COMMUNICATIONS

The Common Core and the Potential for Mathematicians to Improve the Teaching of School Mathematics

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My parents' diner on the outskirts of Detroit served a boisterous clientele of truck drivers, bookies, and laborers, yet there were mathematical moments to be had there for a boy always underfoot. Once when I was six or seven years old, a burly man not long out of the Appalachian hills played against type by pulling apart a book of matches and introducing me to the game of Nim. (The man actually looked a little like John H. Conway, now that I think of it.) The student of game theory could also usually find pinochle or chess underway in the end booth, and an all-night poker game on Fridays. Some other ways in which a kid could sharpen his math skills, amidst clanking dishes and a swirling fug, were to figure the tax on customers' tabs and, once a week, to charge a ten percent commission for running to the liquor store to buy lottery tickets. Naturally a curriculum this eccentric could lead to misconceptions: having learned to count as a child by watching customers play cards, for a time I insisted to my parents that the numbers immediately following ten had to be J, Q, K, A.

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DOI: http://dx.doi.org/10.1090/noti1327

My younger daughter was born right when the standards development was getting started.

Like most of their customers, my parents weren't formally educated beyond high school, but they read challenging books, watched informative programs on television, and believed in education for its own sake. Now that I have kids of my own, my wife and I emphasize education at home just as our parents did. And after spending several years as a mathematical physicist, today I work full time to improve mathematics education in America's public schools—a mission that arose from my role in developing the Common Core State Standards for Mathematics [1].

The Standards

Many experts contributed to the development of the standards, including mathematicians, teachers, state education leaders, and researchers in mathematics education. My two coauthors on the writing team were Phil Daro, a longtime mathematics educator, and William McCallum, a mathematician at the University of Arizona who chaired the mathematics working group.

Research about previous state standards, international mathematics performance, readiness for college and careers, and mathematics education informed the development of the standards. One of the most widely praised features of the resulting document is its mathematical coherence. For example, whereas historically the US curriculum has made the study of fractions the study of round food, the grade 3 standards (Figure 1) emphasize that fractions are numbers, so that when it comes time to multiply fractions in later grades, the students are not being asked to multiply pizzas [2]. Other topics that have been recognized for their mathematically coherent treatment in the standards are the properties of operations; the relationship of subtraction to addition and of division to multiplication; the relationship between fractions and decimals; measurement concepts; the slope of a nonvertical line in the coordinate plane; and the number line. See Figure 2 for an additional excerpt from the standards.

The K–8 standards also revise the previous "strand model" of mathematics content in order to emphasize arithmetic, algebra, and the connections between them. Thanks to its architecture, the Common Core matches the standards of high performing countries more closely than previous state standards did [3].

Students are expected to know the addition and multiplication tables from memory, and they are expected to be fluent with the standard algorithm for each of the four basic operations with whole numbers and decimals. In addition, the standards expect students to use important mathematical concepts such as place value and the properties of operations. Why was it important to include concepts in the standards? The Learning Processes Task Group convened by the National Mathematics Advisory Panel in 2008 concluded that "American students have a poor grasp of most core arithmetical concepts…. Mastery of these core concepts is a necessary component of learning arithmetic and is needed to understand novel problems and to use previously learned procedures to solve novel problems [4]". So it is important for the sake of mathematics achievement that students learn concepts in adequate depth.

Number and Operations—Fractions[5] **3.NF**

Develop understanding of fractions as numbers.

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   - **a.** Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
   - **b.** Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   - **a.** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   - **b.** Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   - **c.** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
   - **d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Figure 1. The grade 3 fractions standards in the Common Core [1, p. 24]. Note the presence of the number line, of fractions greater than 1, and of fractions equal to whole numbers.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   - **a.** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
   - **b.** A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

(a)

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
   - **a.** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
   - **b.** A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

(b)

Figure 2. One way the standards promote mathematical coherence is by giving parallel treatment to analogous concepts. (a) Two introductory standards about area [1, p. 25]. (b) Two introductory standards about volume [1, p. 37].

Rising Expectations for Teaching, Too

Mathematical concepts have always appeared on standardized tests, albeit usually superficially (Figure 3a).

Students can easily be trained to answer item 3(a) without having to think about fractional quantities—just tell them a rule that says, “To answer fraction-of-a-set questions, put the number of desired objects above the line, and put the total number of objects below the line.” Indeed, a teacher with weak mathematical knowledge and a long list of topics to cover might regularly present mathematics this way. Training the students was usually enough to deliver proficient scores on the old tests, but bringing students to proficiency on the new tests will likely require a substantial shift in the way topics are taught. The best way to prepare students for items like 3(b) and 3(c) is probably to teach fractions as the numbers they are—but this will be difficult, if the teacher’s own grasp of fractions is weak.

Having shown some test items, I should be careful to clarify that the Common Core is not a test; it is a list of learning goals. The Common Core is also not a suite of accountability policies: a state adopting the standards does not thereby impose accountability on schools or teachers. The implementation of the standards varies from state to state, and some states adopting the standards have maintained their existing accountability policies, while other states have relaxed these policies in order to give teachers time to adjust to the new standards. Nor is a state adopting the standards thereby required to use The Partnership for Assessment of Readiness for College and Careers (PARCC) or Smarter Balanced tests. However, the items shown in Figure 3 do serve to illustrate the higher demands the standards make on teachers’ mathematical knowledge. Millions of students will have taken these tests in 2015, and policymakers are paying attention to the results. This creates an opportunity.

On a panel discussion for teachers and instructional leaders.

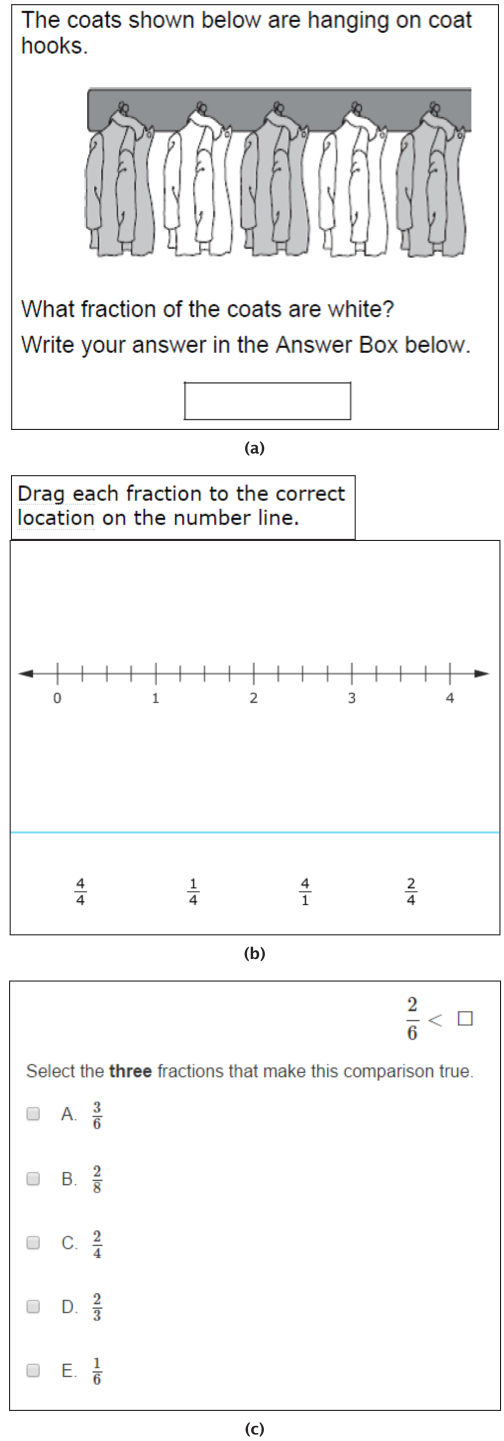

Figure 3. (a) Grade 3 item from one of the most highly regarded pre-Common Core assessments, the Massachusetts MCAS test [5]. (b) Grade 3 item from a Smarter Balanced practice test. (c) Grade 3 item from a PARCC practice test. PARCC and Smarter Balanced are assessment systems developed over the past several years to assess the Common Core [6].

What Mathematicians Can Do

In 2010, the Conference Board of the Mathematical Sciences jointly with AMS and MAA published *The Mathematical Education of Teachers II* (MET II). This report found that "Prospective teachers need mathematics courses that develop a solid understanding of the mathematics they will teach." To be clear, what the report is describing here is not college algebra, abstract algebra, calculus, liberal arts mathematics, or mathematical modeling. Rather, I would encourage mathematicians to work with education faculty to create (where it doesn't already exist) a course that: (1) is required for prospective elementary teachers (undergraduate elementary-education majors and prospective elementary teachers who come into the profession through a postgraduate certification); (2) can be taken as an elective by undergraduates in any major; (3) counts toward general-education quantitative course requirements; (4) has no prerequisites beyond those required for undergraduate admission; (5) is mathematically rigorous. Note, "rigorous" here refers to quality of mathematical thought, not sophistication of topics, techniques, or notation. Unfortunately, as MET II notes, "In some cases, mathematicians do not see the deep study of elementary mathematics content as worthy of college credit. They may try to make the course content 'harder' by introducing higher-level mathematics or teach it as a skills course. Or they may ask elementary teachers to take courses such as calculus or other college mathematics courses in lieu of courses on elementary mathematics." Having taught elementary mathematics at a deep level to college students, however, I agree with mathematician Sybilla Beckmann [7]: "Mathematics courses that explore elementary school mathematics in depth can be genuinely college-level intellectual experiences, which can be interesting for instructors to teach and for teachers to take." (6) imparts, as the stated goal of the course, 'profound understanding' of school arithmetic and early algebra. Those taking the course also learn how to explain topics from arithmetic and early algebra (such as the "invert and multiply" rule of fraction division, or the regrouping steps in the calculation 3014–658) in elementary but mathematically sound ways.

A proto-curriculum for such a course already exists in the Progressions documents published by the Institute for Mathematics and Education at the University of Arizona. These open-source documents are keyed to the Common Core, which makes them directly applicable to school systems and hence of interest to schools of education.

Mathematics won't be high on education schools' agenda unless mathematicians put it there. MET II observes that colleges of education "face increasing pressure to add courses related to English Language Learners, special education, educational policy, assessment, and other contemporary issues, which sometimes leads to the elimination or reduction of mathematics courses for prospective teachers." Hung-Hsi Wu, a Berkeley mathematician long active in mathematics education, says that "Research mathematicians have their work cut out for them: consult with education colleagues, help design new mathematics courses for teachers, teach those courses, and offer constructive criticisms in every phase of this reorientation in preservice professional development" [2]. There are also ways for mathematicians to work with school districts. In Hawai'i, the state education agency partnered with the University of Hawai'i at Hilo so that mathematicians and high school teachers could write curriculum materials together. At UCLA's Philip C. Curtis Jr. Center for Mathematics and Teaching, mathematicians and K–12 educators are collaborating to design professional learning tools for teachers. MET II urges that "Mathematics departments need to encourage and reward faculty for these efforts."

For the first time, because of the Common Core, the country has a widely shared picture of the content of mathematics and a new spur to the demand side of teacher content knowledge. Teachers who used to shy away from mathematics now ask me about such things as unit fractions, the distributive property, and the laws of exponents. A mathematician active in K–12 education for decades told me, "For the first time in my professional career, many teachers seem to realize they need more content knowledge." States and districts must respond to this demand by providing current teachers with learning opportunities in high-impact topics like fractions.

My last recommendation to readers is to find out more. Contact authors cited in the references to this article, and ask for ideas and advice about ways to improve the mathematical education of teachers in your university and community. Substantial work on this problem has been done in scattered places, but with the widespread adoption of the Common Core, successful models could more likely spread. And because more challenging tests are currently being administered for the first time, policymakers and elected officials will be paying attention to the results. Now may be an especially good time to advocate for policies to improve the quality of teachers' mathematical knowledge. This is hard work, but if there is any constituency that has the authority, the responsibility, and ultimately the self-interest in improving mathematics teaching in America, it is the community of professional mathematicians.

References and Further Reading

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