Multiplication and the Meaning of the Factors

Lesson by Great Minds, as featured on EngageNY,
annotation by Student Achievement Partners

GRADE LEVEL Third

IN THE STANDARDS 3.OA.A.1 (3.OA.A.3)

WHAT WE LIKE ABOUT THIS SET OF LESSONS

Mathematically:

- Uses repeated addition as an *entrance point* (not an ending point) for students learning the meaning of multiplication as equal groups (3.OA.A.1)
- Devotes significant time to understanding the concept of multiplication as opposed to the operation of multiplication
- Starts the year right away with the major work of the grade, and begins the journey towards an important fluency expectation (3.OA.A.2)
- Connecting repeated addition number sentences to multiplication number sentences gives students opportunities to look for and make use of structure (MP.7)

In the classroom:

- Uses multiple concrete representations and visual models to make the mathematics explicit
- Prompts students to share their developing thinking and understanding (Student Debrief and throughout lessons)
- Provides opportunities and suggestions for differentiation
- Gives formal and informal opportunities for teachers to check for understanding

MAKING THE SHIFTS[1]

Focus	Belongs to the major work[2] of third grade
Coherence	Builds upon second grade work with equal groups of objects (2.OA.C.4) and lays the foundation for much of the work in third grade
Rigor[3]	Conceptual Understanding: primary in these lessons (3.OA.A.1)
	Procedural Skill and Fluency: secondary in these lessons
	Application: secondary in these lessons (3.OA.A.3)

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Three.
[3]Lessons may target one or more aspect(s) of rigor.

ADDITIONAL THOUGHTS

It's important to note that these sample lessons are the first in a 21-lesson unit called Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10. It is not intended for students to meet the full expectations of the grade-level standards addressed in these lessons through only these two selected lessons. These sample lessons lay a strong foundation for the work that is to come in third grade.

The structure of these lessons and the unit/curriculum overall have some interesting aspects to highlight. The units make explicit the coherence within the fully developed curriculum. Each topic (a set of lessons) is connected to prior learning and also points to the next lesson that follows in the learning progression. Within individual lessons, there are a number of components that add to their strength including daily fluency practice, variety in questioning techniques, and daily opportunities for students to debrief about their learning.

Table of Contents

GRADE 3 • MODULE 1

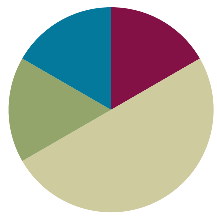
Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

Lesson 1

Objective: Understand *equal groups* of as multiplication.

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Application Problem	(10 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (10 minutes)

- Group Counting **3.OA.1** (10 minutes)

Group Counting (10 minutes)

Note: Basic skip-counting skills from Grade 2 shift focus in this Grade 3 activity. Group-counting lays a foundation for interpreting multiplication as repeated addition. When students count groups in this activity, they add and subtract groups of two when counting up and down.

T: Let's count to 20 forward and backward. Watch my fingers to know whether to count up or down. A closed hand means stop. (Show signals as you explain.)

T: (Rhythmically point up until a change is desired. Show a closed hand then point down.)

S: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0.

T: Let's count to 20 forward and backward again. This time whisper every other number. Say the other numbers in a regular voice.

S: (Students whisper then speak every other number to 20 forward and backward.)

T: Let's count to 20 forward and backward again. This time, hum every other number instead of whispering. As you hum, think of the number.

S: (Hum), 2, (hum), 4, (hum), 6, etc.

T: Let's count to 20 forward and backward again. This time, think every other number instead of humming.

S: (Think), 2, (think), 4, (think), 6, etc.

T: What did we just count by? Turn and talk to your partner.

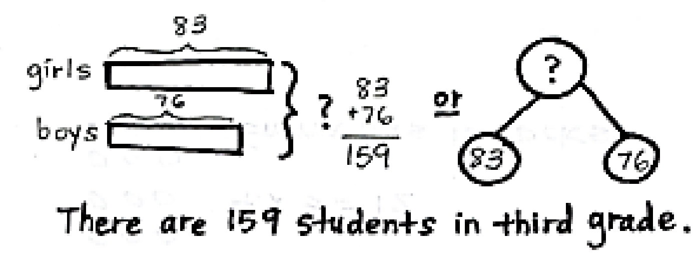
S: Twos.

T: Let's count by twos. (Direct students to count forward to and backward from 20, changing directions at times.)

Application Problem (10 minutes)

There are 83 girls and 76 boys in the third grade. How many total students are in the third grade?

Note: Students may choose to use a tape diagram or a number bond to model the problem. They are also likely to solve today's application problem in less than 10 minutes. Ten minutes have been allotted in order for you to review the RDW (Read, Draw, Write) procedure for problem-solving.



Directions on the Read, Draw, Write (RDW) Process: Read the problem, draw and label, write a number sentence, and write a word sentence. The more students participate in reasoning through problems with a systematic approach, the more they internalize those behaviors and thought processes.

(Excerpted from "How to Implement *A Story of Units*.")

Concept Development (30 minutes)

Materials: (S) 12 counters per student, personal white boards.

Problem 1: Skip-count to find the total number of objects.

T: (Select 10 students to come to the front.) At the signal, say how many arms you have. (Signal.)

S: 2 arms!

T: Since we each represent a group of 2 arms, let's skip-count our volunteers by twos to find how many arms they have altogether. To keep track of our count, the students will raise up their arms when we count them.

S: (Count 2, 4, 6…20.)

T: How many raised arms do we have in all?

S: 20.

T: Arms down. How many twos did we count to find the total? Turn and whisper to your partner.

S: 10 twos.

T: What did you count to find the number of twos?

S: I counted the number of students in the front because each person represents a group of two.

T: Skip-count to find the total number of arms.

S: (Students say 2, 4, 6…. As they count, write 2 + 2 + 2….)

T: Look at our addition sentence. Show thumbs up if you see the correct number of twos.

S: (Show thumbs up.)

T: (Under the addition sentence write 10 twos.) Clap your hands if you agree that 10 groups of two is 20.

S: (Clap hands.)

T: (Write '10 groups of two is 20' under the other expressions.)

Sample teacher board:

2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 20
10 twos
10 groups of two is 20.

Problem 2: Understand the relationship between repeated addition, counting groups in unit form, and multiplication equations.

Seat students at tables with personal white boards and 12 counters each.

T: You have 12 counters. Use your counters to make **equal groups** of two. How many counters will you put in each group? Show with your fingers.

S: (Students hold up 2 fingers and begin to make groups of two.)

T: How many equal groups of two did you make? Tell at the signal. (Signal.)

S: 6 groups.

T: 6 equal groups of how many counters?

S: 6 equal groups of 2 counters.

T: 6 equal groups of 2 counters equal how many counters altogether?

S: 12 counters.

T: Write an addition sentence to show your groups on your personal white board.

S: (Write 2 + 2 + 2 + 2 + 2 + 2 = 12.)

T: (Record the addition sentence on the board.) How many twos did we add to make 12?

S: 6 twos.

T: (Record 6 twos = 12 under the addition sentence.) 6 **×** 2 is another way to write 2 + 2 + 2 + 2 + 2 + 2 or 6 twos. (Record 6 × 2 = 12 under 6 twos = 12 on the board.) These number sentences are all saying the same thing. Another name for number sentence is **equation**.

T: Turn and talk to your partner. How do you think 6 × 2 = 12 relates to the other equations?

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

For some classes it may be necessary to clearly connect the word *times* and the symbol *x*. Have students analyze the model. "How many times do you see a group of three?" Have them count the groups, write the equation, and say the words together.

- "4 groups of three equals 12."
- "4 times three equals 12."

Sample teacher board:

2 + 2 + 2 + 2 + 2 + 2 = 12
6 twos = 12
6 x 2 = 12

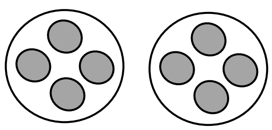
S: They all have twos in them and the answer is 12. → I think the 6 shows how many twos there are. → You have to count two 6 times because there are 6 groups of them. That's how you get 6 times 2. → 6 × 2 might be an easier way to write a long addition sentence.

T: Ways that are easier and faster are efficient. When we have equal groups, **multiplication** is a more efficient way of showing the total than repeated addition.

Repeat the process with 4 threes, 3 fours and 2 sixes with the objective of getting students comfortable with the relationship between repeated addition, counting groups in unit form, and multiplication equations. In this lesson avoid emphasis on finding solutions.

Problem 3: Write multiplication sentences from equal groups.

(Draw or project the following picture.)



T: These are equal groups. Turn and tell your partner why they are equal.

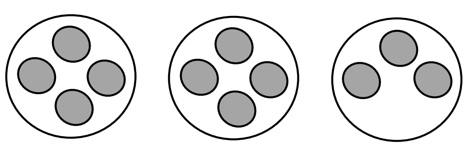
S: There is the same number of grey circles in each group. → All of the grey circles are the same size and shape, and there are 4 in each group.

T: Work with your partner to write a repeated addition and a multiplication sentence for this picture.

S: (Write 4 + 4 = 8, and either 2 × 4 = 8 or 4 × 2 = 8.)

MP.3

T: (Project or draw the following image.) Look at my new drawing and the multiplication sentence I wrote to represent it. Check my work by writing an addition sentence and counting to find the total number of objects.



3 x 4 = 12

S: (Write 4 + 4 + 3 = 11.)

T: Use your addition sentence as you talk in partners about why you agree or disagree with my work.

S: I disagree because my addition sentence equals 11, not 12. → It's because that last group doesn't have 4 circles. → You can do multiplication when the groups are equal. → Here the groups aren't equal, so the drawing doesn't show 4 × 3.

T: I hear most students disagreeing because my groups are not equal. True, to **multiply** you must have equal groups.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

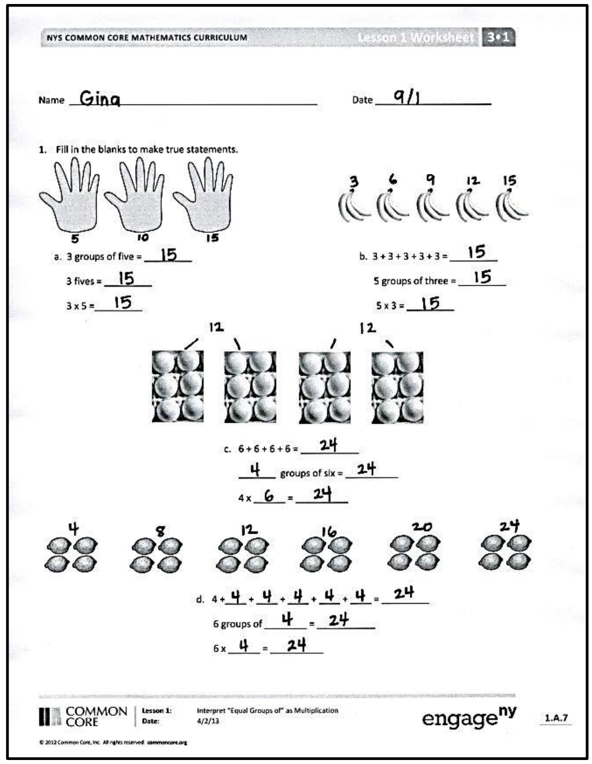
Some students may need more scaffolding to realize that you can't use multiplication to find the total amount of items in groups that are not equal. You might use the following questions to scaffold.

- "Does the drawing show groups of 4 modeled 3 times?"
- "Does 4 *times* 3 represent this drawing?"
- "How might we redraw the picture to make it show 4 x 3?"

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the careful sequencing of the Problem Set guide your selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Assign incomplete problems for homework or at another time during the day.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 1 Worksheet 3•1

Name Gina Date 9/1

1. Fill in the blanks to make true statements.

a. 3 groups of five = 15
3 fives = 15
3 x 5 = 15

b. 3 + 3 + 3 + 3 + 3 = 15
5 groups of three = 15
5 x 3 = 15

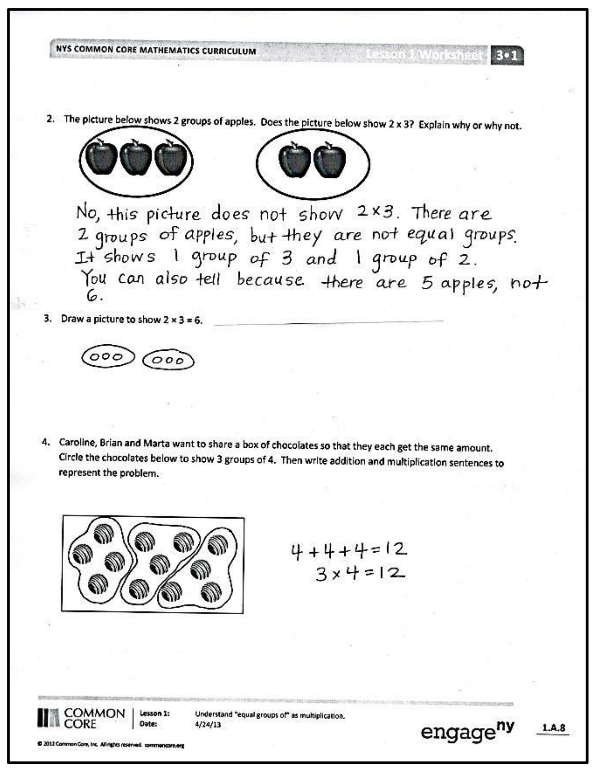
c. 6 + 6 + 6 + 6 = 24
4 groups of six = 24
4 x 6 = 24

d. 4 + 4 + 4 + 4 + 4 + 4 = 24
6 groups of 4 = 24
6 x 4 = 24

COMMON CORE Lesson 1: Interpret "Equal Groups of" as Multiplication Date: 4/2/13 engageny 1.A.7

Student Debrief (10 minutes)

Lesson Objective: Understand *equal groups of* as multiplication.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 1 Worksheet 3•1

2. The picture below shows 2 groups of apples. Does the picture below show 2 x 3? Explain why or why not.

No, this picture does not show 2x3. There are 2 groups of apples, but they are not equal groups. It shows 1 group of 3 and 1 group of 2. You can also tell because there are 5 apples, not 6.

3. Draw a picture to show 2 x 3 = 6.

4. Caroline, Brian and Marta want to share a box of chocolates so that they each get the same amount. Circle the chocolates below to show 3 groups of 4. Then write addition and multiplication sentences to represent the problem.

4+4+4=12
3x4=12

COMMON CORE Lesson 1: Understand "equal groups of" as multiplication. Date: 4/24/13 engageny 1.A.8

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- On the first page, what did you notice about the answers to your problems?
- Discuss the relationship between repeated addition and the unit form *2 groups of three* or *3 groups of two*, depending on the drawing.
-

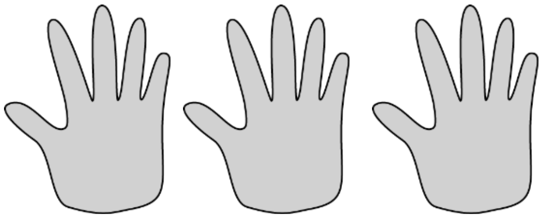
- Discuss the relationship between repeated addition, unit form, and the **multiplication equation** 3 × 2 = 6.
- Review the new vocabulary presented in the lesson, including **equal groups**, **multiply**.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name ____ Date ____________________

1. Fill in the blanks to make true statements.



a. 3 groups of five = _________

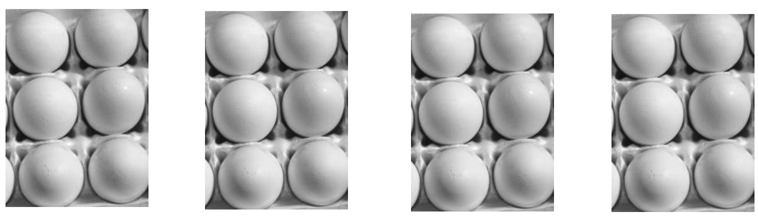
3 fives = _________

3 × 5 =_________

b. 3 + 3 + 3 + 3 + 3 = _________

5 groups of three = _________

5 × 3 = _________



c. 6 + 6 + 6 + 6 = ___________

_______ groups of six = __________

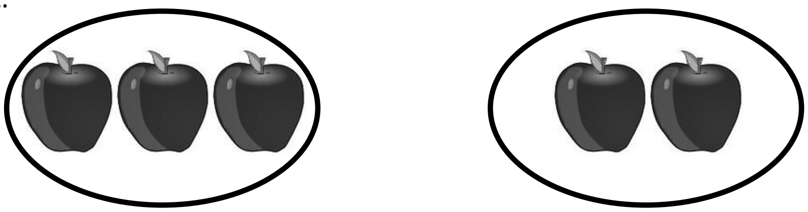
4 × ______ = __________

d. 4 +____ + ____ + ____ + ____ + ____ = _________

6 groups of ________ = ___________

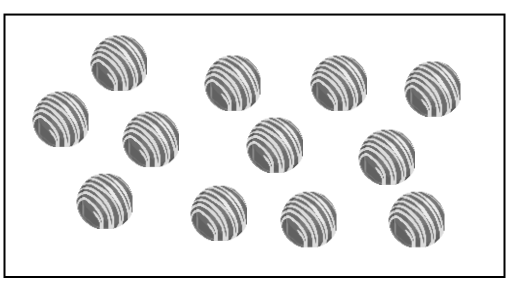
6 × ______ = __________

2. The picture below shows 2 groups of apples. Does the picture below show 2×3? Explain why or why not.



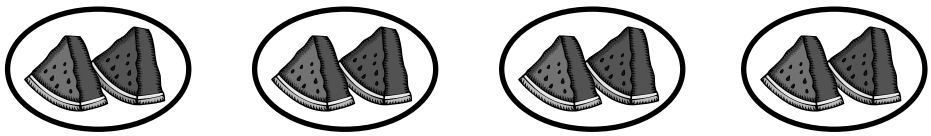
3. Draw a picture to show $2 \times 3 = 6$.

4. Caroline, Brian and Marta want to share a box of chocolates so that they each get the same amount. Circle the chocolates below to show 3 groups of 4. Then write addition and multiplication sentences to represent the problem.



Name ____ Date ________________________

1. The picture below shows 4 groups of 2 slices of watermelon. Write repeated addition and multiplication sentences to represent the picture.



2 + ____ + ____ + ____ = ___________

4 × ______ = __________

2. Draw a picture to show 3 + 3 + 3 = 9. Then write a multiplication sentence to represent the picture.

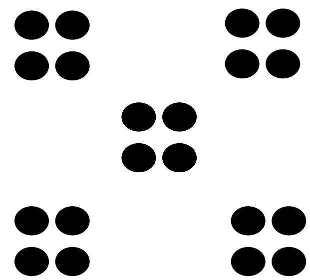
Name ____ Date ______________________

1. Fill in the blanks to make true statements.

a. 4 groups of five = _________

4 fives = _________

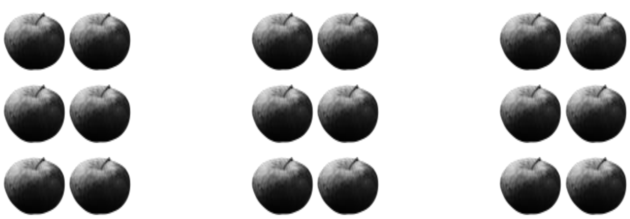
4 × 5 = _________



b. 5 groups of four = _________

5 fours = _________

5 × 4 = _________



c. 6 + 6 + 6 = ___________

_______ groups of six = __________

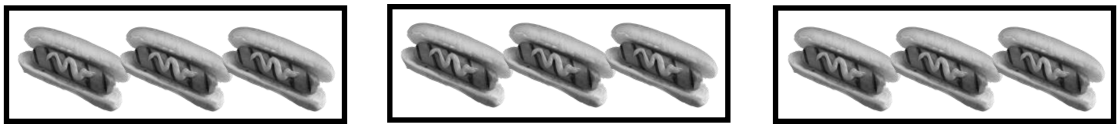
3 × ______ = __________

d. 3 + ____ + ____ + ____ + ____ + ____ = _______

6 groups of ________ = ___________

6 × ______ = __________

2. The picture below shows 3 groups of hot dogs. Does the picture below show 3 × 3? Explain why or why not.



3. Draw a picture to show 4 × 2 = 8.

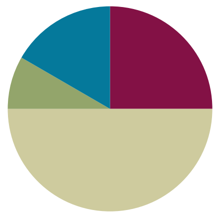
4. Circle the pencils below to show 3 groups of 6. Write addition and multiplication sentences to represent the problem.

Lesson 2

Objective: Relate multiplication to the array model.

Suggested Lesson Structure

Fluency Practice	(15 minutes)
Application Problem	(5 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (15 minutes)

- Add and Subtract by 2 **3.OA.1** (8 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Add Equal Groups **3.OA.1** (3 minutes)

Sprint: Add and Subtract by 2 (8 minutes)

Materials: (S) Add and Subtract by 2 Sprint

Note: This sprint supports group counting skills that are foundational to interpreting multiplication as repeated addition.

Directions for Administration of Sprints

One sprint has two parts with closely related problems on each. Each part is organized into 4 quadrants that move from simple to complex. This builds a challenge into each sprint for every learner. Before the lesson, cut the sprint sheet in half to create *Sprint A* and *Sprint B.* Students complete the two parts of the sprint in quick succession with the goal of improving on the second part, even if only by one more. With practice the following routine takes about 8 minutes.

Sprint A

(Put Sprint A face down on desks with instructions to not look at problems until signal is given.)

T: You will have 60 seconds to do as many problems as you can.

T: I do not expect you to finish all of them. Just do as many as you can, your personal best.

T: Take your mark! Get set! THINK! (When you say THINK, students turn papers over and work furiously to finish as many problems as they can in 60 seconds. Time precisely.)

(After 60 seconds.)

T: Stop! Circle the last problem you did. I will read just the answers. If you got it right, call out “Yes!” and give a fist pump. If you made a mistake, circle it. Ready?

(Repeat to the end of Sprint A, or until no one has any more correct.)

T: Now write your number correct at the top of the page. This is your personal goal for Sprint B.

T: How many of you got 1 right? (All hands should go up.)

T: Keep your hand up until I say a number that is 1 more than the number you got right. So, if you got 14 right, when I say 15 your hand goes down. Ready?

T: (Quickly.) How many got 2 right? 3? 4? 5? (Continue until all hands are down.)

(Optional routine, depending on whether or not your class needs more practice with Sprint A.)

T: Take one minute to do more problems on this half of the sprint.

(As students work you might have the person who scored highest on Sprint A pass out Sprint B.)

T: Stop! I will read just answers. If you got it right, call out “Yes!” and give a fist pump. If you made a mistake, circle it. Ready?

(Read the answers to the first half again as students stand.)

Movement: To keep the energy and fun going, do a stretch or a movement game in between Sprints.

Sprint B

(Put Sprint B face down on desks with instructions to not look at the problems until the signal is given. Repeat the procedure for Sprint A up through the show of hands for how many right.)

T: Stand up if you got more correct on the second Sprint than on the first.

S: (Students stand.)

T: Keep standing until I say the number that tells how many more you got right on Sprint B. If you got 3 more right on Sprint B than on Sprint A, when I say 3 you sit down. Ready?

(Call out numbers starting with 1. Students sit as the number by which they improved is called.) Students may take sprints home.

Group Counting (4 minutes)

Note: Basic skip-counting skills from Grade 2 shift focus in this Grade 3 activity. Group-counting lays a foundation for interpreting multiplication as repeated addition. When students count groups in this activity, they add and subtract groups of three when counting up and down.

T: Let’s count to 18 forward and backward. I want you to whisper, whisper, and then speak numbers.

T: Watch my fingers to know whether to count up or down. A closed hand means stop. (Show signals as you explain.)

T: (Rhythmically point up until a change is desired. Show a closed hand then point down.)

S: (Whisper 1, whisper 2, speak 3, etc.)

T: Let’s count to 18 forward and backward again. This time, think every number instead of whispering.

S: (Think), (think), 3, (think), (think), 6, (think), (think), 9, etc.

T: What did we just count by? Turn and talk to your partner.

S: Threes.

T: Let's count by threes. (Direct students to count forward and backward to 18, periodically changing directions. Emphasize the 9 to 12 transition.)

Add Equal Groups (3 minutes)

Materials: (S) Personal white boards

Note: This activity reviews Lesson 1. Students directly relate repeated addition to multiplication. They interpret products as the number of equal groups times the number of objects in each group.

T: (Project a picture array with 3 groups of 2 circled.) How many groups are circled?

S: 3.

T: How many are in each group?

S: 2.

T: Write this as an addition sentence.

S: (Write 2 + 2 + 2 = 6.)

T: Write a multiplication sentence for 3 twos equals 6.

S: (Write 3 × 2 = 6.)

Continue with possible sequence: 3 groups of 5, 5 groups of 10, and 3 groups of 4.

Application Problem (5 minutes)

Jordan uses 3 lemons to make 1 pitcher of lemonade. He makes 4 pitchers. How many lemons does he use altogether? Use the RDW process to show your solution.

Note: This problem reviews equal groups multiplication from Lesson 1. It also leads into today's concept development of relating multiplication to the array model.

4 x 3 = 12

Jordan uses 12 lemons altogether.

Concept Development (30 minutes)

Materials: (S) Personal white boards, Threes Array Template inside personal boards (pictured below), lemons image from application problem, 1 sheet of blank paper per student

Problem 1: Relate equal groups to arrays.

T: Look back at Jordan's lemons. Compare the way his lemons are organized with the groups of 3 circles on your template.

S: The lemons in one group are touching each other, but the circles have space between them. → Each line on the template shows three, like each group of lemons. → The template is organized with everything in straight lines.

T: Many students are noticing straight lines on the template. Let's call a straight line going across a **row**. Use your blank paper to cover all but the top row.

S: (Cover all but the top row.)

T: Uncover 1 row at a time in the picture. As you uncover each row, write the new total number of circles to the right of it.

S: (Skip-count by three using the Threes Array Template.)

T: At the signal say the total number of circles you counted. (Signal.)

S: 30 circles!

T: Take 10 seconds to find how many rows of 3 you counted. At the signal say how many. (Signal.)

S: 10 rows!

T: True or false: 10 rows of 3 circles equals 30 circles?

S: True!

T: (Write $10 \times 3 = 30$ on the board.) Use the picture on your template to talk with your partner about why this equation is true.

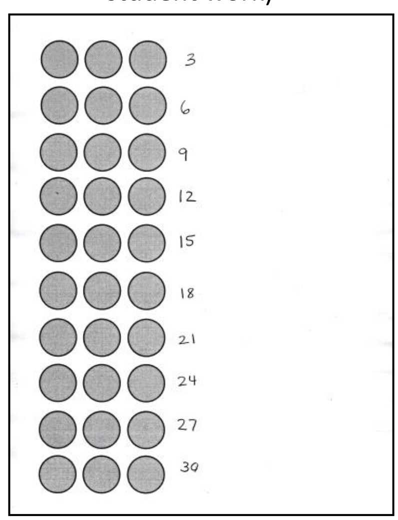
S: Yesterday we learned that we can multiply equal groups. → We skip-counted 10 rows of 3 circles each and the total is 30. → It just means 10 groups of 3 and when you add 10 threes, you get 30! → Yeah, but writing 10×3 is a lot easier than writing out $3 + 3 + 3 + 3$ …

T: We call this type of organized picture an **array**.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

As you teach the vocabulary rows, have students trace a row on the array with a finger while saying the word. You may also have them identify other rows around the room. For example, students may be seated in rows or you may have books arranged in rows.

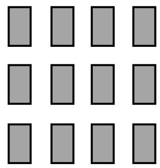
Threes Array Template (with student work)



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

As you teach the vocabulary *array*, you may want to ask students to turn and talk, describing or defining an array for their partner.

T: (Project or draw the following image.) Take a look at this array.

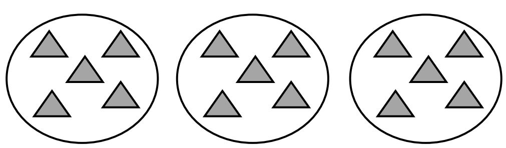


T: At the signal tell how many rectangles are in the top row. (Signal.)
S: 4 rectangles.
T: The size of 1 row is 4 rectangles.
T: At the signal tell how many groups of four are in the array. (Signal.)
S: 3 groups of four.
T: To write the multiplication **fact**, we first write the **number of groups**. How many groups?
S: 3 groups!
T: (Write 3 × _____.) Next we write the **size of the group**. How many rectangles are in each group?
S: 4 rectangles!
T: (Fill in the fact to read 3 × 4.) Skip-count to find the total number of rectangles in the array.
S: 4, 8, 12!
T: (Complete the equation to read 3 × 4 = 12.) We just found the answer to the multiplication fact that represents the array.

Show an array of 2 rows of 6 and repeat the process.

Problem 2: Redraw equal groups as arrays.

T: (Project or draw the following image.) The drawing shows 3 equal groups of 5.

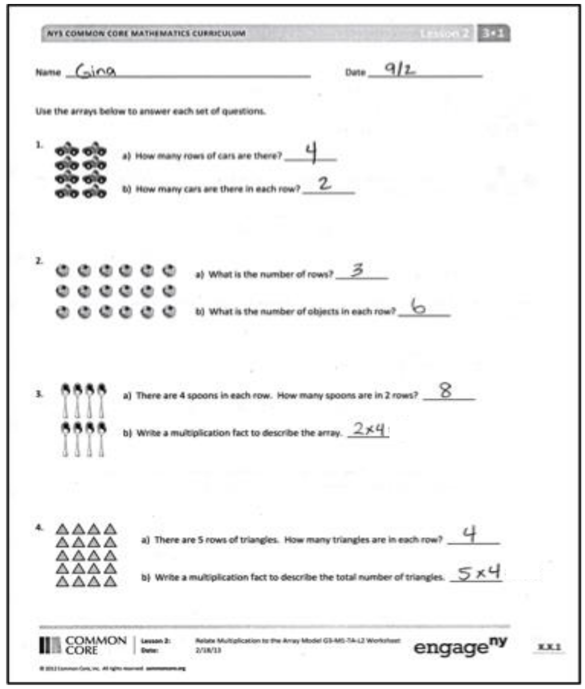


T: Re-draw the picture as an array with 3 rows of 5 on your personal board.
S: (Draw a 3 by 5 array.)
T: Write a multiplication fact to describe your array.
S: (Write 3 × 5.)
T: Skip-count to find the answer to the multiplication fact.
S: 5, 10, 15. (Write 3 × 5 = 15.)

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Provide a challenge in this part of the lesson by giving a multiplication sentence (e.g., 5 x 4 = _____) and no picture. Have students draw both the equal groups and array to represent the sentence. Then they skip-count to find the total.

Show 6 groups of 2 and repeat the process.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 2 3•1

Name Gina Date 9/2

Use the arrays below to answer each set of questions.

1. a) How many rows of cars are there? 4
   b) How many cars are there in each row? 2

2. a) What is the number of rows? 3
   b) What is the number of objects in each row? 6

3. a) There are 4 spoons in each row. How many spoons are in 2 rows? 8
   b) Write a multiplication fact to describe the array. 2×4

4. a) There are 5 rows of triangles. How many triangles are in each row? 4
   b) Write a multiplication fact to describe the total number of triangles. 5×4

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

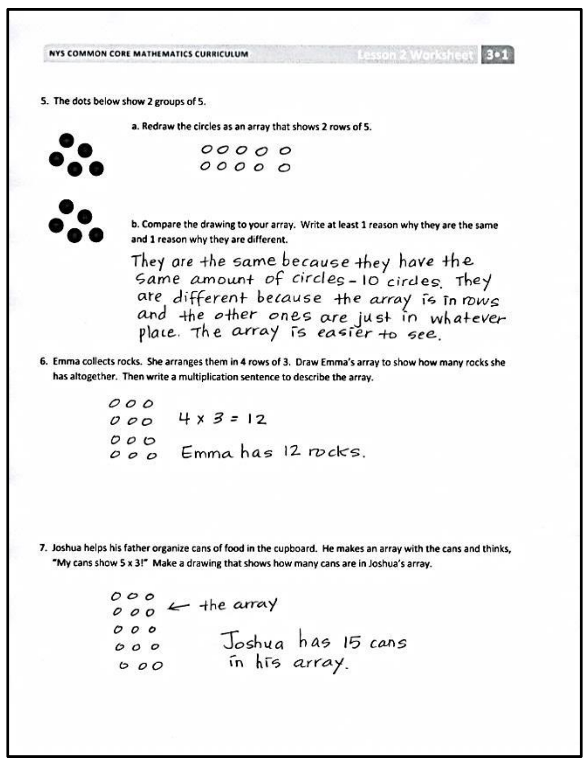
Student Debrief (10 minutes)

Lesson Objective: Relate multiplication to the array model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- In Problems 5 and 6 how do the arrays represent equal groups?
- Compare equal groups in scattered configurations and arrays.
- Review the vocabulary: **row**, **array**, **fact**, **number of groups**, **size of groups**.
- Prompt students to notice arrays around the room and possibly think of arrays in real world situations.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 2 Worksheet 3•1

5. The dots below show 2 groups of 5.

a. Redraw the circles as an array that shows 2 rows of 5.

b. Compare the drawing to your array. Write at least 1 reason why they are the same and 1 reason why they are different.

They are the same because they have the same amount of circles - 10 circles. They are different because the array is in rows and the other ones are just in whatever place. The array is easier to see.

6. Emma collects rocks. She arranges them in 4 rows of 3. Draw Emma's array to show how many rocks she has altogether. Then write a multiplication sentence to describe the array.

4 x 3 = 12

Emma has 12 rocks.

7. Joshua helps his father organize cans of food in the cupboard. He makes an array with the cans and thinks, "My cans show 5 x 3!" Make a drawing that shows how many cans are in Joshua's array.

← the array

Joshua has 15 cans in his array.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

A # Correct ______

Add or subtract.

1	0 + 2 =		23	2 + 4 =	
2	2 + 2 =		24	2 + 6 =	
3	4 + 2 =		25	2 + 8 =	
4	6 + 2 =		26	2 + 10 =	
5	8 + 2 =		27	2 + 12 =	
6	10 + 2 =		28	2 + 14 =	
7	12 + 2 =		29	2 + 16 =	
8	14 + 2 =		30	2 + 18 =	
9	16 + 2 =		31	0 + 22 =	
10	18 + 2 =		32	22 + 22 =	
11	20 - 2 =		33	44 + 22 =	
12	18 - 2 =		34	66 + 22 =	
13	16 - 2 =		35	88 - 22 =	
14	14 - 2 =		36	66 - 22 =	
15	12 - 2 =		37	44 - 22 =	
16	10 - 2 =		38	22 - 22 =	
17	8 - 2 =		39	22 + 0 =	
18	6 - 2 =		40	22 + 22 =	
19	4 - 2 =		41	22 + 44 =	
20	2 - 2 =		42	66 + 22 =	
21	2 + 0 =		43	888 - 222 =	
22	2 + 2 =		44	666 - 222 =	

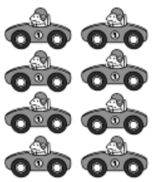
B Improvement _____ # Correct _____

Add or subtract.

1	2 + 0 =		23	4 + 2 =	
2	2 + 2 =		24	6 + 2 =	
3	2 + 4 =		25	8 + 2 =	
4	2 + 6 =		26	10 + 2 =	
5	2 + 8 =		27	12 + 2 =	
6	2 + 10 =		28	14 + 2 =	
7	2 + 12 =		29	16 + 2 =	
8	2 + 14 =		30	18 + 2 =	
9	2 + 16 =		31	0 + 22 =	
10	2 + 18 =		32	22 + 22 =	
11	20 - 2 =		33	22 + 44 =	
12	18 - 2 =		34	66 + 22 =	
13	16 - 2 =		35	88 - 22 =	
14	14 - 2 =		36	66 - 22 =	
15	12 - 2 =		37	44 - 22 =	
16	10 - 2 =		38	22 - 22 =	
17	8 - 2 =		39	22 + 0 =	
18	6 - 2 =		40	22 + 22 =	
19	4 - 2 =		41	22 + 44 =	
20	2 - 2 =		42	66 + 22 =	
21	0 + 2 =		43	666 - 222 =	
22	2 + 2 =		44	888 - 222 =	

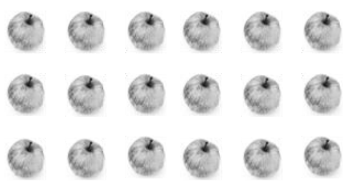
Name ____ Date ____________________

Use the arrays below to answer each set of questions.

1. 

a. How many rows of cars are there? __________

b. How many cars are there in each row? __________

2. 

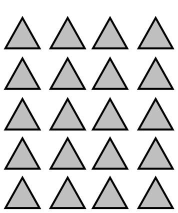
a. What is the number of rows? __________

b. What is the number of objects in each row? __________

3. 

a. There are 4 spoons in each row. How many spoons are in 2 rows? __________

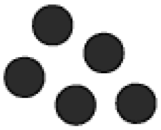
b. Write a multiplication fact to describe the array. __________

4. 

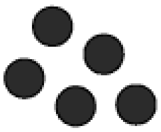
a. There are 5 rows of triangles. How many triangles are in each row? __________

b. Write a multiplication fact to describe the total number of triangles. __________

5. The dots below show 2 groups of 5.



a. Redraw the circles as an array that shows 2 rows of 5.



b. Compare the drawing to your array. Write at least 1 reason why they are the same and 1 reason why they are different.

6. Emma collects rocks. She arranges them in 4 rows of 3. Draw Emma's array to show how many rocks she has altogether. Then write a multiplication sentence to describe the array.

7. Joshua helps his father organize cans of food in the cupboard. He makes an array with the cans and thinks, "My cans show 5 × 3!" Make a drawing that shows how many cans are in Joshua's array.

Name ____ Date ________________

1. 

a. There are 4 rows of stars. How many stars are in each row? __________

b. Write a multiplication fact to describe the total number of stars. _______________

2. Judy collects seashells. She arranges them in 3 rows of 6. Draw Judy's array to show how many seashells she has all together. Then write a multiplication sentence to describe the array.

Name ____ Date ______________________

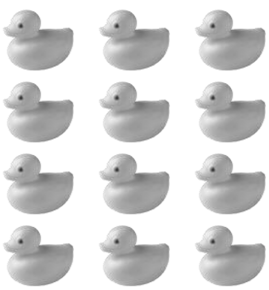
Use the arrays below to answer each set of questions.

1.

a. How many rows of erasers are there? __________

b. How many erasers are there in each row? __________

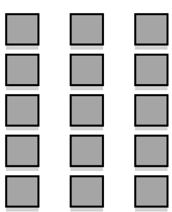
2.



a. What is the number of rows? __________

b. What is the number of objects in each row? __________

3.



a. There are 3 squares in each row. How many squares are in 5 rows? __________

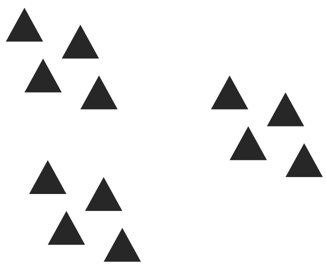
b. Write a multiplication fact to describe the array. __________

4.

a. There are 6 rows of stars. How many stars are in each row? __________

b. Write a multiplication fact to describe the array. __________

5. The triangles below show 3 groups of 4.

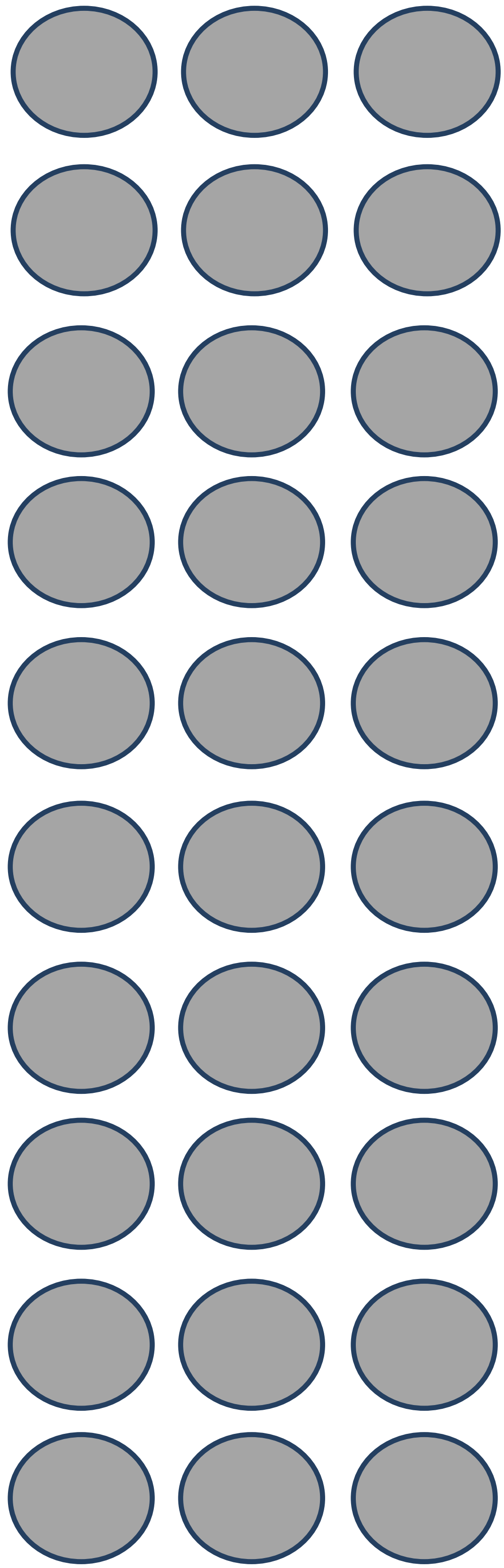


a. Redraw the triangles as an array that shows 3 rows of 4.

b. Compare the drawing to your array. How are they the same? How are they different?

6. Roger has a collection of stamps. He arranges the stamps into 5 rows of 4. Draw an array to represent Roger's stamps. Then write a multiplication sentence to describe the array.

7. Kimberly arranges her 18 markers in an array. Draw an array that Kimberly might make. Then write a multiplication sentence to match your array.

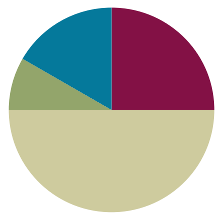


Lesson 3

Objective: Interpret the meaning of factors – the size of the group or the number of groups.

Suggested Lesson Structure

Fluency Practice	(15 minutes)
Application Problem	(5 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (15 minutes)

- Add Equal Groups **3.OA.1** (9 minutes)
- Group Counting **3.OA.1** (3 minutes)
- Add to Multiply **3.OA.1** (3 minutes)

Sprint: Add Equal Groups (9 minutes)

Materials: (S) Add Equal Groups Sprint

Note: This sprint reviews the Lesson 1 Objective. See *Directions for Administration of Sprints* in Lesson 2.

Group Counting (3 minutes)

Note: Basic skip-counting skills from Grade 2 shift focus in this Grade 3 activity. Group-counting reviews interpreting multiplication as repeated addition. Counting by twos and threes in this activity anticipates work with those factors in Topic B.

T: Let's count by twos. (Direct students to count forward and backward to 20, periodically changing directions.)

T: Let's count by threes. (Direct students to count forward and backward to 21, periodically changing directions. Emphasize the 9 to 12 and 18 to 21 transitions.)

Add to Multiply (3 minutes)

Materials: (S) Personal white boards

Note: This activity reviews Lesson 2. Students directly relate repeated addition to multiplication. They interpret products using the array.

T: (Project a picture array with 3 groups of 5 circled.) How many groups are circled?

S: 3.

T: How many are in each group?

S: 5.

T: Write it as an addition sentence.

S: (Write 5 + 5 + 5 = 15.)

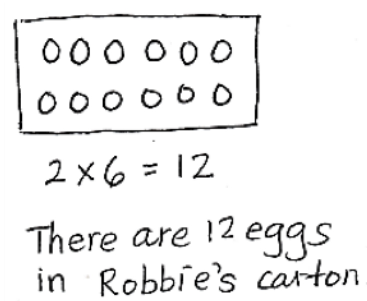
T: Write a multiplication sentence representing 3 fives equals 15.

S: 3 × 5 = 15.

Continue with possible sequence: 3 groups of 10, 3 groups of 4, and 7 groups of 2.

Application Problem (5 minutes)

Robbie sees that a carton of eggs shows an array with 2 rows of 6 eggs. What is the total number of eggs in the carton? Use the RDW process to show your solution.



Note: This problem reviews writing multiplication sentences from arrays learned in Lesson 2. The egg carton provides a natural array for students to see 2 rows of 6.

Concept Development (30 minutes)

Materials: (S) Personal white boards

Problem 1: Equal groups activity.

The following activity should take about 5 minutes.

T: Here are the rules for our opening activity.

1. **Divide** yourselves into 4 equal groups.
2. Each group will stand in a corner of the room.
3. Divide silently. You can use body movements to gesture, but no words.

T: Show thumbs up when your group is ready. Be sure to look around the room to double check that all 4 groups are equal before showing you're ready.

NOTES ON EQUAL GROUPS ACTIVITY:

Adjust the directions for the equal groups activity depending on the total number of students in your class.

- If the total number is not a multiple of four, that adds complexity. Either some students will not be in a group or you can use an adult, teddy bear, etc. to stand in.
- If there are 16–19 students, you will have 4 groups of four. This not advisable. Use stand-ins to compensate.

S: (Students move around the room silently until there are 4 equal groups, 1 in each corner.)

T: At the signal tell how many equal groups we've made. (Signal.)

S: 4 equal groups.

T: (Write 4 × ___ = total number of students in the class.) At the signal tell the size of each group. (Signal.)

S: (Students respond depending on class numbers.)

T: (Complete the equation on the board.) These numbers—the number of groups and the number in each group—are called **factors**.

Problem 2: Interpret the meaning of factors in number bonds.

Students transition back to their seats with personal boards.

T: Use the multiplication sentence on the board to draw an array. Make sure that your board is vertical.

S: (Draw a 4 × ____ array.)

T: Let's draw a number bond for our multiplication sentence. Draw a circle with our class total.

S: (Students draw.)

T: Draw parts coming from the total. Make 1 part to represent each row in our array.

S: (Draw 4 circles coming from the total.)

T: Show the size of 1 row with your fingers.

S: (Show fingers.)

T: Write the factor representing the size of the group inside the circle that represents each group.

T: Look back at the multiplication sentence. How is the factor 4 represented in the number bond?

S: It's in the number of parts. → Groups are like parts. → In the number bond the 'part' circles actually represent equal groups, so there are 4. The number inside is the size of the group.

T: Here is an analysis of our number sentence.

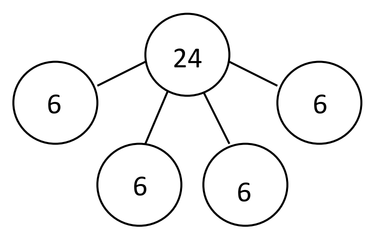
4	×	6	=	24
↓		↓		↓
number of groups		size of groups		total

NOTES ON NUMBER BONDS:

The number bond is a pictorial representation of part–part–whole relationships and shows that within a part–whole relationship, smaller numbers (the parts) make up larger numbers (the whole). The number bond may be presented as shown below.

(Excerpted from "How to Implement *A Story of Units*.")

Sample number bond for a class of 24:



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

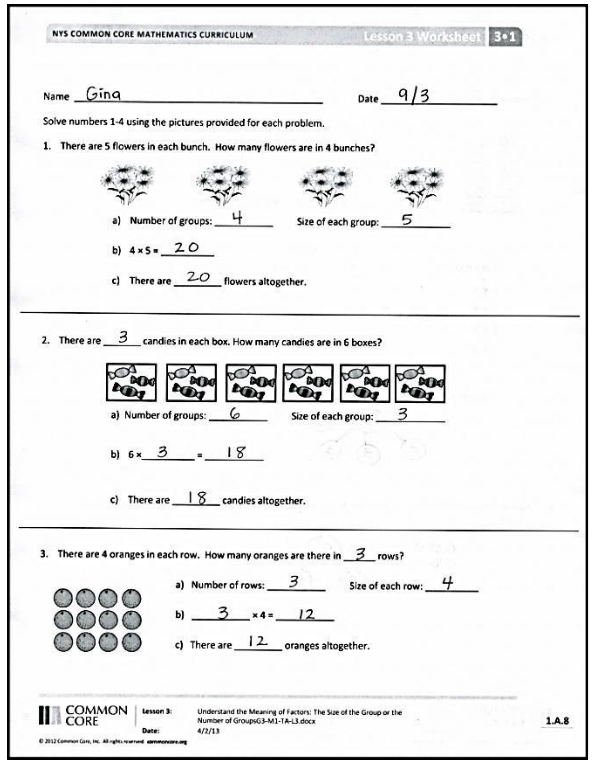
The number bond is another way for students to explore the relationship between factors in multiplication. Have them compare it with its uses in addition to distinguish the importance of equal groups in multiplication.

Continue with the following possible suggestions:

- 2 groups of 8
- 3 rows of 5
- A number bond showing 6 groups of 3
- The equation 5 × 4 = 20

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 3 Worksheet 3•1

Name Gina Date 9/3

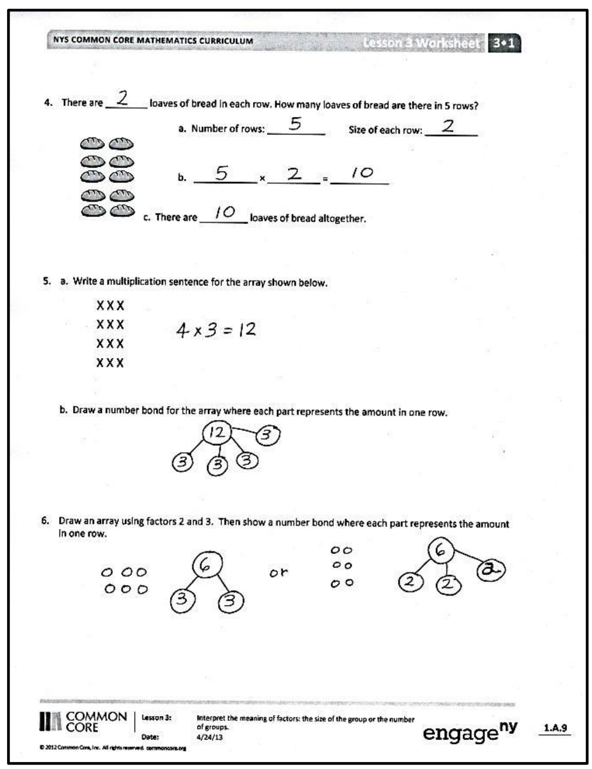
Solve numbers 1-4 using the pictures provided for each problem.

1. There are 5 flowers in each bunch. How many flowers are in 4 bunches?
   a) Number of groups: 4 Size of each group: 5
   b) 4 × 5 = 20
   c) There are 20 flowers altogether.

2. There are 3 candies in each box. How many candies are in 6 boxes?
   a) Number of groups: 6 Size of each group: 3
   b) 6 × 3 = 18
   c) There are 18 candies altogether.

3. There are 4 oranges in each row. How many oranges are there in 3 rows?
   a) Number of rows: 3 Size of each row: 4
   b) 3 × 4 = 12
   c) There are 12 oranges altogether.

COMMON CORE Lesson 3: Understand the Meaning of Factors: The Size of the Group or the Number of Groups Date: 4/2/13 1.A.8



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 3 Worksheet 3•1

4. There are 2 loaves of bread in each row. How many loaves of bread are there in 5 rows?
   a. Number of rows: 5 Size of each row: 2
   b. 5 × 2 = 10
   c. There are 10 loaves of bread altogether.

5. a. Write a multiplication sentence for the array shown below.

   XXX
   XXX
   XXX
   XXX

   4 × 3 = 12

   b. Draw a number bond for the array where each part represents the amount in one row.

   12 — 3, 3, 3, 3

6. Draw an array using factors 2 and 3. Then show a number bond where each part represents the amount in one row.

   ooo ooo — 6 — 3, 3 or oo oo oo — 6 — 2, 2, 2

COMMON CORE Lesson 3: Interpret the meaning of factors: the size of the group or the number of groups. Date: 4/24/13 engage^ny 1.A.9

Student Debrief (10 minutes)

Lesson Objective: Interpret the meaning of factors—the size of the group or the number of groups.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Why do you think I started the lesson by asking you to **divide** yourselves into equal groups in the corners of the room?
- Identify the **factors** and their meanings from each image on the Problem Set from 1 to 5.
- In Problem 6, discuss the two ways to draw the array and number bond with factors 2 and 3.
- Module 1 introduces many new vocabulary words: row, array, number of groups, size of groups, and factor. You may want to have students make a vocabulary page in their math journals.

- Relate factors to their meaning: the size of the group or the number of groups. Have students share the definition in pairs. Then ask students to write the word and a definition or example next to it in their journals.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

A # Correct _____

Solve.

1	2 + 2 =		23	7 + 7 =	
2	2 twos =		24	2 sevens =	
3	5 + 5 =		25	9 + 9 =	
4	2 fives =		26	2 nines =	
5	2 + 2 + 2 =		27	8 + 8 =	
6	3 twos =		28	2 eights =	
7	2 + 2 + 2 + 2 =		29	3 + 3 + 3 =	
8	4 twos =		30	3 threes =	
9	5 + 5 + 5 =		31	4 + 4 + 4 =	
10	3 fives =		32	3 fours =	
11	5 + 5 + 5 + 5 =		33	3 + 3 + 3 + 3 =	
12	4 fives =		34	4 threes =	
13	2 fours =		35	4 fives =	
14	4 + 4 =		36	4 + 4 + 4 + 4 + 4 =	
15	2 threes =		37	3 sixes =	
16	3 + 3 =		38	6 + 6 + 6 =	
17	2 sixes =		39	3 eights =	
18	6 + 6 =		40	8 + 8 + 8 =	
19	5 twos =		41	3 sevens =	
20	2 + 2 + 2 + 2 + 2 =		42	7 + 7 + 7 =	
21	5 fives =		43	3 nines =	
22	5 + 5 + 5 + 5 + 5 =		44	9 + 9 + 9 =	

B Improvement _____ # Correct _____

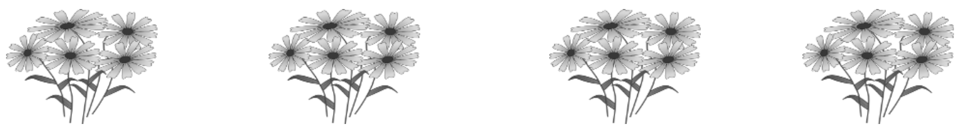
Solve.

1	5 + 5 =		23	8 + 8 =	
2	2 fives =		24	2 eights =	
3	2 + 2 =		25	7 + 7 =	
4	2 twos =		26	2 sevens =	
5	5 + 5 + 5 =		27	9 + 9 =	
6	3 fives =		28	2 nines =	
7	5 + 5 + 5 + 5 =		29	3 + 3 + 3 + 3 =	
8	4 fives =		30	4 threes =	
9	2 + 2 + 2 =		31	4 + 4 + 4 =	
10	3 twos =		32	3 fours =	
11	2 + 2 + 2 + 2 =		33	3 + 3 + 3 =	
12	4 twos =		34	3 threes =	
13	2 threes =		35	4 fives =	
14	3 + 3 =		36	4 + 4 + 4 + 4 + 4 =	
15	2 sixes =		37	3 sevens =	
16	6 + 6 =		38	7 + 7 + 7 =	
17	2 fours =		39	3 nines =	
18	4 + 4 =		40	9 + 9 + 9 =	
19	5 fives =		41	3 sixes =	
20	5 + 5 + 5 + 5 + 5 =		42	6 + 6 + 6 =	
21	5 twos =		43	3 eights =	
22	2 + 2 + 2 + 2 + 2 =		44	8 + 8 + 8 =	

Name ____ Date ____________________

Solve numbers 1–4 using the pictures provided for each problem.

1. There are 5 flowers in each bunch. How many flowers are in 4 bunches?



   a. Number of groups: ___________ Size of each group: _____________

   b. 4 × 5 = __________

   c. There are _________ flowers altogether.

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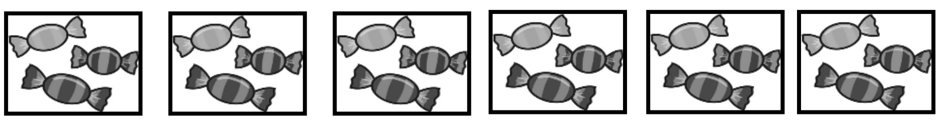
2. There are _______ candies in each box. How many candies are in 6 boxes?



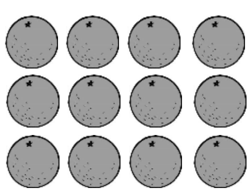
   a. Number of groups: ___________ Size of each group: _____________

   b. 6 ×__________ = ___________

   c. There are _________ candies altogether.

---

3. There are 4 oranges in each row. How many oranges are there in ______ rows?

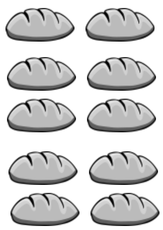


   a. Number of rows: ___________ Size of each row: __________

   b. ___________ × 4 = ___________

   c. There are _________ oranges altogether.

4. There are ________ loaves of bread in each row. How many loaves of bread are there in 5 rows?



a. Number of rows: ___________ Size of each row: __________

b. ____________ × __________ = ___________

c. There are _________ loaves of bread altogether.

5. a. Write a multiplication sentence for the array shown below.

X X X
X X X
X X X
X X X

b. Draw a number bond for the array where each part represents the amount in one row.

6. Draw an array using factors 2 and 3. Then show a number bond where each part represents the amount in one row.

Name ____ Date ______________________

Draw an array that shows 5 rows of 3 squares. Then show a number bond where each part represents the amount in one row.

Name ____ Date _____________________

Solve problems 1–4 using the pictures for each problem.

1. There are 5 pineapples in each group. How many pineapples are there in 5 groups?

   a. Number of groups: ____________ Size of each group: ___________

   b. 5 × 5 = ___________

   c. There are _________ pineapples altogether.

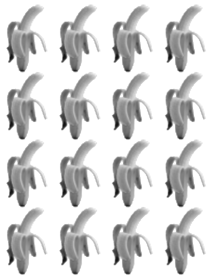
2. There are ______ oranges in each basket. How many oranges are there in 6 baskets?

   a. Number of groups: ____________ Size of each group: ___________

   b. 6 × _________ = ___________

   c. There are _________ oranges altogether.

3. There are 4 bananas in each row. How many bananas in ________ rows?



a. Number of rows: ___________ Size of each row: __________

b. _________ × 4 = __________

c. There are __________ bananas altogether.

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4. There are ___________ peppers in each row. How many peppers are there in 6 rows?

a. Number of rows: ___________ Size of each row: __________

b. _________ × __________ = __________

c. There are __________ peppers altogether.

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5. Draw an array using factors 4 and 2. Then show a number bond where each part represents the amount in one row.