STUDENT ACHIEVEMENT PARTNERS

Experience the Content of the Lesson: Facilitation Guide
Equivalent Expressions

Prepare for the activity:

- Do the math yourself.
- Plan to lead the discussion in a way that allows you to model the instructional practices in the IPG. With an emphasis on:
    - 2A: The teacher makes the mathematics of the lesson explicit through the use of explanations, representations, tasks, and/or examples. The mathematics presented is clear and correct.
    - 2C: The teacher strengthens all students' understanding of the content by strategically sharing variety of students' representations and solution methods.
    - 3C: The teacher establishes a classroom culture in which students explain their thinking.
    - 3E: The teacher connects and develops students' informal language to precise mathematical language appropriate to their grade.
    - 3G: The teacher asks students to explain and justify work and provides feedback that helps students revise initial work.

Some ideas that may emerge from the conversation:
Facilitators should choose which points to address based on the needs of the participants and the time allotted for this activity

- The directions for the first activity state "Complete the graphic organizer to represent the pattern." This language does not clearly state that the algebraic expression and description in words should represent the number of toothpicks in the pattern. *A participant might write something like "the shape has two toothpicks on the bottom and then adds two toothpicks on the side each time." Although this satisfies the directions as written, it does not lead to algebraic thinking required by the grade 6 standards.*
- It is important to define the meaning of the variable. *It may be helpful to list all of the participants Algebraic Expressions on the board and discuss whether they represent the same thing. (e.g., if participants come up with 2x + 2 and 2n + 2, this provides an opportunity to discuss how each participant defined their variables and to note that the symbol used for the variable is not important but the meaning of the variable is essential.)*
- For some participants, there may be a direct connection between the visual model and the expressions they create. *It may be helpful to ask participants how they saw the pattern growing and then to help them relate that to the expressions they wrote. For example, you might get these responses:*
    - *a participant writes 2n+2 because they see that a vertical row of toothpicks is equal to the shape number and that they need to double that and then add the 2 toothpicks at the bottom of the shape to get the total number of toothpicks*
    - *a participant writes 2(n+1) because they split the shape in half along the line of symmetry and see that each side has a number of vertical toothpicks equal to the shape number plus 1 horizontal toothpick and they need to double that number to find the total number of toothpicks*

- Algebraic and written expressions generated by participants may be recursive or explicit. *It may be helpful to list all of the participants Descriptions in Words on the board and then discuss which expressions can help determine the number of toothpicks in any shape.*
    - Recursive expressions rely on knowing the number of toothpicks in the prior shape.
        - This may be more natural for students to articulate in words (e.g., "each shape has 2 more toothpicks than the prior shape")
        - In mathematical symbols, this is trickier. The description might look like: number of toothpicks in prior shape + 2. *It may be helpful to ask participants: What issue arises if we try to replace "number of toothpicks in prior shape" with a variable? (This algebraic expression wouldn't allow us to find the number of toothpicks in the 100^{th} shape unless we know the number of toothpicks in the 99^{th} shape.)*
    - Explicit expression can be used to determine the total number of toothpicks for any shape in the pattern if the shape number is known.
        - 2n+2; where n represents the shape number
        - the definition of the variable is more clearly tied to the 6th grade work on expressions and equations
- There are two different ways that you can justify the problems on p.2: substitution or applying properties in order to create the same expression. These relate to the standards 6.EE.A.4 and 6.EE.A.3 respectively. The problems on p.2 allow both of these ideas to be surfaced.
    - Substitution can only be used to disprove equivalence, not to prove equivalency. It is important to substitute using more than one value. *It may be helpful to discuss #3. If you substitute n=2 or n=0, then n^2+2 appears to be equivalent to 2n+2. Substituting any other value will show that the two expressions are not equivalent.*
    - The associative and distributive property are useful to prove equivalency in many of the expressions on the worksheet. *For example, #4 and #5 require the distributive property while #7 and #8 require the associative property.*
- Grade 6 students work with whole number exponents but do not learn operations with signed numbers until grade 7. *It may be helpful to look at 'Itzel's expression' on the exit slip. Since students have not yet learned to distribute a negative, students will need to use substitution to determine equivalency.*

Note: There is a revised version of this lesson plan available at: http://achievethecore.org/page/2851/modeling-equivalent-expressions. This resource may be helpful to facilitators as they prepare to present the IPT. In addition, facilitators may choose to share the plan with participants towards the end of the IPT experience.