## Applications of Multiplication \& Division (Grade 3)

TEACHER: Yesterday we did a lesson in math where we were discovering some things. Does anybody remember what we were discovering yesterday in math? Isabelle, what were we discovering?

STUDENT: When we were multiplying, if we would get an even or odd number when we multiplied with two even numbers, two odd numbers, or an even and an odd number.

TEACHER: OK. So we were figuring out the factors. When we took an even factor and multiplied it with an even factor. And what did we discover? What do we get with the product, Skya?

STUDENT: It was still even.
TEACHER: The product was even, wasn't it? What happened when we took an even factor and we multiplied it by an odd factor? Josh B.

STUDENT: It's an even product.
TEACHER: It's an even product. So then what happened when we took an odd factor times an odd factor. What did we get, Mathias?

STUDENT: We got an odd product.
TEACHER: We got an odd product. So some of the vocabulary-- that we were working on yesterday-- was the word factor and was the word product. We're going to continue with that today, exploring more with factors and products with an activity that you're going to be doing.

Now yesterday we talked, just a little bit, about some of the things that we like to see when we're working in math. And so we have a few learning goals of what we're going to be doing. And the first one, like we always do in every math lesson, is we're always trying to persevere through math.

And then we have two other big goals of what I'm looking for when you are working on your math today. The first one is-- I'm going to be looking for students that are using drawings, possibly equations, or even math tools to help you when you're solving those math problems. When you're trying to find equal groups. And this isn't anything new. We've been working with equal groups for a while. But we haven't talked about it-- in probably a month or so. So we're coming back to it and really starting to rediscover and connect some of those learnings and discoveries that we had earlier in the school year and putting them to use now.

And our next goal is-- to possibly use some math vocabulary as we're explaining our thinking. And I already went ahead and put up the two math terms that we had from yesterday. But-there's an empty space here. So as we go through the lesson, if we feel that there's some other
math vocabulary that we would like to add-- or we think we should add-- based on our conversations. Do you think we should add it here?

## STUDENT: Yes.

TEACHER: And I'm going to give you permission, sometimes if you do come up with a word, I might say, oh, go put that on the board. So this is the spot where you would put that. All right, are you guys ready for your task?

STUDENT: Yes.
TEACHER: Awesome. All right, we are going to be working on this. Our task says-- a teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students? Now I'm going to come around and I'm going to give you a sticker that's going to go into your math blue journal. It's the same problem that you see out there. And I'm going to let you start to solve it.

So what are you thinking when you're putting this?
STUDENT: [INAUDIBLE]
TEACHER: What are you going to do, Ulysses?
STUDENT: I'll put-- you can put them into [INAUDIBLE] if you make four groups of them with 96.

TEACHER: 96 ? Why are we trying to make 96 ?
STUDENT: 36, yeah.
TEACHER: How much?

STUDENT: 36.
TEACHER: 36, why are you worried about 36 ?
STUDENT: Because it says on the math
TEACHER: Right. So we're worried about 36 students and what are we trying to do with those 36 students?

STUDENT: We can put them into groups.
TEACHER: We're trying to put them in?

STUDENT: Groups.

TEACHER: What kind of groups?
STUDENT: Equal.
TEACHER: Equal groups. All right, keep going.
STUDENT: Put this into 6 [INAUDIBLE] and I found something, I looked here. And 6 times 6 is 36 .

TEACHER: Oh, so you're using another tool. What tool is this?
STUDENT: A multiplication chart.
TEACHER: That's right. And so here-- you're using-- how did you know to put two dots here? How did you start with that thought?

STUDENT: I just put one dot on each and six times when I get back and one to each of-- and then [INAUDIBLE] a little more. And then I count and they all add up to six.

TEACHER: OK, so where's this 24 number coming from?
STUDENT: Oh, this was just when I went to go get the cubes. When I sculpted them.
TEACHER: So what's your next step? So now that you've found that there can be six groups of 6 , what's going to be your next thought with an equal group?

STUDENT: Hm. I don't know yet.
TEACHER: All right. So are you going to continue with your tool this way? Are you going to use the cubes?

STUDENT: Well, I can't do threes. I can't do it in groups of threes.
TEACHER: You can't?
STUDENT: No, I can.
TEACHER: Oh, why don't you try it?
Boys and girls, as I'm walking around, be sure that you are drawing those models of what you're discovering in your journal. Because when the lesson is over, the math tools will be put away, but we need to know your thinking. So that's why you're using your journal to record your thoughts.

So tell me about your x's here. What are you working on with these x's on your journal?

STUDENT: To show that 18 times 2 equals 36.
TEACHER: Hm. And did you just know that already or did you use the cubes to help you?
STUDENT: I used the cubes.
TEACHER: I like how you're making a representation there. All right, so now that we've found one way, can you find a different way? I think you're coming up with one, aren't you? What are you discovering?

STUDENT: 6 times 6.
TEACHER: All right, let's make sure we're recording that in our journal.
So, I'm curious. When you look here, how many are you putting in the groups?
STUDENT: 18.
TEACHER: So how did you know to put 18 in? What were you thinking?
STUDENT: (WHISPERS) I just put 18 in there. I just put 18 in there. [INAUDIBLE]
TEACHER: How did you know that? What made you think to split them in half? So you're just kind of trying?

STUDENT: Yeah, I'm just kind of trying [INAUDIBLE]
TEACHER: And then seeing if it works-- and if it doesn't work, then you just move on to the next one? All right. So you've tried 2. So two groups of 18. What other groups have you tried?

STUDENT: I've tried groups of 2, groups of 6, and then groups of 12 . And now, I'm going to try groups of 9.

TEACHER: Do you think groups of 9 will work? OK, keep going.
I'm curious, how did you know to put 12 in each group?
STUDENT: Well, I took one and I thought it would be 3, so I put one in each one and I just kept on trying to finish.

TEACHER: Oh, I see. So then you just kept evenly disbursing them and that's what gave you 12 in each group.

STUDENT: Yeah.

TEACHER: But I'm curious, why do you have a plus sign in between each one where it's saying 12 plus 12 plus 12? What are you doing from up here? I think it has to do with your picture up here. Bless you.
[SNEEZING]
STUDENT: Thank you.
TEACHER: Are these correlated? How are they correlated? Are they the same?
STUDENT: Yeah.
TEACHER: OK. So we pretty much-- it looks like you took the three groups that you had and you added them together. And when you added them together, what did you get?

STUDENT: 36.
TEACHER: OK. All right. And then what are you doing down here?
STUDENT: I did 36 minus 12.
TEACHER: So why are we taking 36 and subtracting 12? Let's go back to our problem. What is our problem asking of us?

STUDENT: An equal number in each.
TEACHER: So an equal number in each group. So if I subtract 36 minus 12, am I looking for equal groups? OK, so let's keep this where it is, but let's go back to this. So you tried three groups of 12, how else could you use these to help you figure out another equal group? I don't know. What other number do you think you could try? Because you tried 3 and you tried 12. What's another number?

STUDENT: 4.
TEACHER: OK, so let's try 4 . Oh, you did try 4, and so when you were adding the 4's. How much did you get?

STUDENT: 36.
TEACHER: So how many 4's did you have to add together to get to 36 ?
STUDENT: Nine.
TEACHER: So you had to add 9. OK. So then what does that tell us, how many would be in each-- how many groups of 4 would there be to make 36 ? You just told me. Do you remember?

STUDENT: Three.
TEACHER: How many groups of 4 do you have to give you 36 ?
STUDENT: Ms. Scherr.
TEACHER: Just a minute. Let's count them.
STUDENT: Nine.
TEACHER: Yeah, so nine-- nine groups of 4 give us 36 .
All right, so boys and girls, I'm going to have you write down one thought. And we're going to do a check in right now. And see where we are. OK, each of you are sitting next to somebody, Nick.

Each of you are sitting next to someone that is your partner. And what I would like you to do right now, is for each of you to take a minute to think about your thinking in solving our problem of equal groups. And as you're having those discussions-- what I need to happen is when we do have a conversation, one person has to be talking and what does the other person have to be doing?

STUDENT: Listening.
TEACHER: Listening. And if the person listening has a question about the mathematics, is that OK to ask?

STUDENT: Yes.
TEACHER: It is. Because I know as I'm walking around and asking some of you to explain, I can't understand what you're thinking with it. So I might ask a question that helps clarify what you're thinking. So it is definitely OK to ask your partner a clarifying question with the mathematics so you can understand it. Because part of our goal is to understand what other people are thinking.

Then you're going to switch and let your partner share. And again, if you have questions about that then you would ask that at this time. Does anyone have any questions of what I'm asking you to do?

Sophia, I'm going to have you and Will share. Who has the longest hair?
STUDENT: Sophia
TEACHER: Sophia does. OK, so Sophia can you explain a couple things that you were discovering? And then you need to be a good listener while you're listening.

STUDENT: I'm stage fright.
TEACHER: You're stage fright?
STUDENT: Yeah.
TEACHER: So do you think you could go up and help him explain? Since you listened to how he was explaining? So you both are going to come up together, OK? Bring your journal with you.

Boys and girls, if I could have your attention in $5,4,3,2$, and 1 . All right, you've had a minute to share with your partner. What I've asked is for Carson to come up with Colin, and he is going to share-- although Colin is going to help Carson speak because Carson is feeling that he's not sure he can come up and really share. Now what I noticed, when I was going around, is this was one way Carson was solving that. So Carson and Colin, I'm going to let you both go together and can you explain your thinking with that?

Can you speak in a loud voice for me?
STUDENT: All right. He said he did 6 times 6 and then went up and the up part equals 36 . He said he did mental math.

TEACHER: So he did mental math. What does that mean, Carson, when you're doing mental math?

STUDENT: Doing math in your head.
TEACHER: OK. But I'm curious, you used tally marks, can you explain the tally marks to us?
STUDENT: [INAUDIBLE].
TEACHER: I know but Carson, you were doing a great job explaining it to me earlier. And you, basically-- instead of using cubes-- you were using tally marks each time you went through and you were doing it in equal fashion, weren't you? OK, thank you very much. Isabelle, can you bring yours up and share with us what you are doing in your thinking?

All right, if everyone can put their attention up to the board, Isabelle's going to be explaining her thinking.

STUDENT: So, when I was doing it, I just did a bunch of numbers and I used the cubes to see if it worked. And groups of 4 did. So I did groups of 4. And groups of 4 worked. So here's 4, and that makes 8 . And so $2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36$.

TEACHER: So I noticed how you were counting by 2's when you did that. Wait, can you leave those for just a minute? Because the one thing that I noticed is when she did this, she was
basically doing circles and inside the circles she was then placing the cubes. And how many cubes were going inside the circle?

## STUDENT: 4.

TEACHER: Now, it's very similar to what Carson was doing, but instead of using the cubes he was using what instead?

STUDENT: Tally marks.
TEACHER: Tally marks. OK, both great different tools to use. However, adding to this-- thank you very much, Isabelle. Carly, can you bring yours up? Because I want you to see what Carly was using, same answer, but just showing it in a different way.

OK, I want you to look at how Carly put hers together with the four groups of 9. And Carly, can you explain what you did?

STUDENT: Yes. I used the cubes and I decided-- Mrs. Scherr helped me do something-- because I wasn't really good at doing boxes all by myself-- and I found out that you could do 4 with cubes. And I said or because you could do it a diagonal way-- well not in diagonal, but going down, instead of just going sideways. And that's how come I put 4.

TEACHER: All right, any questions for Carly? Clarifying questions. I have one, Carly. What do you mean when you're going sideways, what does that mean?

STUDENT: Well, it means that you're going-- basically-- how could I say this?
TEACHER: Maybe someone in our audience could help. Can you call on someone, Carly, to help explain what you're thinking?

## STUDENT: Mia.

It means that you're going this way. The one at the top is going both ways instead of going up and down, it's going side to side.

TEACHER: OK, can you come up and point? And use those words as you're pointing.
STUDENT: This one's going up and down, this one's going side to side.
TEACHER: OK, so one is more horizontal and the other one is more vertical?

STUDENT: Yes.
TEACHER: OK.

STUDENT: We should write those words up on the board.

TEACHER: We should, shouldn't we. So we were talking about horizontal and vertical. OK, what is different with what Carly did as opposed to what Carson and Isabelle did? Remember, they came up with the same answer. 4 times 9 equals 36. What is different, Frankie?

STUDENT: Instead of using tally marks or dots or cubes she used arrays.
TEACHER: She used arrays. Hm, should that be a math vocabulary word we put up here?
STUDENT: Yeah.
TEACHER: OK. So, Carly used an array. And, Carly, I noticed, the partner that you sit by, Ariel, also used arrays. Would you raise your hand if you used arrays to help you solve this problem? Thank you. Would you raise your hand if you were using something similar to what Isabelle or Carson was using, where you were putting them in groups but they weren't really making the arrays.

OK. Thank you, Carly. Now, adding on to this, because all of these are different ways we can solve the equal groups, although Lucas had an interesting thought. So Lucas, can you bring up your journal please?

What was that?
STUDENT: So when you add the 6 and 6 you get the 12 .
TEACHER: Brandon, are you listening? Josh, are you listening?
STUDENT: And then, subtract the 3 from the 6 , you get 3 .
TEACHER: OK, so he was doing something with the 6 and the 12 . What do we know about the relationship between the 6 and the 12? Jenna?

STUDENT: When you double the 6 , it equals 12 .
TEACHER: When we double the 6, it equals 12 . So I'm just going to add some words here. When we double the 6 , it gives us 12 . All right, and then what about this relationship between 6 and the 3 ? Hm, what do you notice? Nick?

STUDENT: If you double 3, you get 6 .
TEACHER: OK, so if we double 3, we're going to get 6 . That's interesting. What do you guys think about that?

STUDENT: It's really cool.
TEACHER: Why does that work? Because it works, doesn't it?

STUDENT: Yeah.
TEACHER: Why does that work?
STUDENT: I see a pattern, like when you do 3 times 12 . To split the 12 into-- split the three 12 's in to 6 and then you get six 6's.

TEACHER: OK can you come up and draw in on this chart paper here for me, because I am having a hard time following that.

STUDENT: OK, so you have like--
TEACHER: OK, you have to write a little bigger.
STUDENT: OK. Is this OK?
TEACHER: Yes, that's good.
STUDENT: So you-- this goes like--
TEACHER: OK, you have to turn and face your audience, so they can hear you.
STUDENT: So you-- this is a 12, split that into-- just split the 12's
TEACHER: So take your markers and split them in half.
STUDENT: Um, here--
TEACHER: OK, let's split them in half bigger. 'Cause we want to see what you're talking about. OK?

STUDENT: And now, it splits-- now the 12's are into 6, and then if you can see [INAUDIBLE]
TEACHER: OK, so he said now the 12 's are into 6 's. Do you see how he made that go into 6 's?
STUDENT: And then there's six 6's.

TEACHER: So he took a line, and he halved the 12 . And what is half of 12 ?
STUDENT: 6.
TEACHER: OK. And then he took a horizontal line and he split up the two halves. And he split them into equal groups of 3's. Do you see what he did?

STUDENT: No--

TEACHER: No, you didn't do that?
STUDENT: No, what I meant was just-- you split What three 12's into 6's and then you get six 6's.

TEACHER: And you get six 6's. Interesting. Any questions for Kevin? Go ahead and call on some people, Kevin.

STUDENT: How does that make 36? Because I thought it was 6 times 6 that only makes 36 . Because Lucas did it and it's 6 times 6, I never heard of 6 times 3 is 36 .

I didn't say 6 times 3 equals 36.
TEACHER: So what did you say? Can you explain?
STUDENT: I said-- I said that there were three 12's and I split the three 12's into 6's. And I get six 6's, which means it is 6 times 6 . Yes, Colin.

I looked at it and then I saw you could split into 3's too. So 3's in that row.
TEACHER: What do you mean you could split into 3's?
STUDENT: Oh, yeah! You can split it into groups of 3's.
TEACHER: Can you come up and show us? Making all kinds of interesting discoveries.
STUDENT: You split the 12 into 3's.
The 12 right here. You can do that and it will still equal 36.
TEACHER: What do you mean it would still equal 36 ?
STUDENT: If you took these groups away, this would still be 6 . This would still be 6 .
TEACHER: OK.

STUDENT: And then, he meant by [INAUDIBLE] you just split the 6's. The six 6's are [INAUDIBLE] into nine 3's.

TEACHER: Into nine 3's? So nine 3's gives me 36?
STUDENT: No, no, no. Not nine 3's. How many 3's? Twelve 3's.
TEACHER: So twelve 3's equals 36.
STUDENT: And six double 6's equals 12, which is [INAUDIBLE].

TEACHER: Is everybody following Kevin because he's talking really lightly?
STUDENT: Yeah
TEACHER: OK, so he's saying-- what Colin is doing is he split them into groups of 3 to show that there are three groups of 12-- that equal 36. And then, Kevin, what was the last thing you said about 9's? Did you say something about 9's?

STUDENT: No, I--
TEACHER: You were saying something about 6's, what were you saying about 6's?
STUDENT: Yeah, he split the 6's that I made into 3's.
TEACHER: OK, so he split the 6's into 3's, which 3 is half of 6, isn't it? OK, I think you have some other questions, thank you, Colin.

STUDENT: Yes, Josh.
You can split them into 2's.
How?

Because 36 is an even number and so is 2 , so you can split them into 2's like $2,4,6,8,10,12$, 14--

Yes, we did split it in 2 . This vertical line splits it in 2.
Yeah, you can split two of those x's into just two. And then split another row of 2, 2, 2.
TEACHER: OK so how many 2's are actually in 36 then? Did anybody figure that one out?
STUDENT: 18.
TEACHER: 18, are you sure it's 18 ?
STUDENT: Yeah!
TEACHER: I think it's interesting. Josh used some words we used yesterday. He said, 36 is an even number. And earlier in the year, we talked about if it's an even number, we can always half it. Can't we, Tiernan?

So we have a lot to think about now, don't we?
STUDENT: Yeah.

TEACHER: Right, and I'm not sure, did we find all the answers-- to how many equal groups we can make with 36 ?

STUDENT: No, I found another way.
TEACHER: OK, you found another way, what do you mean?
STUDENT: Well, I did dots in this part of--
TEACHER: OK, so similar to the tally marks? OK. All right, I think what we're going to do next, as we move forward, is we're going to continue exploring equal groups as we go. Let's go back to our vocabulary here. Were we talking about factors?

STUDENT: Yes.
TEACHER: And how about products?
STUDENT: Yes.

TEACHER: So what numbers became our factors? Can you turn to your neighbor and share with them-- what were some of the numbers that were factors?

STUDENT: [CHATTER]
TEACHER: Come back to me in $5,4,3,2,1$. OK, I heard a lot of different numbers. I heard 3 and 12. I heard 9 and 4, 6 and 6 . What about the product? Was there a lot of product numbers?

STUDENT: No.
TEACHER: What was the product?
STUDENT: 36.
TEACHER: 36, it stayed constant throughout the whole entire exploration that we had. Were we multiplying?

STUDENT: Yes, yes.
TEACHER: Were some of you dividing?
STUDENT: Yes, I was.
TEACHER: And taking those cubes and dividing them out into equal groups.
STUDENT: I did that.

TEACHER: We added some things in here. What was interesting to me was you all had different ideas of how to solve this. Did we come up with similar answers? We did. Frankie?

STUDENT: I have something else.
TEACHER: I understand, but our time is up, I'm sorry. When we come back to this tomorrow, we'll take a look and see if we can add on to some other thoughts as we move forward with our arrays. So were you persevering through your math?

STUDENT: Yes.
TEACHER: Were you using some type of argument for your reasoning with your math?

STUDENT: Yes.
TEACHER: OK. And then let's go back, did you use drawings or equations or math tools?
STUDENT: Yes.
TEACHER: All right. And how about using some math vocabulary?
STUDENT: Yes.

TEACHER: How many of you think that we should still continue to work on this math vocabulary? 'Cause it's still-- I agree, as well. All right, I'm going to ask you to go ahead and close your math journals. You guys did a great job today.

