td>27</td> </tr> <tr> <td>Just sisters</td> <td>22</td> </tr> <tr> <td>Brothers and sisters</td> <td>14</td> </tr> <tr> <td>No siblings</td> <td>3</td> </tr> </tbody> </table> <p>If her school has 700 students, what is the best estimate of the number of students who have both brothers and sisters?</p> <ul style="list-style-type: none"> <li>a) About 68 students</li> <li>b) About 150 students</li> <li>c) About 200 students</li> <li>d) About 330 students</li> </ul> <p>Rationale: The cluster on using sampling data provides a natural and important connection to the standards in the Ratio and Proportion domain in 7th grade. This item allows students to use proportions to draw conclusions based on sampling data collected, whereas the original item only focuses on sampling methods and misses the opportunity to connect to the Major Work of the RP domain.</p>	Siblings	Number of students	Just brothers	27	Just sisters	22	Brothers and sisters	14	No siblings	3
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<p>8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.</p>	<p>What is the fraction notation for <math>0.\overline{22}</math>?</p> <p>a) <math>\frac{1}{2}</math></p> <p>b) <math>\frac{1}{9}</math></p> <p>c) <math>\frac{2}{9}</math></p> <p>d) <math>\frac{2}{11}</math></p>	<p>4. Most items aligned to a single content standard should assess the central concern of the standard.</p>	<p>Which number is irrational?</p> <p>a) <math>\frac{1}{3}</math></p> <p>b) <math>\sqrt{.45}</math></p> <p>c) <math>\sqrt{16}</math></p> <p>d) 0.241567...</p> <p>Rationale: The central concern of the standard is understanding the difference between rational and irrational numbers. In high school, students will be required to compute and reason with both types of numbers, so it is important for students to develop an understanding of the differences in grade 8. In the original item, the requirement for students is to convert between fractions and decimals, which is not at the heart of the standard.</p>
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8.EE.C.7b

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Solve for x. On each line, explain the step you used to create an equivalent equation.

Equation	Justification
$\frac{1}{5}(3x - 9) + .8 = \frac{5}{6}x + 10$	Original equation

2. Items are designed to address the aspect(s) of rigor (conceptual understanding, procedural skill, and application) evident in the language of the content standards.

Solve for x:

$$\frac{1}{5}(3x - 9) + .8 = \frac{5}{6}x + 10$$

Rationale: By grade 8, solving equations should be a procedural skill that builds on the conceptual understanding students have developed in previous grades. In order to assess the procedural skill, it is no longer necessary to ask students to explain each step in the procedure.

MP.4 Model with mathematics.  
(7th grade)

This table represents the cost of renting a truck from Moving Company X and Moving Company Y. Each company charges a one-time rental fee plus a charge for each mile driven.

Moving Company	One-time Rental Fee	Charge per Mile
X	\$150	\$0.25
Y	\$ 50	\$0.75

Graph two linear equations that represent the cost of using each moving company given a number of miles driven.

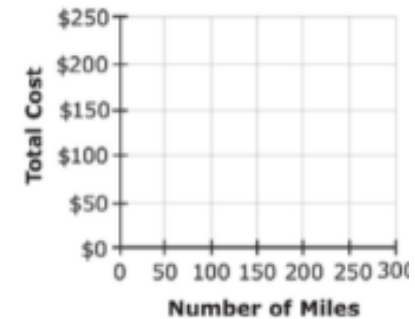


7. The demands of items measuring the Standards for Mathematical Practice are appropriate to the targeted grade level.

This table represents the cost of renting a truck from Moving Company X and Moving Company Y. Each company charges a one-time rental fee plus a charge for each mile driven.

Moving Company	One-time Rental Fee	Charge per Mile
X	\$150	\$0.25
Y	\$ 50	\$0.75

Graph two linear equations that represent the cost of using each moving company given a number of miles driven.





Given the map below, with a scale of 1 cm = x miles, name two cities for which a move between them would be less expensive with Moving Company Y.



*Adapted from Smarter Balanced Grades 6-8 Claim 4 Item Specifications; accessed July 2016*

**Rationale:** In the middle school grades, students should be making assumptions as they are using mathematics to model real-world problems. The original item could be a good way to assess 7.EE.B.4. However, to meet the expectations of the MP for the grade, the item should require students to deal with more complexity, as they would need to in Part B of the revised item. The revised item also takes advantage of connecting mathematical ideas across domains within the 7th grade standards.