

STUDENT ACHIEVEMENT PARTNERS

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# Introduction

**What is this guidance?**

Based on research and the progression of the disciplines, the 2020–21 Priority Instructional Content names the priorities in high school mathematics that should be the focus of instruction for educators in the 2020–21 academic year. This document provides guidance for the field about content priorities by leveraging the structure and emphases of college- and career-ready mathematics standards. It is intended to help publishers, other designers of instructional materials, and instructional leaders find new efficiencies in the curriculum that are critical for the unique challenges that have resulted from school closures and anticipated disruptions in the year ahead, keeping at the forefront principles of equitable instruction that support all students.

**Why create this guidance?**

The 2020–21 school year presents a unique set of opportunities and challenges due to the disruption to instruction in spring 2020 as well as the uncertainty associated with what the “return to school” will look like. Educators know that every school year there are students who require support in addressing unfinished learning from prior grades, a challenge that will be felt more prominently in the 2020–21 school year. Most critically, the pandemic has further illuminated inequities that have always existed. Our position is that it is entirely possible to hold high expectations for all students, address unfinished learning in the context of grade-level work, and dial into the assets students bring with them in order to unlock the creativity and energy they bring to the joyful work of learning something new. Since time is a scarce commodity in classrooms—made more limited by anticipated closures and distance or hybrid learning models in the fall of 2020—strategic instructional choices about which content to prioritize and how to assess student learning must be made.

This guidance names the content that should be of focus for all students, recognizing that intentional instructional choices will be essential for supporting all students to mastery, and that this is especially true for students with specialized learning needs. This document does not address the many considerations of instruction, but recognizes that it is critical for those using the guidance and supporting English learners to ensure that students have the instructional supports and scaffolds that supplement, and do not supplant, core instruction and thereby ensure students’ access to grade-level content. As emphasized by the Council of the Great City Schools (CGCS) in *Addressing Unfinished Learning After COVID-19 School Closures*, "Teachers should therefore resist the inclination to ‘water down’ instruction and assignments for ELL students—and other students with specialized learning needs. These students require the same challenging work and cognitive demands as their peers in order to develop academic skills and grow as scholars” (CGCS, 2020). Note that for English learners, language and content development are simultaneous and should be considered in context of math instruction. For more specific guidance about adjusting curricular content to meet the needs of English learners, refer to the Appendix for resources created by the English Learner Success Forum including activities and scaffolds that can be strategically built into lessons and units to deepen and accelerate English

learners’ learning in mathematics, as well as a reference to the framework from the Council of the Great City Schools for Re-envisioning Mathematics Instruction.

**What is the purpose of this guidance?**

The intention of this guidance is to support the work of the following groups:

* *Publishers of instructional materials*: to design modifications to mathematics instructional materials for the 2020–21 school year.
* *District mathematics leaders*: to design modifications to scope and sequence documents, to design professional learning scope and sequence for teachers, to design modifications to district-created instructional materials where used, and to support administrators in implementing equitable instruction and equitable structures.
* *State education agencies*: to support districts in planning and decision-making for instruction.
* *Providers of professional learning for teachers*: to design modifications to professional learning curricula for the summer of 2020 and the 2020–21 school year.

This guidance has been developed in response to current conditions. These documents are not criteria, and they do not revise college- and career-ready state standards. This guidance does not stand alone but is to be used in conjunction with those standards. This guidance does not attempt to repeat what standards already say, nor does it mention every opportunity the standards afford to make coherent connections within a course or between one course and another. Further, leveraging the focus and coherence of high-quality instructional materials aligned to college- and career-ready state standards is more important than ever.

This guidance was developed with additional principles specific to current needs:

* *Generalizability and usability.* The recommendations should allow a variety of decision makers to implement valuable changes to instructional materials and instructional planning.
* *Flexibility.* The 2020–21 school year is uncertain in terms of what schooling looks like; therefore, guidance should not specify pedagogy or make assumptions that learning is happening in physical classrooms with a designated content teacher.
* *Social, emotional, academic considerations.* While this guidance does not address the full context of “schooling” in 2020–21, there is a section of the document describing considerations for attending to students' social-emotional development in the

context of teaching the academic content described. Emotional health and well-being of students is a central concern of educators, particularly given the pandemic, and these suggestions demonstrate ways in which social, emotional, and academic development can be fostered in the context of grade-level college- and career-ready content.

This guide is intended to complement resources being released by various other organizations, including the Council of the Great City Schools and the Council of Chief State School Officers, that also address the challenges of prioritizing instruction and addressing unfinished learning and the social-emotional and mental health needs of students. The common messages found across these materials illustrate a consensus in the field around the importance of safeguarding equity and access in the wake of the COVID-19 crisis.

# Mathematics High School Priority Instructional Content for the 2020–21 School Year

As the 2020–21 school year approaches, mathematics educators are more interested than ever in knowing which topics or standards are most important. This document provides guidance for the field about content priorities by leveraging the structure and emphases of

college- and career-ready mathematics standards. As in previous years, students will need to engage deeply with grade-level mathematics by justifying claims, sharing their thinking and responding to the thinking of others, and solving well-chosen problems that connect to their world and advance them mathematically. This need is especially pronounced in high school mathematics where changes have been slower to take shape. As noted in *Catalyzing Change in High School Mathematics: Initiating Critical Conversations*:

Despite the progress that the mathematics education community has made to improve mathematics instruction and learning in kindergarten through grade 8 (NCES, 2015), an implementation gap persists between the calls for change and the comprehensive actions needed to support all high school students to learn and appreciate mathematics, to prepare them sufficiently for postsecondary education opportunities or a career (particularly in STEM), and to equip them with the quantitative skills and critical mathematical reasoning skills necessary to make sound decisions in their lives and as members of a democratic society. (NCTM, 2018, p. 2)

Instead of viewing this observation as an additional layer, or one more thing to “get done” while also navigating the recent and ongoing interruptions to schooling, it is possible to elevate the word “catalyzing” in the title of NCTM’s book and use this moment to deliver on some of the changes in high school mathematics that have been slow to unfold despite the known implications for just and equitable instruction that such changes could yield. In other words, we can prioritize content in a way that pushes our systems toward more equitable pathways that do a better job than today’s pathways do at connecting students to their desired postsecondary opportunities. Because of greater than usual variability in the recent mathematics experiences of returning students, educators will be looking for ways to accelerate learning and “catch up,” but students are unlikely to benefit from simply increasing the pace. Indeed, in guidance from the Council of the Great City Schools, *Addressing Unfinished Learning After COVID-19 School Closures*, a key recommendation is to:

Focus on the depth of instruction, not on the pace… [A]void the temptation to rush to cover all of the ‘gaps’ in learning from the last school year. The pace required to cover all of this content will mean rushing ahead of many students, leaving them abandoned and discouraged. It will also feed students a steady diet of curricular junk food: shallow engagement with the content, low standards for understanding, and low cognitive demand—all bad learning habits to acquire. Moreover, at a time when social emotional wellbeing, agency, and engagement are more important than ever, instructional haste may eclipse the patient work of building academic character and motivation. (CGCS, 2020)

But where will the time for in-depth teaching come from? The guidance in this document is intended to help publishers, other designers of instructional materials, and mathematics instructional leaders find new efficiencies in the curriculum that are critical for the unique challenges that have resulted from school closures and anticipated disruptions in the year ahead. In the sections that follow, the most important priorities in high school mathematics are clearly signaled. Opportunities are highlighted for reducing the normal emphasis of particular topics and, in some cases, there are suggestions to omit certain mathematical topics entirely or almost entirely for the 2020–21 school year. In this high school document, all of these suggestions are grounded in the larger national conversation about re-focusing high school mathematics programming and dismantling the longstanding tradition of tracking students by ability, moving instead towards instructional sequences more strongly associated with postsecondary success across a broad spectrum of college and career options. So while the primary purpose of this document is to inform decisions about which content to prioritize for the 2020–21 school year, it may also serve as a catalyst for the larger structural and content changes described in *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM, 2018). For additional information on high school to postsecondary mathematics pathways, see the “Additional Resources” section of the Appendix.

**How do we remain focused on equitable teaching that responds to students’ social, emotional, and academic development?**

As noted in *Addressing Unfinished Learning After COVID-19 School Closures*, “social emotional well being, agency, identity, and belonging are more important than ever” (CGCS, 2020). Indeed as focus narrows and there is recommitment to what matters most academically, research tells us that four learning mindsets are particularly important in supporting students’ academic development, specifically students’ sense of 1) belonging and safety, 2) efficacy, 3) value for effort and growth, and 4) engagement in work that is relevant and culturally responsive (Aspen Institute, 2019; The University of Chicago Urban Education Institute, 2018). Regardless of the mode of learning for the upcoming school year, attention must be given to restoring relationships and a sense of community, so students feel safe, engage fully, and work hard. Students need help knowing that caring adults believe in them and that their ability and competence will grow with their effort. And more than ever, students need to see value and relevance in what they are learning to their lives and their very beings. Investing in students' social-emotional development is done by the entire system of adults.

Confidence about the coming school year will come not only from recognizing the power and dedication of educators across the country, but also from investing in our nation’s students. Our beliefs about our students will matter greatly to our success. In *Catalyzing Change in High School Mathematics: Initiating Critical Conversations*, there is a valuable list of equitable mathematics teaching practices. Some of these practices are especially relevant today -- even as we make adjustments to the modes of instructional delivery (Table 1).

Table 1. Selected equitable mathematics teaching practices from *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM, 2018).

|  |
| --- |
| Selected Equitable Mathematics Teaching Practices from*Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM, 2018) |
| Create structures to position each and every student as a full participant in mathematics and recognize that participation builds agency (Turner, 2013 as cited in NCTM, 2018). |
| Use tasks that require reasoning, problem solving and modeling (i.e., tasks with high cognitive demand) to build positive student orientation toward mathematics allowing them to see themselves as doers of mathematics (Boaler & Staples, 2008 as cited in NCTM, 2018). |
| Elicit and use students’ ideas and pose purposeful questions to ensure that students see value in their own mathematical thinking and resist pedagogies that reinforce mathematics as a discipline focused solely on right and wrong responses. |

Remember that “...equitable mathematics teaching practices are inclusive when they acknowledge that students bring knowledge and resources from their community and make community-based knowledge and resources an integral part of mathematics teaching” (NCTM, 2018). As educators navigate the 2020–21 school year, these teaching practices can provide the necessary grounding to ensure that even as adaptations are made to the mode of instructional delivery that all students are positioned as knowers and doers of mathematics.

Mathematics has seldom been as prominent in the public square as it is now. Fewer citizens are saying, “I’m not a math person.” Instead they are reading the news about COVID-19 and contemplating rates, percentages, denominators, and time lags in order to know better how they can safely conduct their lives. Today, mathematics offers students both the empowerment that comes from using mathematical tools to understand and confront an epidemic, as well as the emotional escape that can come from permitting oneself to entertain abstract but beautiful questions at such a time. But caution should be taken here, as the topic of the pandemic is not one that should be tossed around casually or as a way to simply meet a particular mathematics standard without the deep intellectual preparation necessary to engage in conversations about our own humanity and that of our students. Venet (2020) provides some specific, thoughtful guidance for educators to

reflect on before they consider how to approach the topic of the pandemic with students in her blog post, “Is the Pandemic a Teachable Moment?”

**How should mathematics assessment be considered in light of this instructional guidance?**

Uncovering and addressing unfinished learning in the context of course-level work will require teachers to know what students know and can do at the beginning of and throughout the school year. This document is not intended to serve as a guide for assessment products. However, the instructional guidance has implications for assessment in service of equitable course-level instruction. Assessment should:

1. Be used to determine *how* to bring students into a unit of course-level instruction, not whether to bring them into it.
2. Center *formative* practices (FAST SCASS, 2018). Leverage such sources of information as exit tickets, student work, and student discussions. Use these sources of information to inform instructional choices in connection with high-quality instructional materials.
3. Employ *targeted* checks for very specific subject and course-level instructional purposes.

In mathematics in particular, assessment will be more useful, efficient, and supportive of social, emotional, and academic development when it takes place at the instructional triangle of teacher, student, and (course-level) subject. For example, unit-level assessments that publishers provide to accompany high-quality instructional materials are preferable to district-administered interim assessments. In mathematics, we can better understand students’ thinking even on assessments by engaging them in discussions of the problems they worked on.

Assessment should be used to determine how to bring students into a unit of course-level instruction, not whether to bring them into it. The point isn’t to generate data about what students get right and wrong; it’s to understand how to support students as they work. A single multiple choice item will not provide that, nor will a single numerical score. In mathematics, sometimes a couple of well-selected problems do the job of providing the right information to understand how to support students. In a distance learning scenario, one-on-one check-ins with students are likely the best way to understand how they are thinking about some of the important particulars and to help them understand how those particulars connect to the current course-level content they are about to engage with.

Pre-assessment is not needed for every unit in a curriculum. In some cases the prerequisites to a unit are few. Indeed some topics are well thought of as making their first appearance in a given course, and diagnosing about such topics is inappropriate. In many cases, the prerequisites for a unit are naturally and efficiently prompted by the content of the unit itself (remediating just-in-time, not just-in-case). And in some cases, students’ entry is based on a longer trajectory over multiple years.

This approach is being proposed as a deliberate alternative to assessment choices that have the potential to serve as a gatekeeper to course-level content. It also deliberately recognizes the very real social-emotional needs of students—particularly students who have been disproportionately affected by the pandemic. After such major disruptions, it is essential that students engage immediately and consistently in the affirmative act of learning new ideas, not be deemed deficient because of events outside of their control. Regarding administering tests too soon, the Council of the Great City Schools notes in *Addressing Unfinished Learning After COVID-19 School Closures* that “testing appears to put the onus of learning losses on the students themselves—the resulting label of ‘deficient’ or academically behind may very well further alienate and isolate the students who most need our support” (CGCS, 2020).

**Where to focus high school mathematics?**

This 2020–21 Support for Instructional Content Prioritization in High School Mathematics (High School Mathematics Instructional Priorities) is designed to provide guidance for decisions about how to elevate some of the most important mathematics in typical high school mathematics courses in the coming school year while reducing time and intensity for topics that are less integral to the overall coherence of college- and career-ready standards.

The High School Mathematics Instructional Priorities document differs in structure from the K–8 document due primarily to the structural difference in the standards themselves: namely, that high school mathematics standards are not organized by grade level, and the ways in which states and/or districts organize standards into courses vary widely. However, similar to the K–8 document, this guidance suggests ways to reduce or sometimes eliminate topics in a way that minimizes the impact to overall coherence and thereby creates some additional time in the school year for supporting students in accessing and engaging with the most important high school mathematics content. In using this guidance, decision makers should thoughtfully consider in their unique context the likely implications of the spring 2020 disruption as decisions are made to select supports to ensure that students are able to successfully engage with the course-level content. Decision makers should also bear in mind that while this document articulates content priorities, elevating the Standards for Mathematical Practice in connection with course-level content is always a priority.

The 2020–21 school year presents a unique set of opportunities and challenges due to the disruption to instruction in spring 2020 as well as the uncertainty associated with the 2020–21 school year. The High School Mathematics Instructional Priorities are provided in response to these conditions. They are not criteria, and they do not revise the standards. Rather, they are potential ways, and not the only ways possible, to help students engage deeply with course-level mathematics in the 2020–21 school year.

The High School Mathematics Instructional Priorities do not stand alone but are to be used in conjunction with college- and career-ready standards. One reason for this is that codes such as F-IF.A must be traced back to the standards in order to see the language to which they refer. The High School Mathematics Instructional Priorities do not reiterate what the standards already say—even in cases where the specific language of a standard is fundamentally important to a high-quality aligned curriculum. Therefore, the High School Mathematics

Instructional Priorities will be used most powerfully by educators who know the standards well and can use existing resources such as those listed in the Appendix.

In constructing the recommendations for the High School Mathematics Instructional Priorities, several resources were consulted to gain an understanding of how the standards are typically organized into courses as well as to make determinations about which standards to prioritize, which standards to de-emphasize, and which standards could reasonably be eliminated under the current circumstances. In addition to the information obtained from the resources listed below, some decisions required professional judgment of the document’s lead writers, who also serve in district roles where such guidance for the upcoming school year will be greatly needed.

Resources consulted to inform the assignment of standards to courses:

1. Utah Core Standards: Major Works (Utah State Board of Education, n.d.)
2. Achieve the Core’s High School Coherence Map (Student Achievement Partners, n.d.)
3. *Common Core State Standards for Mathematics Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards* (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010b)

Resources consulted to inform the prioritization of standards for 2020–21 school year:

1. *Common Core State Standards for Mathematics* [for standards-designated modeling] (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010a)
2. Achieve the Core’s Widely Applicable Prerequisites (Student Achievement Partners, n.d.)
3. *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM, 2018)
4. High School Core Math Content (Oregon Department of Education, in press)

For the 2020–21 school year, prioritization of mathematical concepts and skills will be essential to support all students in meeting course- level expectations. Since the vast majority of high schools across the United States still use either an Algebra 1, Geometry, Algebra 2 sequence or some form of Integrated Mathematics I, II, and III sequence, the standards listed on the pages that follow have been coded in a way that corresponds to these courses. The tables use the following codes associated with each course: Algebra 1 (A1); Geometry (G); Algebra 2 (A2); Integrated Mathematics 1 (M1); Integrated Mathematics 2 (M2); and Integrated Mathematics 3 (M3).

**How to Read the Content Prioritization Tables**

The tables are first organized by conceptual category and cluster; then below each cluster heading, the associated standards each receive a designation to indicate the recommended level of emphasis within a particular course for the 2020–21 school year. The designations below represent the codes used to communicate this emphasis:

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

For standards coded with “P” for a particular course, users should interpret that to mean that no special considerations should be made for curricula well aligned to the particulars of that standard, or that the emphasis should be comparable to what it typically is for that course. Standards coded with “R” have suggestions for either reducing the emphasis on certain parts of the standard or for reducing the overall time and attention to the entire standard, or some combination of these adaptations. For these cases, there will be a note accompanying the standard to provide additional guidance related to the particular reduction in emphasis that is being suggested by the coding. Standards coded with “E” are eligible to be eliminated for the upcoming school year to make room for additional support that may be needed to ensure that students can engage successfully with the most important content of each course and to recognize that some of the modes of learning being discussed for the upcoming year simply require more time on fewer topics. The designation “--” indicates that the standard is typically taught in a different course.

One additional set of codes in the tables is designed to help users understand in part how levels of prioritization were determined. These codes are assigned to individual standards and carry the following meanings:

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

Standards that are considered “widely applicable prerequisites” are those with relatively wide applicability across a wide range of postsecondary work and often not taught for course credit in postsecondary settings. Modeling standards are those that lend themselves to developing and analyzing mathematical models for real world phenomena and generally have greater overall importance in the high school sequence of courses. Finally, standards identified as essential in *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM, 2018), are also marked as indicated above.

As a final thought, it is important to understand that these tables will not provide a one-to-one correspondence between standards and any particular scope and sequence or set of instructional materials. Well-designed mathematics curricula are structured to communicate mathematical ideas in a coherent, logical manner and often integrate standards in ways that cannot be seen when standards are shown as a list. Professional judgment, local context considerations, and flexible decision-making throughout the 2020–21 school year will be essential to effectively using the information presented on the pages that follow.

# High School Standards Prioritization Tables

|  |
| --- |
| **Conceptual Category: Number and Quantity Domain: The Real Number System** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Extend the properties of exponents to rational exponents. |
| HS.N.RN.A.1^~ | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5*. | E | -- | P | -- | P | -- |
| HS.N.RN.A.2^~*See Note* | Rewrite expressions involving radicals and rational exponents using the properties of exponents.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | E | -- | R | -- | R | -- |
| **Cluster:** Use properties of rational and irrational numbers. |
| HS.N.RN.B.3^~ | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | E | -- | -- | -- | E | -- |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

|  |
| --- |
| **Conceptual Category: Number and Quantity Domain: Quantities** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Reason quantitatively and use units to solve problems.*Note: All standards in this cluster require students to work with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.* |
| HS.N.Q.A.1^~ | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | P | -- | -- | P | -- | -- |
| HS.N.Q.A.2^~ | Define appropriate quantities for the purpose of descriptive modeling. | P | -- | E | P | -- | -- |
| HS.N.Q.A.3^~ | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | P | -- | -- | P | -- | -- |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

|  |
| --- |
| **Conceptual Category: Number and Quantity Domain: The Complex Number System** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Perform arithmetic operations with complex numbers. |
| HS.N.CN.A.1*See Note* | Know there is a complex number *i* such that *i*2 = -1, and every complex number has the form *a + bi* with *a* and *b* real.*Note: Combine lessons with* N.CN.C.7 *and* A.REI.B.4b *to address key concepts and reduce the amount of time spent on this standard.* | -- | -- | R | -- | R | -- |
| HS.N.CN.A.2*See Note* | Use the relation *i*2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | R | -- |
| HS.N.CN.A.3 | (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Represent complex numbers and their operations on the complex plane. |
| HS.N.CN.B.4 | (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | -- | -- | -- | -- | -- | -- |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

|  |
| --- |
| **Conceptual Category: Number and Quantity Domain: The Complex Number System** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Represent complex numbers and their operations on the complex plane. (continued) |
| HS.N.CN.B.5 | (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, (-1 + √3 i)3 = 8 because (- 1 + √3 i) has modulus 2 and argument 120°.* | -- | -- | -- | -- | -- | -- |
| HS.N.CN.B.6 | (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Use complex numbers in polynomial identities and equations. |
| HS.N.CN.C.7*See Note* | Solve quadratic equations with real coefficients that have complex solutions.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | R | -- |
| HS.N.CN.C.8 | (+) Extend polynomial identities to the complex numbers. *For example, rewrite x2 + 4 as (x + 2i)(x - 2i)*. | -- | -- | E | -- | E | E |
| HS.N.CN.C.9 | (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | -- | -- | E | -- | E | E |

Note: Vector Quantities and Matrices are not included in AGA or M1M2M3

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

|  |
| --- |
| **Conceptual Category: Algebra****Domain: Seeing Structure in Expressions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Interpret the structure of expressions. |
| HS.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. |
| HS.A-SSE.A.1a^\* | Interpret parts of an expression, such as terms, factors, and coefficients. | P | -- | P | P | P | P |
| HS.A-SSE.A.1b^\**See Note* | Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)2 as the product of P and a factor not depending on P*.*Note: Reduce overall emphasis, but retain focus on interpreting expressions to shed light on a quantity in context (as described in parent standard A- SSE.A.1).* | R | -- | R | R | R | R |
| HS.A-SSE.A.2^~*See Note* | Use the structure of an expression to identify ways to rewrite it. *For example, see x2 - y4 as (x2)2 - (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 - y2)(x2 + y2)*.*Note: Reduce overall emphasis in earlier algebra-focused courses.* | R | -- | P | -- | R | P |
| **Cluster:** Write expressions in equivalent forms to solve problems. |
| HS.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Algebra****Domain: Seeing Structure in Expressions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Write expressions in equivalent forms to solve problems. (continued) |
| HS.A-SSE.B.3a^\* | Factor a quadratic expression to reveal the zeros of the function it defines. | P | -- | -- | -- | P | -- |
| HS.A-SSE.B.3b^\**See Note* | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic and emphasize the value of the form of the expression over fluency with the specific process of completing the square. Connect to students’ work on A-REI.B.4a.* | R | -- | -- | -- | R | -- |
| HS.A-SSE.B.3c^\* | Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15t can be rewritten as (1.151/12)12t**≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%*. | P | -- | E | -- | P | -- |
| HS.A-SSE.B.4\*^*See Note* | Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**Note: Combine with F-BF.A.2.* | -- | -- | R | -- | -- | R |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Algebra****Domain: Arithmetic with Polynomials & Rational Expressions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Perform arithmetic operations on polynomials. |
| HS.A-APR.A.1^*See Note* | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*Note: A-APR.1 - Less emphasis on adding/subtracting and more prioritize multiplying. Combine lessons with A-SSE 2 to address key concepts and reduce the amount of time spent on this standard.* | R | -- | P | -- | R | P |
| **Cluster:** Understand the relationship between zeros and factors of polynomials. |
| HS.A-.APR.B.2^*See Note* | Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x).*Note: Reduce overall emphasis and the number of repetitious practice problems.* | -- | -- | R | -- | -- | R |
| HS.A-APR.B.3^ | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | E | -- | P | -- | -- | P |
| **Cluster:** Use polynomial identities to solve problems. |
| HS.A-APR.C.4^ | Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity (x2 + y2)2 = (x2 - y2)2 + (2xy)2 can be used to generate Pythagorean triples.* | -- | -- | E | -- | -- | E |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Algebra****Domain: Arithmetic with Polynomials & Rational Expressions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Use polynomial identities to solve problems. (continued) |
| HS.A-APR.C.5^ | (+) Know and apply the Binomial Theorem for the expansion of (*x* + *y*)*n* in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. | -- | -- | E | -- | -- | E |
| **Cluster:** Rewrite rational expressions. |
| HS.A-APR.D.6^*See Note* | Rewrite simple rational expressions in different forms; write *a*(*x*)/*b*(*x*) in the form *q*(*x*) + *r*(*x*)/*b*(*x*), where *a*(*x*), *b*(*x*), *q*(*x*), and *r*(*x*) are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic. Connect to A-APR.B.2.* | -- | -- | R | -- | -- | R |
| HS.A-APR.D.7^ | (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | -- | -- | E | -- | -- | E |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Algebra Domain: Creating Equations** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Create equations that describe numbers or relationships. |
| HS.A-CED.A.1^\* | Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. | P | -- | P | P | P | P |
| HS.A-CED.A.2^\* | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | P | -- | P | P | P | P |
| HS.A-CED.A.3^\* | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*. | P | -- | P | P | -- | P |
| HS.A-CED.A.4^\**See Note* | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R*.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | P | -- | P | P | P | R |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Algebra****Domain: Reasoning with Equations and Inequalities** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand solving equations as a process of reasoning and explain the reasoning. |
| HS.A-REI.A.1^~*See Note* | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.*Note: Lessen the normal emphasis on problem types related to explaining each step and elevate the importance of constructing viable arguments.* | R | -- | E | R | -- | -- |
| HS.A-REI.A.2^ | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | -- | -- | P | -- | -- | P |
| **Cluster:** Solve equations and inequalities in one variable. |
| HS.A-REI.B.3^*See Note* | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | R | -- | -- | R | -- | -- |
| HS.A-REI.B.4 | Solve quadratic equations in one variable. |

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| **Conceptual Category: Algebra****Domain: Reasoning with Equations and Inequalities** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Solve equations and inequalities in one variable. (continued) |
| HS.A-REI.B.4a^*See Note* | Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form (*x* - *p*)2 = *q* that has the same solutions. Derive the quadratic formula from this form.*Note: Lessen the normal emphasis on deriving the quadratic formula and reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | R | -- | -- | -- | R | -- |
| HS.A-REI.B.4b^~*See Note* | Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*.*Note: Lessen the emphasis on completing the square and emphasize solving by inspection, taking square roots, quadratic formula, and factoring; recognize when quadratic formula gives non-real solutions but reduce emphasis on this case.* | R | -- | R | -- | R | -- |
| **Cluster:** Solve systems of equations. |
| HS.A-REI.C.5^ | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | E | -- | -- | E | -- | -- |

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| **Conceptual Category: Algebra****Domain: Reasoning with Equations and Inequalities** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Solve systems of equations. (continued) |
| HS.A-REI.C.6^ | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | P | -- | E | P | -- | -- |
| HS.A-REI.C.7^*See Note* | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line *y* = -3*x* and the circle *x*2 + *y2* = 3.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | R | -- | E | -- | R | -- |
| HS.A-REI.C.8 | (+) Represent a system of linear equations as a single matrix equation in a vector variable. | -- | -- | -- | -- | -- | -- |
| HS.A-REI.C.9 | (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | -- | -- | -- | -- | -- | -- |
| **Cluster:** Represent and solve equations and inequalities graphically. |
| HS.A-REI.D.10^ | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | P | -- | -- | P | -- | -- |

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| **Conceptual Category: Algebra****Domain: Reasoning with Equations and Inequalities** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Represent and solve equations and inequalities graphically. (continued) |
| HS.A-REI.D.11^\*~ | Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*)= *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | P | -- | P | P | -- | P |
| HS.A-REI.D.12^~*See Note* | Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.*Note: Emphasize problems that ground the mathematics in real world contexts.* | P | -- | -- | P | -- | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Functions Domain: Interpreting Functions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand the concept of a function and use function notation. |
| HS.F-IF.A.1^~ | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). | P | -- | -- | P | -- | -- |
| HS.F-IF.A.2^ | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | P | -- | -- | P | -- | -- |
| HS.F-IF.A.3^*See Note* | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n**≥ 1*.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | R | -- | R | R | -- | -- |
| **Cluster:** Interpret functions that arise in applications in terms of the context.*M1 - Linear, exponential, and quadratic**M2 - Emphasize selection of appropriate models* |
| HS.F-.IF.B.4^\*~ | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. | P | -- | P | P | P | P |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Functions Domain: Interpreting Functions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Interpret functions that arise in applications in terms of the context. (continued)*M1 - Linear, exponential, and quadratic**M2 - Emphasize selection of appropriate models* |
| HS.F-IFB.5^\* | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*\* | P | -- | P | P | P | P |
| HS.F-IF.B.6^\* | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | P | -- | P | P | P | P |
| **Cluster:** Analyze functions using different representations. |
| HS.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| HS.F-IF.C.7a^\* | Graph linear and quadratic functions and show intercepts, maxima, and minima. | P | -- | -- | P | P | -- |
| HS.F-IF.C.7b^\**See Note* | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*Note: Eliminate step functions; emphasize square root and cube root.* | R | -- | P | -- | P | R |
| HS.F-IF.C.7c^\* | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | -- | -- | P | -- | -- | P |

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| **Conceptual Category: Functions Domain: Interpreting Functions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Analyze functions using different representations. (continued) |
| HS.F-IF.C.7d | (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | -- | -- | -- | -- | -- | -- |
| HS.F-IF.C.7e^\* | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | P | -- | P | P | -- | P |
| HS.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. |
| HS.F-IF.C.8a^*See Note* | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.*Note: Reduce the number of repetitious practice problems related to factoring trinomials over the integers, and emphasize using the factored form to draw conclusions. Connect to* HS.A-SSE.B.3b. | R | -- | R | -- | R | -- |
| HS.F-IF.C.8b^ | Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)ᵗ, y = (0.97)ᵗ, y = (1.01)12ᵗ, y = (1.2)t/10, and classify them as representing exponential growth or decay.* | E | -- | E | -- | E | -- |

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| **Conceptual Category: Functions Domain: Interpreting Functions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Analyze functions using different representations. (continued) |
| HS.F-IF.C.9^*See Note* | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | P | -- | R | P | P | P |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Functions Domain: Building Functions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Build a function that models a relationship between two quantities. |
| HS.F-BF.A.1 | Write a function that describes a relationship between two quantities. |
| HS.F-BF.A.1a^\**See Note* | Determine an explicit expression, a recursive process, or steps for calculation from a context.*Note: Combine with F-BF.A.2, F-LE.A.2 and F-IF.A.3 to address key concepts and reduce the amount of time spent on this standard.* | R | -- | E | R | R | E |
| HS.F-BF.A.1b^\* | Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*. | E | -- | E | E | E | E |
| HS.F-BF.A.1c | (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*. | -- | -- | -- | -- | -- | -- |
| HS.F-BF.A.2\**See Note* | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*Note: Combine with F-BF.A.1b and F-LE.A.2 to address key concepts and reduce the amount of time spent on this standard.* | R | -- | R | R | -- | -- |

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| **Conceptual Category: Functions Domain: Building Functions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Build new functions from existing functions. |
| HS.F-BF.B.3 | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*)for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. | P | -- | P | P | P | P |
| HS.F-BF.B.4 | Find inverse functions. *For example, f(x) =2*𝑥*3 or f(x) = (x+1)/(x-1) for x ≠ 1*. |
| HS.F-BF.B.4a*See Note* | Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | E | -- | R | -- | E | R |
| HS.F-BF.B.4b*See Note* | (+) Verify by composition that one function is the inverse of another. *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | -- | -- | -- | R |
| HS.F-BF.B.4c | (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. | -- | -- | -- | -- | E | E |
| HS.F-BF.B.4d | (+) Produce an invertible function from a non-invertible function by restricting the domain. | -- | -- | -- | -- | E | E |
| HS.F-BF.B.5 | (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. | -- | -- | -- | -- | -- | -- |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Functions****Domain: Linear, Quadratic, and Exponential** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Construct and compare linear, quadratic, and exponential models and solve problems. |
| HS.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. |
| HS.F-LE.A.1a^\*~ | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | P | -- | -- | P | -- | -- |
| HS.F-LE.A.1b^\*~ | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | P | -- | -- | P | -- | -- |
| HS.F-LE.A.1c^\*~ | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | P | -- | -- | P | -- | -- |
| HS.F-LE.A.2\*~ | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | P | -- | E | P | -- | -- |
| HS.F-LE.A.3\**See Note* | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*Note: Combine with F-LE.A.1b and F-LE.A.1c to address key concepts and reduce the amount of time spent on this standard.* | R | -- | -- | R | R | R |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Functions****Domain: Linear, Quadratic, and Exponential** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Construct and compare linear, quadratic, and exponential models and solve problems. (continued) |
| HS.F-LE.A.4\**See Note* | For exponential models, express as a logarithm the solution to *abct* = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | -- | R |
| **Cluster:** Interpret expressions for functions in terms of the situation they model. |
| HS.F-LE.B.5\*~ | Interpret the parameters in a linear or exponential function in terms of a context. | P | -- | E | P | -- | -- |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Functions Domain: Trigonometric Functions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Extend the domain of trigonometric functions using the unit circle. |
| HS.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | -- | -- | P | -- | -- | P |
| HS.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | -- | -- | P | -- | -- | P |
| HS.F-TF.A.3 | (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for *x*, π + *x*, and 2π - *x* in terms of their values for *x*, where *x* is any real number. | -- | -- | -- | -- | -- | -- |
| HS.F-TF.A.4 | (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Model periodic phenomena with trigonometric functions. |
| HS.F-TF.B.5\* | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | -- | -- | P | -- | -- | P |
| HS.F-TF.B.6 | (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | -- | -- | -- | -- | -- | -- |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Functions Domain: Trigonometric Functions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Model periodic phenomena with trigonometric functions. (continued) |
| HS.F-TF.B.7 | (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Prove and apply trigonometric identities. |
| HS.F-TF.C.8 | Prove the Pythagorean identity𝑠𝑖𝑛*2*(θ) + 𝑐𝑜𝑠*2*(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. | -- | -- | E | -- | E | E |
| HS.F-TF.C.9 | (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | -- | -- | -- | -- | -- | -- |

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| **Conceptual Category: Geometry Domain: Congruence** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Experiment with transformations in the plane. |
| HS.G-CO.A.1^*See Note* | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.*Note: Combine with G-CO.A.4 to address key concepts and reduce the amount of time spent on this standard.* | -- | R | -- | R | -- | -- |
| HS.G-CO.A.2^~ | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | -- | P | -- | P | -- | -- |
| HS.G-CO.A.3^~*See Note* | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.*Note: Combine with G-CO.A.2 to address key concepts and reduce the amount of time spent on the standard.* | -- | R | -- | R | -- | -- |
| HS.G-CO.A.4^ | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | -- | P | -- | P | -- | -- |
| HS.G-CO.A.5^~ | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | -- | P | -- | P | -- | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Geometry Domain: Congruence** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand congruence in terms of rigid motions. |
| HS.G-CO.B.6^~ | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | -- | P | -- | P | -- | -- |
| HS.G-.CO.B.7^~ | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | -- | P | -- | P | -- | -- |
| HS.G-CO.B.8^ | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | -- | P | -- | P | -- | -- |
| **Cluster:** Prove geometric theorems. |
| HS.G-CO.C.9^~ | Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*. | -- | P | -- | -- | P | -- |
| HS.G-CO.C.10^~*See Note* | Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point*.*Note: Reduce overall time spent on proving theorems.* | -- | R | -- | -- | R | -- |

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| **Conceptual Category: Geometry Domain: Congruence** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Prove geometric theorems. (continued) |
| HS.G-CO.C.11*See Note* | Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*.*Note: Reduce overall time spent on proving theorems.* | -- | R | -- | -- | R | -- |
| **Cluster:** Make geometric constructions. |
| HS.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line*. | -- | P | -- | P | -- | -- |
| HS.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | -- | E | -- | E | -- | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Geometry****Domain: Similarity, Right Triangles, and Trigonometry** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand similarity in terms of similarity transformations. |
| HS.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: |
| HS.G-SRT.A.1a^ | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | -- | P | -- | -- | P | -- |
| HS.G-SRT.A.1b^*See Note* | The dilation of a line segment is longer or shorter in the ratio given by the scale factor.*Note: Combine with students’ work on G-SRT.A.1a.* | -- | R | -- | -- | R | -- |
| HS.G-SRT.A.2^~ | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | -- | P | -- | -- | P | -- |
| HS.G-SRT.A.3^ | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | -- | P | -- | -- | P | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Geometry****Domain: Similarity, Right Triangles, and Trigonometry** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Prove theorems involving similarity. |
| HS.G-SRT.B.4^ | Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* | -- | P | -- | -- | P | -- |
| HS.G-SRT.B.5^ | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | -- | P | -- | -- | P | -- |
| **Cluster:** Define trigonometric ratios and solve problems involving right triangles. |
| HS.G-SRT.C.6^\* | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | -- | P | -- | -- | P | -- |
| HS.G-.SRT.C.7*See Note* | Explain and use the relationship between the sine and cosine of complementary angles.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | -- | R | -- |
| HS.G-SRT.C.8\*~*See Note* | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | -- | R | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Geometry****Domain: Similarity, Right Triangles, and Trigonometry** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Apply trigonometry to general triangles. |
| HS.G-SRT.D.9 | (+) Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | -- | E | -- | -- | -- | E |
| HS.G-SRT.D.10 ^*See Note* | (+) Prove the Laws of Sines and Cosines and use them to solve problems.*Note: Lessen the normal emphasis on proofs and elevate the importance of solving problem types.* | -- | E | -- | -- | -- | R |
| HS.G-SRT.D.11^*See Note* | (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | E | -- | -- | -- | R |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Geometry Domain: Circles** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand and apply theorems about circles. |
| HS.G-C.A.1 | Prove that all circles are similar. | -- | E | -- | -- | E | -- |
| HS.G-C.A.2*See Note* | Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.**Note: Emphasize primarily the concept of perpendicularity between the radius and any tangent to the circle.* | -- | R | -- | -- | R | R |
| HS.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | -- | E | -- | -- | E | -- |
| HS.G-C.A.4 | (+) Construct a tangent line from a point outside a given circle to the circle. | -- | E | -- | -- | E | -- |
| **Cluster:** Find arc lengths and areas of sectors of circles. |
| HS.G-C.B.5*See Note* | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.*Note: Reduce overall emphasis on the standard but retain the core definition of radian measure as described in the standard.* | -- | R | -- | -- | R | R |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Geometry****Domain: Expressing Geometric Properties with Equations** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Translate between the geometric description and the equation for a conic section. |
| HS.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. | -- | P | -- | -- | P | -- |
| HS.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix. | -- | E | E | -- | E | -- |
| HS.G-GPE.A.3 | (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. | -- | -- | -- | -- | -- | E |
| **Cluster:** Use coordinates to prove simple geometric theorems algebraically. |
| HS.G-GPE.B.4~ | Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* | -- | P | -- | P | P | -- |
| HS.G-GPE.B.5~*See Note* | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | R | -- | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Geometry****Domain: Expressing Geometric Properties with Equations** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Use coordinates to prove simple geometric theorems algebraically. (continued) |
| HS.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio. | -- | E | -- | -- | E | -- |
| HS.G-GPE.B.7\**See Note* | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*Note: Emphasize understanding the formula conceptually, use it to solve real world problems, and reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | R | -- | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Geometry****Domain: Geometric Measurement and Dimension** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Explain volume formulas and use them to solve problems. |
| HS.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*. | -- | E | -- | -- | E | -- |
| HS.G-GMD.A.2 | (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. | -- | E | -- | -- | -- | -- |
| HS.G-GMD.A.3\*~ | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | -- | P | -- | -- | P | -- |
| **Cluster:** Visualize relationships between two-dimensional and three-dimensional objects. |
| HS.G-GMD.B.4*See Note* | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | -- | -- | R |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Geometry Domain: Modeling with Geometry** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Apply geometric concepts in modeling situations. |
| HS.G-MG.A.1\*~ | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).\* | -- | P | -- | -- | -- | P |
| HS.G-MG.A.2\*~ | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).\* | -- | P | -- | -- | -- | P |
| HS.G-MG.A.3\*~ | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).\* | -- | P | -- | -- | -- | P |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Statistics & Probability****Domain: Interpreting Categorical and Quantitative Data** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Summarize, represent, and interpret data on a single count or measurement variable. |
| HS.S-ID.A.1\*~ | Represent data with plots on the real number line (dot plots, histograms, and box plots). | E | -- | -- | E | -- | -- |
| HS.S-ID.A.2^\*~ | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | P | -- | -- | P | -- | -- |
| HS.S-ID.A.3\*~ | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | P | -- | -- | P | -- | -- |
| HS.S-ID.A.4\*~ | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | -- | -- | P | -- | -- | P |
| **Cluster:** Summarize, represent, and interpret data on two categorical and quantitative variables. |
| HS.S-ID.B.5\*~ | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | P | -- | -- | P | -- | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Statistics & Probability****Domain: Interpreting Categorical and Quantitative Data** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Summarize, represent, and interpret data on two categorical and quantitative variables. (continued) |
| HS.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. |
| HS.S-ID.B.6a\*~ | Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. | P | -- | E | P | -- | -- |
| HS.S-ID.B.6b\*~ | Informally assess the fit of a function by plotting and analyzing residuals. | P | -- | -- | P | -- | -- |
| HS.S-ID.B.6c\*~ | Fit a linear function for a scatter plot that suggests a linear association. | P | -- | -- | P | -- | -- |
| **Cluster:** Interpret linear models. |
| HS.S-ID.C.7^\*~ | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | P | -- | -- | P | -- | -- |
| HS.S-ID.C.8\*~*See Note* | Compute (using technology) and interpret the correlation coefficient of a linear fit.*Note: Emphasize interpreting the correlation coefficient.* | R | -- | -- | R | -- | -- |
| HS.S-ID.C.9\*~ | Distinguish between correlation and causation. | P | -- | -- | P | -- | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Statistics & Probability****Domain: Making Inferences and Justifying Conclusions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand and evaluate random processes underlying statistical experiments. |
| HS.S-IC.A.1^\*~ | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | -- | -- | P | -- | -- | P |
| HS.S-IC.A.2^\*~ | Decide if a specified model is consistent with results from a given data- generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*? | -- | -- | P | -- | -- | P |
| **Cluster:** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. |
| HS.S-IC.B.3\*~*See Note* | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*Note: Combine lessons with S-IC.B.4 and S-IC.B.5 to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | -- | R |
| HS.S-IC.B.4\*~*See Note* | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*Note: Combine lessons with S-IC.B.3 and S-IC.B.5 to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | -- | R |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Statistics & Probability****Domain: Making Inferences and Justifying Conclusions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. (continued) |
| HS.S-IC.B.5\*~*See Note* | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*Note: Combine lessons with S-IC.B.3 and S-IC.B.4 to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | -- | R |
| HS.S-IC.B.6\*~*See Note* | Evaluate reports based on data.*Note: Reduce the normal emphasis.* | -- | -- | R | -- | -- | R |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Statistics & Probability****Domain: Conditional Probability and the Rules of Probability** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand independence and conditional probability and use them to interpret data. |
| HS.S-.CP.A.1\*~ | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). | -- | P | E | -- | P | -- |
| HS.S-.CP.A.2\*~*See Note* | Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*Note: Combine with lessons on other S-CP.A standards to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | E | -- | R | -- |
| HS.S-CP.A.3\*~*See Note* | Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.*Note: Combine with lessons on other S-CP.A standards to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | E | -- | R | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Statistics & Probability****Domain: Conditional Probability and the Rules of Probability** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand independence and conditional probability and use them to interpret data. (continued) |
| HS.S-CP.A.4\*~ | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* | -- | P | E | -- | P | -- |
| HS.S-CP.A.5\*~ | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* | -- | P | E | -- | P | -- |
| **Cluster:** Use the rules of probability to compute probabilities of compound events. |
| HS.S-CP.B.6\**See Note* | Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | E | -- | R | -- |
| HS.S-CP.B.7\**See Note* | Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.*Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | E | -- | R | -- |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| --- |
| **Conceptual Category: Statistics & Probability****Domain: Conditional Probability and the Rules of Probability** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Use the rules of probability to compute probabilities of compound events. (continued) |
| HS.S-CP.B.8\* | (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. | -- | E | E | -- | E | -- |
| HS.S-CP.B.9\* | (+) Use permutations and combinations to compute probabilities of compound events and solve problems. | -- | E | E | -- | E | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

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| **Conceptual Category: Statistics & Probability Domain: Using Probability to Make Decisions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Calculate expected values and use them to solve problems. |
| HS.S-MD.A.1 | (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. | -- | -- | -- | -- | -- | -- |
| HS.S-MD.A.2 | (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. | -- | -- | -- | -- | -- | -- |
| HS.S-MD.A.3 | (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* | -- | -- | -- | -- | -- | -- |
| HS.S-MD.A.4 | (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* | -- | -- | -- | -- | -- | -- |

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^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

|  |
| --- |
| **Conceptual Category: Statistics & Probability Domain: Using Probability to Make Decisions** |
| **Standard** | **Language of Standard** | **Courses** |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Use probability to evaluate outcomes of decisions. |
| HS.S-MD.B.5 | (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. |
| HS.S-MD.B.5a | Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* | -- | -- | -- | -- | -- | -- |
| HS.S-MD.B.5b | Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* | -- | -- | -- | -- | -- | -- |
| HS.S-MD.B.6 | (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). | -- | E | E | -- | E | E |
| HS.S-MD.B.7 | (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). | -- | E | E | -- | E | E |

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

# Appendix

**Additional Resources**

Charles A. Dana Center at the University of Texas at Austin. (2019). [*Launch years: Reimagining mathematics education*](https://www.utdanacenter.org/our-work/k-12-education/launch-years)*.*

Daro, P., & Asturias, H. (2019). [*Branching out: Designing high school mathematics pathways for equity*](https://justequations.org/wp-content/uploads/Just-Equations-2019-Report-Branching-Out-Digital.pdf)*.*

Common Core Standards Writing Team. (2013). [*Progressions documents for the Common Core Math Standards*](http://ime.math.arizona.edu/progressions/). Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of the Great City Schools. (2020)*. Addressing unfinished learning after COVID-19 school closures*. Washington, DC: Author. [www.cgcs.org](http://www.cgcs.org/)

Council of the Great City Schools. (n.d.). [*English language learners: Instructional materials*](https://www.cgcs.org/Page/664).

English Learners Success Forum. (n.d.). [*Explore the resources*](https://www.elsuccessforum.org/resources).

Formative Assessment for Students and Teachers (FAST) State Collaborative on Assessment and Student Standards (SCASS). (2018). [*Revising*](https://ccsso.org/sites/default/files/2018-06/Revising%20the%20Definition%20of%20Formative%20Assessment.pdf)[*the definition of formative assessment*](https://ccsso.org/sites/default/files/2018-06/Revising%20the%20Definition%20of%20Formative%20Assessment.pdf). Washington, DC: Council of Chief State School Officers.

National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*.

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National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). [*Common Core State Standards for*](http://www.corestandards.org/Math/)[*Mathematics*](http://www.corestandards.org/Math/). Washington, DC: Author.

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Venet, A. S. (2020). Is the pandemic a teachable moment? [*Unconditional learning*](https://unconditionallearning.org/2020/07/21/%20is-the-pandemic-a-teachable-moment/).

**Social, Emotional, and Academic Development (SEAD) Sources**

Allensworth, E. M., Farrington, C. A., Gordon, M. F., Johnson, D. W., Klein, K., McDaniel, B., & Nagaoka, J. (2018). *Supporting social, emotional, & academic development: Research implications for educators*. Chicago, IL: University of Chicago Consortium on School Research.

Aspen Institute. (2019). *Integrating social, emotional, and academic development (SEAD): An action guide for school leadership teams*. The Aspen Institute Education & Society Program.

Collaborative for Academic, Social, and Emotional Learning. (2013). *CASEL guide: Effective social and emotional learning programs— preschool and elementary school edition*.

The University of Chicago Urban Education Institute. (2018). Cultivating social, emotional, and academic development. In *New knowledge and developments in public education* (pp. 11–16).

Wiener, R. (2020). *Recovery and renewal: Principles for advancing public education post-crisis*. The Aspen Institute Education & Society Program.

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National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010b). [*Common Core State Standards for*](http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf)[*Mathematics, Appendix A: Designing high school mathematics courses based on the Common Core State Standards*](http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf). Washington, DC: Author.

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