[redacted]

Modeling Day 1 – **ALL MODELS ARE WRONG, SOME MODELS ARE USEFUL!**

For each of the following problems,

- Use the TI to find each of the following models (record equation)
  - Linear $y = ax+b$
  - Exponential $y = ab^x$
  - Logarithmic lnreg $y = a+b\ln x$
  - Power $y = ax^b$
- Record r
- Sketch the "line" through each data set
- Sketch the residual plot (residual = observed value – expected value)
- Determine which model is most appropriate

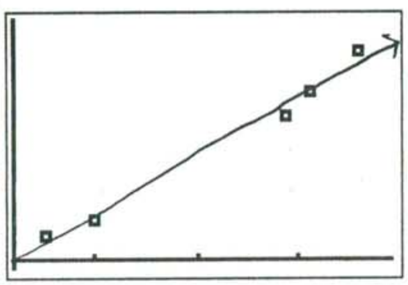
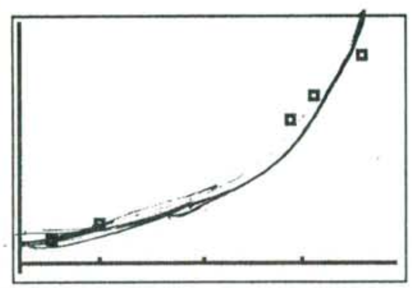
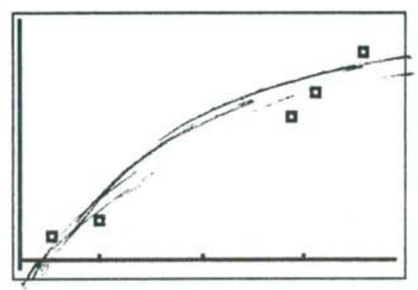
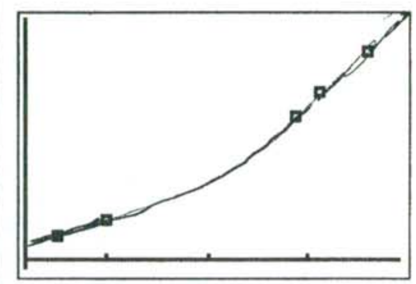
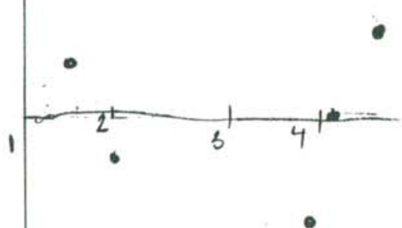
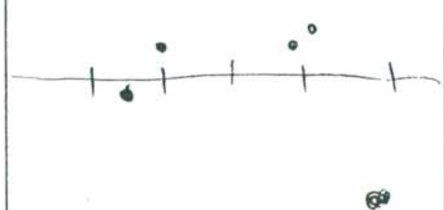
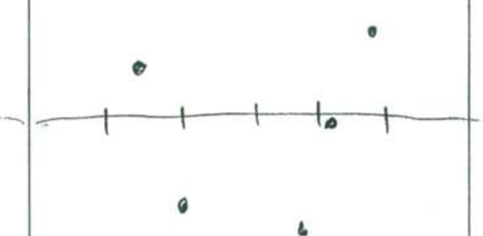
As you work through these problems, make conjectures about

- the value of r
- the role of the residual plot

How can each of these assist you in determining the most appropriate model?

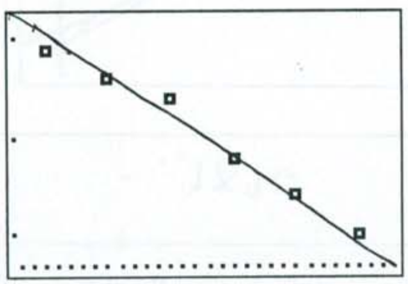
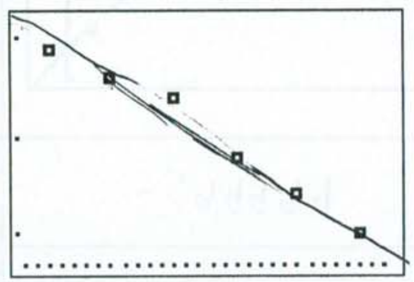
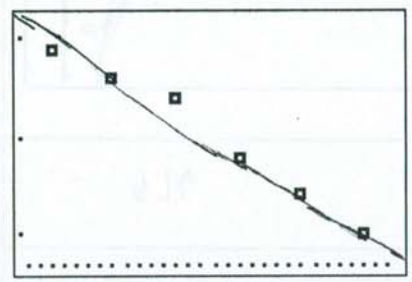
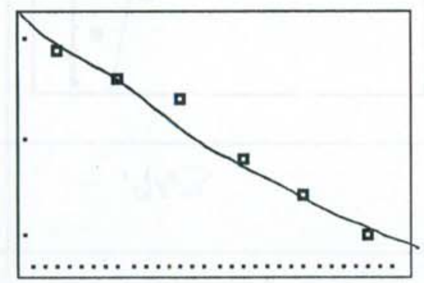
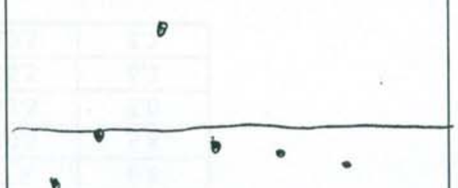
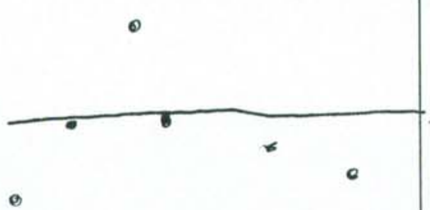
1.

x	y
1.5	11.5
2.0	20.0
3.9	72.4
4.1	84.5
4.6	104.2

	Linear	Exponential	Logarithmic	Power
equation	$y = 29.4x - 36.199$	$y = 4.4(2.03)^x$	$y = -27.96 + 80.12 \ln x$	$y = 5.15x^{1.966}$
r	.994	.996	.979	.9998
graph				
residual plot				

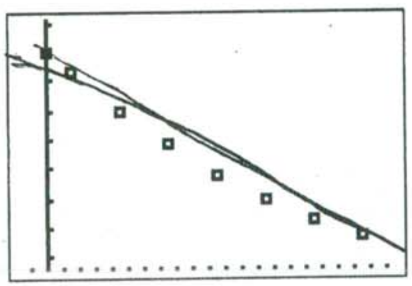
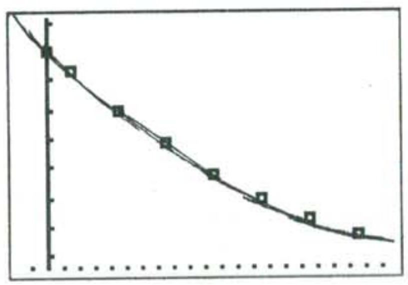
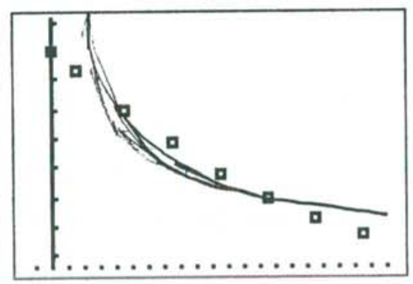
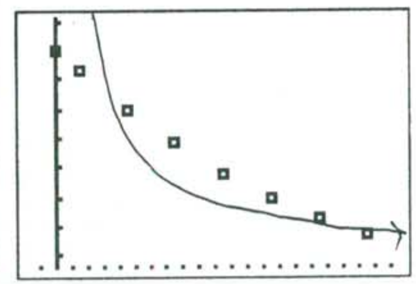
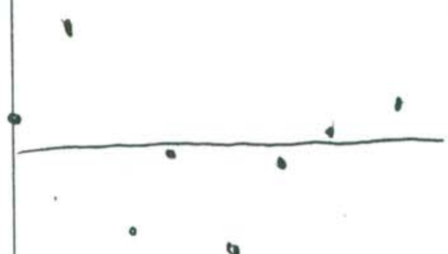
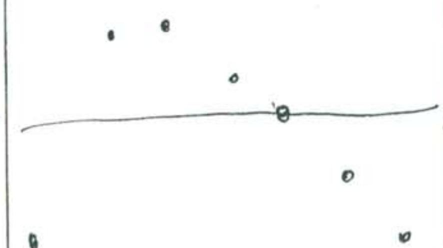
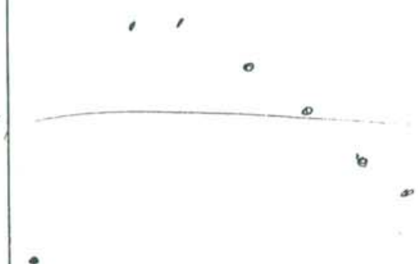
2.

x	y
1980	15.9
1985	15.6
1990	15.4
1995	14.8
2000	14.4
2005	14.0

	Linear	Exponential	Logarithmic	Power
equation	$y = -.078x + 171$	$508631.25(.995)^x$	$y = 1199.8 - 155.91 \ln x$	$y = 3.85x^{-10.4}$
r	$-.9913$	$-.9901$	$-.9912$	$-.9898$
graph				
residual plot				

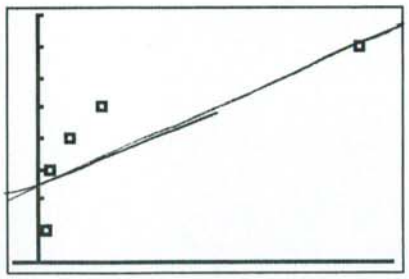
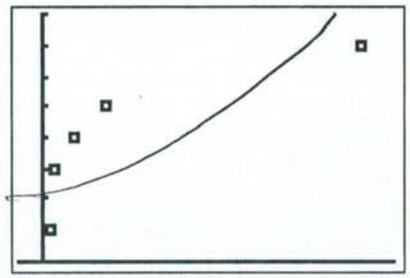
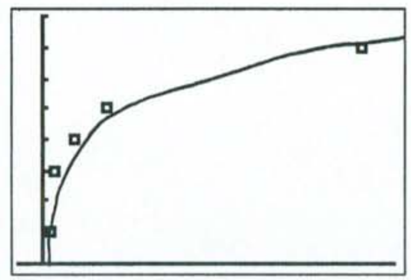
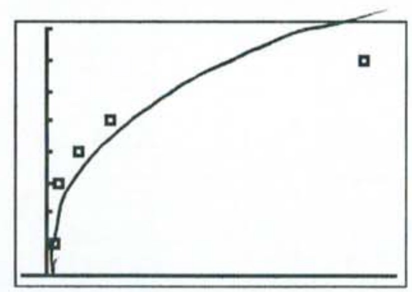
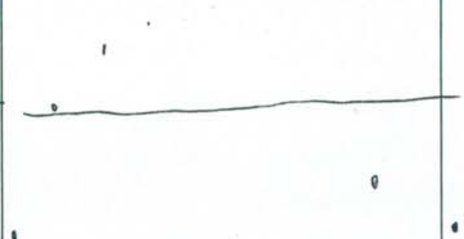
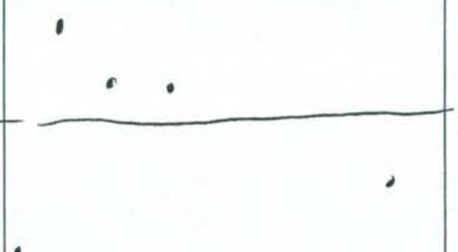
3.

x	y
0	10
1.5	9.3
4.5	7.9
7.5	6.8
10.5	5.8
13.5	5.0
16.5	4.3
19.5	3.7

	Linear	Exponential	Logarithmic	Power
equation	$-.324x + 9.58$	$9.98(.95)^x$	$10.7 - 2.19\ln x$	$12.1x^{-.34}$
r	$-.9896$	$-.99994$	$-.976$	$-.9423$
graph				
residual plot				

4.

x	y
20	4
40	6
100	7
200	8
1000	10

	Linear	Exponential	Logarithmic	Power
equation	$.004x + 5.7$	$5.646(1.0006)^x$	$.159 + 1.4551\ln x$	$2.43x^{.2156}$
r	.839	.75	.9855	.95
graph				
residual plot				

Context for Day 1 Modeling (Day 2) [redacted]

1. Tyler drops a ball from various heights and records the time that it takes for the ball to hit the ground. The independent variable is time (seconds), the dependent variable is distance (meters). $y = 5.15x^{1.966}$
   a) How does this model compare to the physical model $s = \frac{1}{2}gt^2$? What is Tyler's estimate of g? (It is known that the acceleration due to gravity is approximately 9.8 meters/sec^2.)
   b) Predict the height from which the ball was dropped if it took 8.9 seconds to reach the ground.

Extrapolation

382.04 m

2. The table shows the number of live births per 1000 women aged 15-44 years in the United States, starting in 1980.
   a) Identify and interpret the slope of the line. $-.078$
   b) Identify and interpret (if reasonable) the y-intercept. none ~~15.80~~
   c) In 1978 the birthrate was actually 15.0. How close did your model come to this value? 16.7, overprediction
   d) Predict the birthrate for 2015. 13.8
   e) Predict the birthrate for 2030. 12.7

$\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{\text{\# of live births per 1000}}{\text{year}}$

Each year, the # of live births per 1000 women between ages 15-44 in U.S decreases on avg. by .078.
7.8%

$y = 9.98(.95)^x$

3. The table gives the amount of insulin in the blood after a particular amount of time (minutes) has elapsed.
   a) What does the value of *a* in your model tell you? That 9.98 was the amount of insulin at the beginning.
   b) What does the value of *b* in your model tell you? The rate of decay is 5%/m
   c) When would you expect the insulin level to drop to 2.0? $2 = 9.98(.95)^x$ $x = 31.3$
   d) When would you expect the insulin level to drop to 0.0? It won't

$y = .159 + 1.455 \ln x$

4. The table displays the results of the High-Low game. An individual picks a number from 1 to n and the other person guesses until they get the correct number. (After each guess the individual is told whether the guess is too high or too low.) "n" is the independent variable; the dependent variable is the number of guesses made until they were correct. How many guesses would one expect to make if the number was from 1 to 1,000,000?

~~1000000 = .159 + 1.455 ln x~~

extrapolation error

$y = .159 + 1.455 \ln 1000000$

2013

d = rt → linear

[redacted]

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    - Linear
    - Exponential
    - Logarithmic
    - Power
- Record r
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- the value of r
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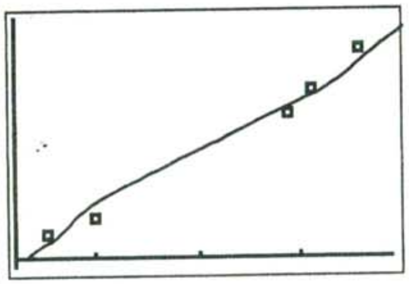
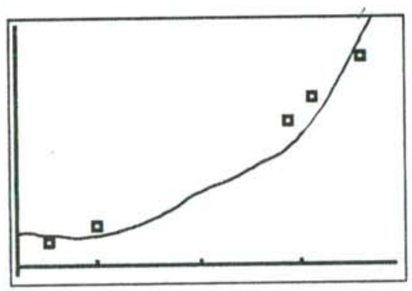
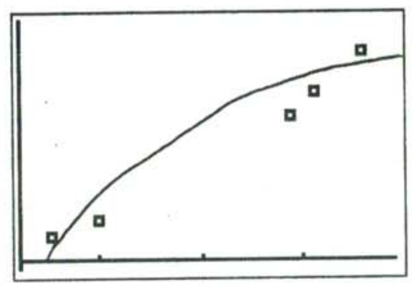
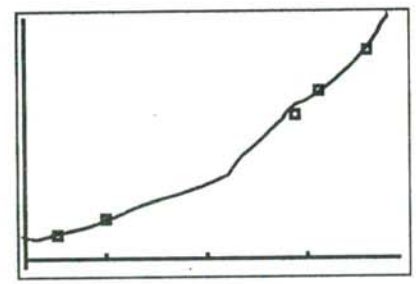
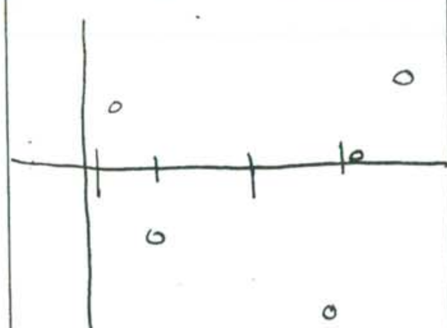
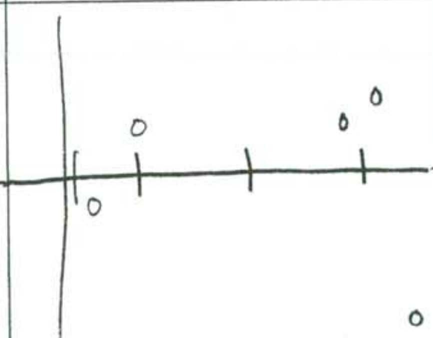
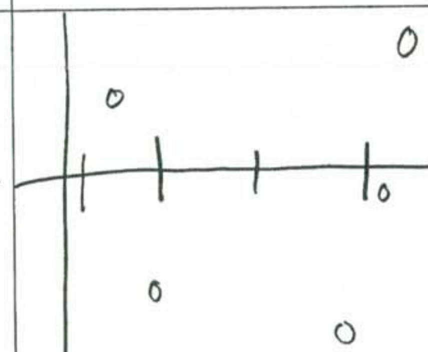
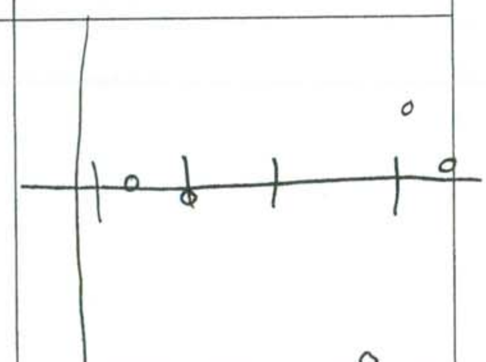
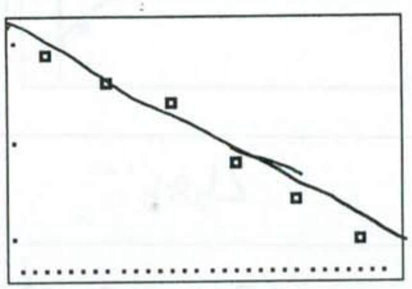
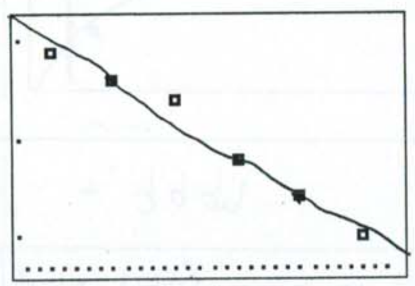
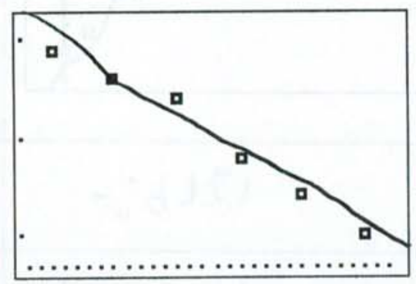
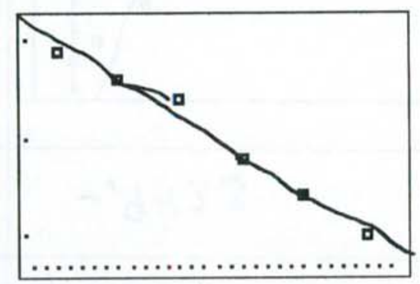
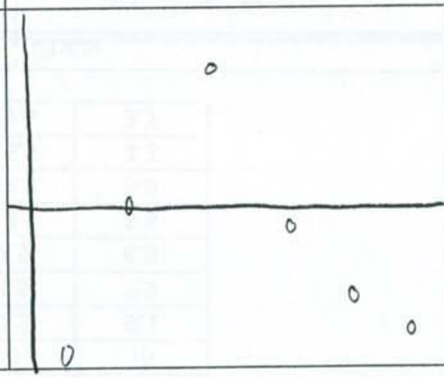
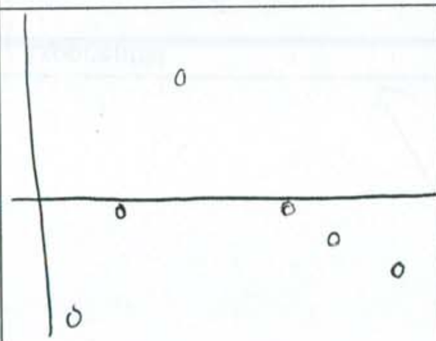
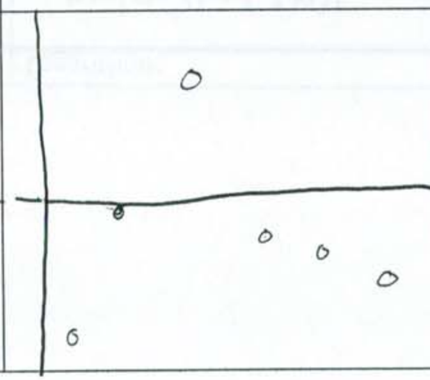
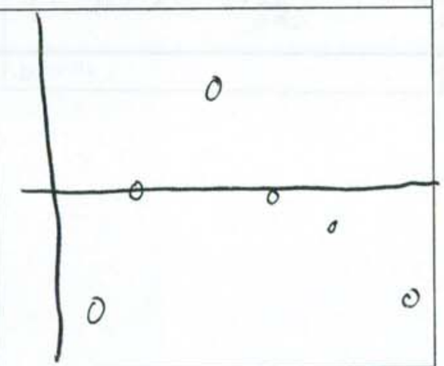
graph				
residual plot				

2.

x	y
1980	15.9
1985	15.6
1990	15.4
1995	14.8
2000	14.4
2005	14.0

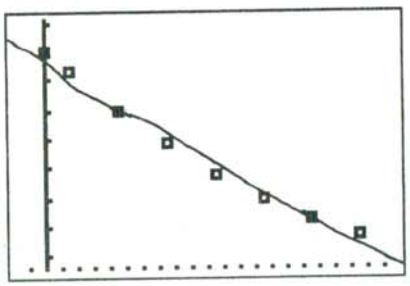
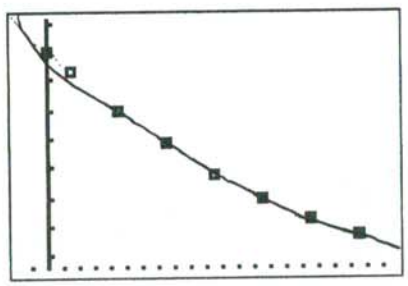
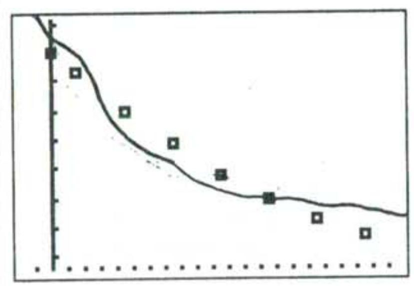
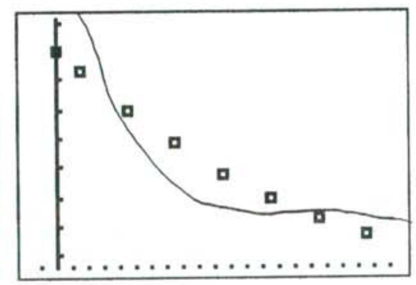
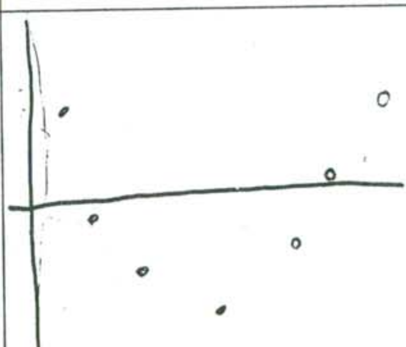
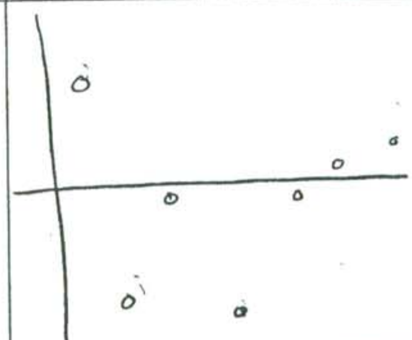
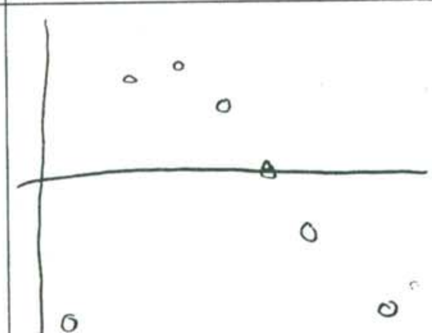
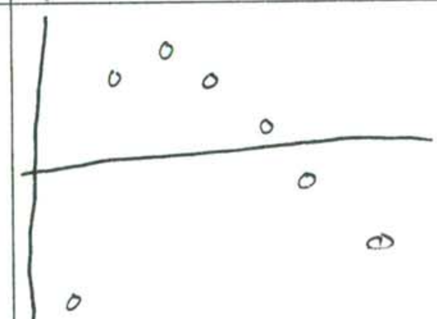
$\Delta x = 5$

-.3
-.2
-.4
-.4
-.4

	Linear	Exponential	Logarithmic	Power
equation	$y = -.08x + 171$	$y = 508631.3(.995)^x$	$y = 1199.85 + (-155.96)\ln x$	$y = 3.86x^{-10.43}$
r	-.9913	-.9901	-.9912	-.9899
graph				
residual plot				

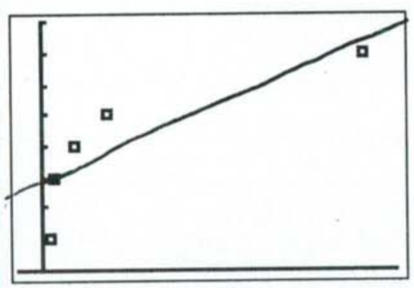
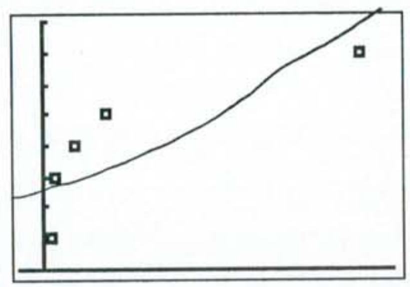
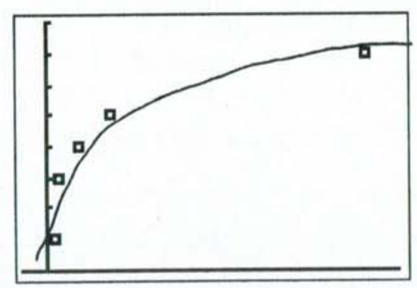
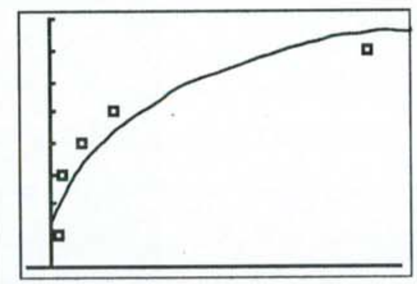
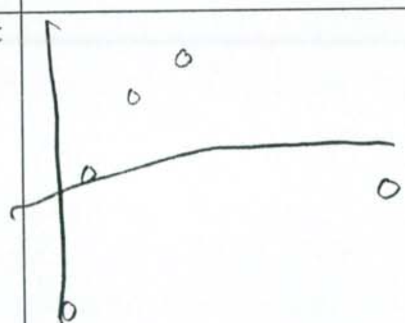
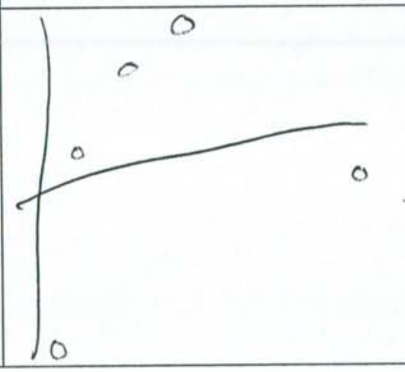
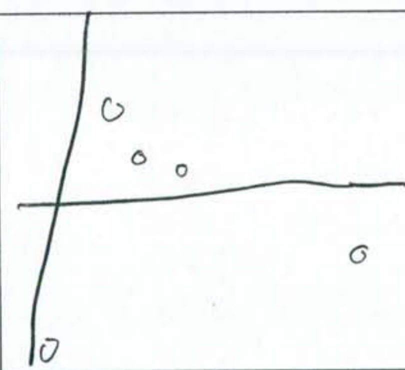
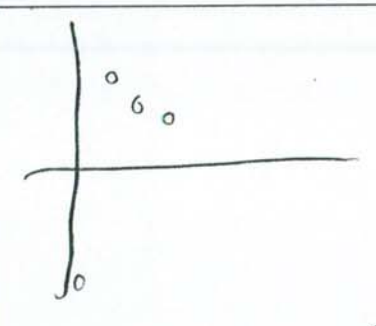
3.

x	y
0	10
1.5	9.3
4.5	7.9
7.5	6.8
10.5	5.8
13.5	5.0
16.5	4.3
19.5	3.7

	Linear	Exponential	Logarithmic	Power
equation	$y=-.31x+9.34$	$y=9.98(.95)^{x}$	$y=10.71+(-2.19)\ln x$	$y=12.11x^{-.35}$
r	$-.9897$	$-.9999$	$-.9761$	$-.9423$
graph				
residual plot				

4.

x	y
20	4
40	6
100	7
200	8
1000	10

	Linear	Exponential	Logarithmic	Power
equation	$y = .065x + 5.764$	$y = 5.65(1.0006)^x$	$y = .159 + (1.456)\ln x$	$y = 2.43x^{.216}$
r	.839	.752	.985	.950
graph				
residual plot				

Context for Day 1 Modeling (Day 2) [redacted]

1. Tyler drops a ball from various heights and records the time that it takes for the ball to hit the ground. The independent variable is time (seconds), the dependent variable is distance (meters).
   a) How does this model compare to the physical model $s = \frac{1}{2}gt^2$? What is Tyler's estimate of g? (It is known that the acceleration due to gravity is approximately 9.8 meters/sec^2.) Good
   b) Predict the height from which the ball was dropped if it took 8.9 seconds to reach the ground.

$y = \boxed{5.15}x^{1.97}$

$s = \boxed{\frac{1}{2}g}t^2$

$\frac{1}{2}g = 5.15$

$g = 10.3$

$y = 5.15(8.9)^{1.97}$

$y = 382\,m$

Error of Extrapolation

2. The table shows the number of live births per 1000 women aged 15-44 years in the United States, starting in 1980.
   a) Identify and interpret the slope of the line. $-.078$ $\frac{\text{\# of live birth/1000 women}}{\text{year}}$
   b) Identify and interpret (if reasonable) the y-intercept. 171 no meaningful interpretation
   c) In 1978 the birthrate was actually 15.0. How close did your model come to this value? 1.716 over (RESIDUAL
   d) Predict the birthrate for 2015. 13.83
   e) Predict the birthrate for 2030. 12.66

$y = -.078x + 171$

$\frac{\Delta y}{\Delta x} = \frac{-.4}{5} = -.08$

Each year the # of live births per 1000 women aged 15-44 years in the US decreases on average by .078 (7.8%).

$-\frac{78}{1000}\ \frac{\text{\#of livebirth}}{\text{years}}$

$y = -.078(1978) + 171$

$y = 16.716$

$-.078(2015) + 171 = 13.83$

$-.078(2030) + 171 = 12.66$

3. The table gives the amount of insulin in the blood after a particular amount of time (minutes) has elapsed.
   a) What does the value of *a* in your model tell you? Initial amount
   b) What does the value of *b* in your model tell you? insulin decreases by .95
   c) When would you expect the insulin level to drop to 2.0? ~~9 min~~ ≈ 31.338 min
   d) When would you expect the insulin level to drop to 0.0? ~~[illegible]~~ Never

$$\cancel{9.98(.95)^2 = 9}$$

$$\cancel{9.98(.95)^{0} = 10}$$

$$9.98(.95)^x = 2$$

$$.95^x = \frac{2}{9.98}$$

$$\log_{.95}\frac{2}{9.98} = x$$

$$x = 31.338$$

$$\log_{.95} 0 = y$$

$$x \neq \varnothing$$

4. The table displays the results of the High-Low game. An individual picks a number from 1 to n and the other person guesses until they get the correct number. (After each guess the individual is told whether the guess is too high or too low.) "n" is the independent variable; the dependent variable is the number of guesses made until they were correct. How many guesses would one expect to make if the number was from 1 to 1,000,000?

$$.159 + (1.456)\ln 1000000 = 20$$

$\approx$ 20 guess