Looking For and Making Use of Structure – Quadratic Equations 1

Sample task from achievethecore.org
Task by CME Project, EDC, annotation by Student Achievement Partners

GRADE LEVEL High School

IN THE STANDARDS A-REI.B.4, MP.7

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Rewards the practice of looking for and making use of structure (MP7).
- Solvable by casting the equation into standard form, although considerable procedural fluency is required for this method. In that case, the task rewards procedural skill as well as perseverance (MP1).

In the classroom:

- Allows individuals, groups, or the class as a whole to suggest ideas for how to proceed.
- Allows the class to compare multiple methods of solving.
- Does not require extensive setup on the part of the teacher.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Elicits evidence of student thinking both verbally and in written form.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.
- Provides opportunities to support students in connecting mathematical language with mathematical representations.

MAKING THE SHIFTS[1]

Focus	A.REI.B.4 belongs to the Widely Applicable Prerequisites for College and Careers[2]
Coherence	Chunking (seeing parts of an expression as a single object) is a key algebraic skill, useful in factoring, completing the square, and other mindful algebraic calculations, and not limited to quadratics alone
Rigor[3]	Conceptual Understanding: primary in this task Procedural Skill and Fluency: primary in this task Application: not targeted in this task

[1] For more information read Shifts for Mathematics.
[2] For more information, see Widely Applicable Prerequisites.
[3] Tasks will often target only one aspect of rigor.

For a direct link, go to: http://www.achievethecore.org/page/847/quadratic-equations-1

INSTRUCTIONAL ROUTINE

Engage students in the Contemplate then Calculate Instructional Routine. This routine focuses students' attention on structural elements of equations and expressions and shifts their attention away from immediately solving for x.

Ask students to notice what is mathematically important about the task (think, pair, share). Record the collective noticings for the full class. Have partners work together to find a shortcut to solve for x using their noticings (for example, "6x - 4 is two times 3x - 2").

Have students share out their shortcuts to the full class. While students share out, point and gesture to the equation to support classmates' comprehension. Another student should then repeat and rephrase what they heard from their classmate while the teacher annotates the equation for the full class. As students share strategies, capitalize on opportunities to highlight structural elements and/or thinking through turn and talks. Identifying 2(3x-2) = (3x-2)^2 can result in students dividing both sides by 3x-2 and allows for the discussion of potential implications of losing a zero.

Finally, facilitate a reflection process that allows students to articulate how they will look for and make use of structure. Create a public record of the reflections generated in the room for future reference.

Note that when using Contemplate then Calculate, students likely won't solve with the quadratic formula (solution #2) because they are not using pencil and paper.

LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Equation
- Evaluate
- Quadratic
- Exponent
- Parentheses
- Equivalent
- Coefficient
- Squared
- Terms
- Variable

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work

COMMENTARY AND SOLUTION

The practice of looking for and making use of structure (MP7) often amounts to deferring evaluation. In this case, instead of immediately expanding $(3x-2)^2$, the student might pause, hold both hands in his or her lap, and examine the structure of the equation. Does any idea come to mind when we see $3x-2$ and $6x-4$ next to one another? Solution 1 below develops this line of thought.

Alternatively, Solution 2 shows the route a student might take if he or she is procedurally fluent and has a habit of perseverance (MP1). There is nothing wrong with this "straight ahead" approach; in fact, it has the virtue of being general. One will note, however, how much longer and more technically difficult that approach is, compared to the approach that exploits the structure of the equation.

Solution 1

$6x-4$ is twice $3x-2$, so the equation is

$$(3x-2)^2 = 2(3x-2).$$

If we put $Q = 3x-2$, then this is $Q^2 = 2Q$ which can be put into standard form as $Q^2 = 2Q = 0$ and factored as $(Q-2) = 0$. This implies $Q = 0$ or $Q = 2$, i.e., $3x-2=0$ or $3x-2=2$. Hence, $x = \frac{2}{3}$ or $x = \frac{4}{3}$. Both values satisfy the equation.

Solution 2

Expanding, we obtain

$$9x^2 - 12x + 4 = 6x - 4\ 9x^2 -$$
$$18x + 8 = 0.$$

The quadratic formula now gives $x = \frac{(18 \pm \sqrt{324-288})}{18} = \frac{(18 \pm 6)}{18} = 1 \pm \frac{2}{3} = \frac{2}{3}$ or $\frac{4}{3}$.
Alternatively, one may factor as $(3x-4)(3x-2) = 0$.

ADDITIONAL THOUGHTS

Quadratic equations come in a variety of forms, such as

$$t^2 = 49$$

$$3a^2 = 4$$

$$7 = x^2$$

$$r^2 = 0$$

$$\frac{1}{2}y^2 = \frac{1}{5}$$

$$y^2 - 8y + 15 = 0$$

$$2x^2 - 16x + 30 = 0$$

$$2p = p^2 + 1$$

$$t^2 = 4t$$

$$7x^2 + 5x - 3 = 0$$

$$\frac{3}{4}c\,(c - 1) = c$$

$$(3x - 2)^2 = 6x - 4$$

If a textbook fails to cover a variety of forms – presenting instead problem after problem in standard forms such as $ax^2 + bx + c = 0$ – then students will not see the true variety of quadratic equations. They may learn stock routines instead of general methods that work for all problems. They may develop "buggy" techniques. And they may not develop the habit of looking for and making use of structure.

For more about the ways in which students should be reasoning with equations, read pages 13 and 14 of the progressions document, *High School, Algebra*, available at www.achievethecore.org/progressions.

Solve the equation

$$(3x-2)^2 = 6x-4$$