# Penny Circle Task

Task by Dan Meyer and Desmos, annotation by Student Achievement Partners

GRADE LEVEL High School

IN THE STANDARDS F-LE, MP.4, F-IF.7, F-BF.3, A-CED.1, A-REI.4

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Requires students to confront the question of whether a model's predictions are reasonable (part of the modeling cycle).
- Sets up a situation with many avenues for mathematical investigation.
- Uses technology efficiently to support the mathematics.

In the classroom:

- Allows for student collaboration and discussion.
- Provides a platform for the teacher to view students' work online.
- Allows teacher to filter results, e.g., to see which students revised their models.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.
- Provides students with support in negotiating written word problems through multiple reads and/or multi-modal interactions with the problem.

MAKING THE SHIFTS[1]

| | | |
|---|---|---|
| Focus | Belongs to the Widely Applicable Prerequisites for College and Careers[2] |
| Coherence | Task helps distinguish the concepts of linear, quadratic, and exponential growth. |
| Rigor[3] | Conceptual Understanding: secondary in this task |
| | Procedural Skill and Fluency: not targeted in this task |
| | Application: Primary in this task |

[1] For more information read Shifts for Mathematics.
[2] For more information, see Widely Applicable Prerequisites.
[3] Tasks will often target only one aspect of rigor.

## LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Exponential Growth
- Quadratic
- Model
- Diameter
- Linear
- Evaluate

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work.

ELLs may need support with the following vocabulary words during the classroom discussion:

- Prediction
- Circumstances

## INSTRUCTIONAL ROUTINE

*The steps in this routine are adapted from the Principles for the Design of Mathematics Curricula: Promoting Language and Content Development.*

Before presenting the task itself, engage students in Co-Crafting Questions Mathematical Language Routine. This allows students to get inside the context before feeling the pressure to produce answers.

1. PRESENT SITUATION: Show this picture to the class:

2. STUDENTS WRITE: Ask students to produce questions using mathematical language that might be asked about this situation. These questions should be answerable by doing math. They may also ask questions about the situation shown in the picture, information they think might be missing, and even about some assumptions they think will be important.
3. PAIRS COMPARE: Ask students to share their questions with another student or in a group of three.

4. STUDENTS SHARE: As they share their questions about this partial circle full of pennies, students can begin to think about mathematical models that might be used to answer these questions. During this step, push for clarity and revoice their ideas as necessary.
5. REVEAL QUESTIONS: Play the video on "Screen 1: Make a Prediction" at the bottom of this page to reveal the task.

## TASK

The task is hosted here. When you go to the page, click "Student Preview" to explore the task. Click "Teacher Guide" to prepare for implementing the task.

## COMMENTARY

Penny Circle

This activity is a project from Dan Meyer and the Desmos Team.

Goals

- Students will understand how to use math as a model, how to use smaller things to make predictions about bigger things.
- Students will understand the difference between linear, quadratic, and exponential models.

Getting Started

If you do not have one already, create a free log-in. Then create a Class Code for your students.

Time Required

45 minutes.

Your Role

- Ask questions that surface your students' understanding.
- Push that understanding around in productive ways.

Question & Answer

- When it comes time to select a model - linear, exponential, or quadratic - make sure students fully understand the implications of that model and how it relates to their initial guess. If a student selects an exponential model, for example, say something like, "So your guess earlier was 500 pennies. The exponential model grows so fast it says the answer is millions of pennies. Either your earlier guess was way off or the exponential model isn't the right model. You decide which." Ask a similar question in the other direction if a student selects a linear model.
- Students need to understand WHY a quadratic is the best model here. It isn't enough to say that its prediction was closest to the actual answer. We have to know WHY it was closest. Ask for ideas. Ask them what happens to the area of a circle when the diameter goes from 3 inches to 6 inches. Some may say it doubles. If that's true, our penny's model should be linear. But it doesn't, it multiplies by $2^2$. There's our quadratic model. (The formula for area of a circle also has a SQUARED term, which helps explain the model too.) One reason the exponential model is incorrect is that when you evaluate $y = ab^x$ for a zero inch circle, you get "a." In other words, the exponential model says a zero-inch circle has more than zero pennies in it. That can't happen.
- Ask students, "Did anybody change their model at all? Like from linear to quadratic or anything else. Why did you change?"
- Ask students to use their model in reverse. Ask them, "What if there were 2,000 pennies? What's the smallest circle that'd fit those pennies?"
- Ask them to think of other circumstances where this kind of modeling would be useful.

## ADDITIONAL THOUGHTS

Another mathematical model for the "Pennies" problem could come from simply thinking about division. To estimate the number of pennies that fit in the circle, simply divide the area of the large circle, $\pi R^2$, by the area of a penny,$_0$:

$$\left(\frac{\pi}{A_0}\right) R^2$$

The quadratic dependence emerges directly from this line of thinking. Students could also express the area of the penny as $\pi r^2$ and show that the model is quadratic in the dimensionless ratio $R/r$. Note that the quotient above is an estimate; to our knowledge, an exact expression for the greatest number of small circles that can fit within a larger circle is not known. More information on this and related problems is here.