Fraction Comparisons with Pictures, Assessment Variation

Sample task from achievethecore.org

By Illustrative Mathematics and Student Achievement Partners

GRADE LEVEL Third

IN THE STANDARDS 3.NF.A.3d

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Presents two fractions with the same numerator so that students can compare the size of an area that is 2 one-third sections with an area that is 2 one-fifth sections.
- Requires student to recognize that two fractions must refer to the same whole in order to be compared (3.NF.A.3d).
- Builds on student understanding of non-unit fractions as multiple parts of unit fractions (3.NF.A.1).
- Uses rectangular fraction models, which can lead to understanding of fractions on the number line, showing coherence within the 3.NF standards.

In the classroom:

- Uses concrete representation to make the mathematics explicit.
- Can lead into related discussions with the other fraction comparison tasks.
- Presents a task in an assessment-ready form that is also easily implemented in the classroom.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Elicits evidence of thinking both verbally and in written form.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.
- Provides opportunities to support students in connecting mathematical language with mathematical representations.

MAKING THE SHIFTS[1]

Focus	Belongs to the Major Work[2] of third grade
Coherence	Lays groundwork for grade 4 fraction equivalence and ordering expectations (4.NF.A)
Rigor[3]	Conceptual Understanding: primary in this task Procedural Skill and Fluency: secondary in this task Application: not targeted in this task

[1] For more information read Shifts for Mathematics.
[2] For more information see Focus in Grade Three.
[3] Tasks will often target only one aspect of Rigor.

For a direct link, go to: http://www.achievethecore.org/page/876/fraction-comparisons-with-pictures-assessment-variation

INSTRUCTIONAL ROUTINE

To use this task during instruction, engage students in the Decide and Defend Instructional Routine for Part A. This routine prompts students to interpret a hypothetical student's fraction comparison statement, decide if it is correct, draft a defense of their decision, share the defense with the class, and reflect on their learning.

Ask students to interpret one of the statements from Ben, Lee, or Mia about $\frac{2}{3}$ and $\frac{2}{5}$. Provide individual think time, partner processing time, and public recording of the meaning of the statement from Ben, Lee, or Mia.

Have students individually and then with partners annotate their work with color, visuals, and words in order to make sense of it (for example, drawing $\frac{2}{5}$ in comparison to $\frac{2}{3}$, etc.) and to decide if they agree or disagree with Ben's, Lee's, or Mia's statement. Partners work together to draft a defense that will make their stance clear to their classmates.

While partners share their defense in the full group, students consider each other's arguments. Choose what ideas to explore with particular attention to the size of the whole, same numerator, and different denominators.

Facilitate a reflection process that allows students to identify new mathematical understandings and articulate aspects of constructing viable arguments. Create a public record of the reflections generated in the room for future reference.

LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Greater than
- Less than
- Equal
- Compare
- Symbols
- Numerator
- Denominator
- Whole
- Fraction

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work

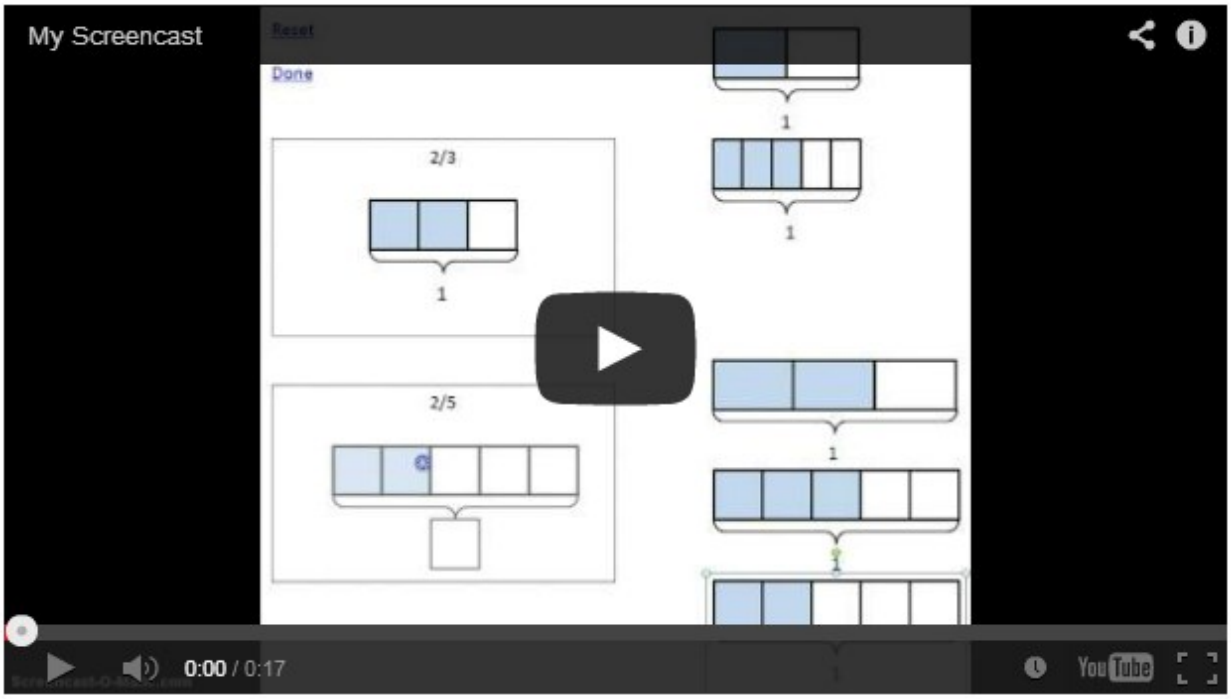
ELLs may need support with the following vocabulary words during the classroom discussion:

- Compare
- Defend
- Justify
- Correct
- Same

ADDITIONAL THOUGHTS

The video below highlights that, in order to compare two fractions, they must refer to the same whole. Notice that the student first moves the "smaller" representation of $\frac{2}{3}$ into the $\frac{2}{3}$ box. However, the student then recognizes that the representation of $\frac{2}{5}$ is "larger" (i.e., represents a different whole) and correctly switches out the representation of $\frac{2}{3}$ for the "larger" one.

For a direct link, go to: http://www.achievethecore.org/page/876/fraction-comparisons-with-pictures-assessment-variation



As noted in the Commentary below, this task is the third in a set of three tasks. Each task is unique, but all assess various aspects of the same standard. This is important to note when thinking about assessing the full breadth of a standard. The other tasks in the set can be found here: Comparing Fractions with the Same Denominator and Comparing Fractions with the Same Numerators.

For more information on fraction comparison, read page 4 of the progression document, *Number and Operations – Fractions, 3–5*, available at www.achievethecore.org/progressions.

For more analysis on this task from an assessment perspective, please read the Cognitive Complexity section on the Illustrative Mathematics website.

3.NF Fraction Comparisons With Pictures, Assessment Variation

a. Who correctly compares the numbers 2/3 and 2/5?

i. Ben said that 2/3 is greater than 2/5.

ii. Lee said that 2/3 is equal to 2/5.

iii. Mia said that 2/3 is less than 2/5.

b. Compare 2/3 and 2/5 using symbols:

2/3 ▾ 2/5

c. Choose the two pictures that best compare 2/3 and 2/5.

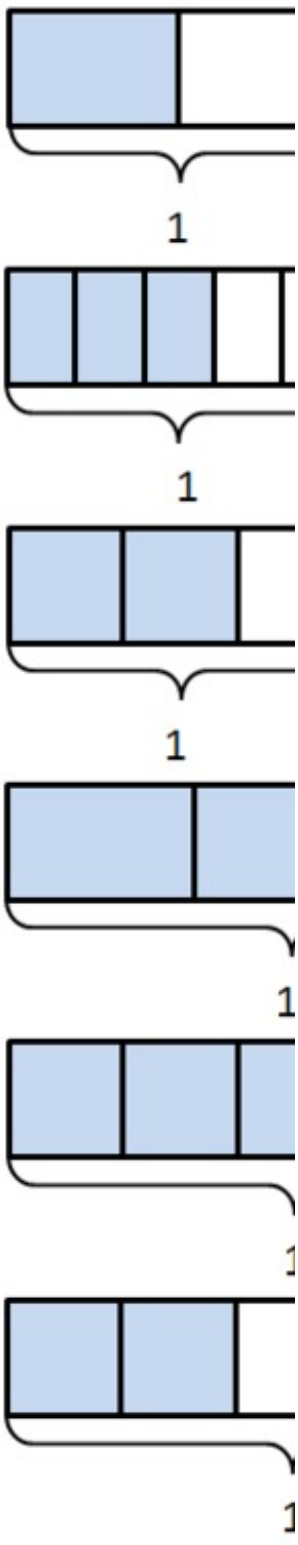
Illustrative Mathematics

Reset

Done

2/3

2/5



Commentary

This task is part of a joint project between Student Achievement Partners and Illustrative Mathematics to develop prototype machine-scorable assessment items that test a range of mathematical knowledge and skills described in the CCSSM and begin to signal the focus and coherence of the standards.

Task Purpose

This task is part of a set of three assessment tasks for 3.NF.3d. This part of the standard is about comparing two fractions with the same numerator or the same denominator by reasoning about their size, and understanding that such comparisons are valid only when the fractions refer to the same whole.

- 3.NF Comparing Fractions with the Same Denominators asks students to compare fractions using words and symbols and then to correctly represent them on the number line. This and the next task are designed to distinguish an understanding of relative magnitude, proficiency with symbolic notation, and proficiency with the number line. For example, if a student correctly chooses the greater fraction and the correct placement on the number line but selects the wrong inequalities, we diagnose that student as having trouble with the symbolic representation but not with their understanding of the relative magnitude of the fractions.

- 3.NF Comparing Fractions with the Same Numerators is identical in structure to the first task.

- 3.NF Fraction Comparisons with Pictures asks students to select pictures that accurately represent the relative magnitude of two fractions. This task also requires that students recognize that in order to compare two fractions, they must refer to the same whole.

Cognitive Complexity

Mathematical Content

Comparing fractions with common denominators should be as easy as comparing 5 oranges and 7 oranges: the greater numerator corresponds to the greater fraction because we are comparing different quantities of the same unit. Comparing fractions with a common numerator is more complex because the smaller denominator corresponds to the greater fraction. Thus, “3.NF Comparing Fractions with the Same Numerator” is more complex than “3.NF Comparing Fractions with the Same Denominator.”

"3.NF Fraction Comparisons with Pictures" requires students to compare two fractions with the same numerators and then select pictures from a set of choices that can be used to accurately represent the comparison. For the first two tasks in this set, students can choose any method or representation they like to compare the fractions. This task is more complex because students must evaluate pictures they did not draw themselves and must understand that the wholes (rather than the pieces) must be the same size. **Mathematical Practice**

The third task partially assesses students' ability to construct viable arguments and critique the reasoning of others (MP 3). Students have to evaluate three different claims and choose the correct one; they also must identify pictures that could be used to support an argument about comparing two fractions.

Linguistic Demand

The linguistic demand for the first two tasks is low. The linguistic demand for the third task is medium for third grade.

Stimulus Material

The first two tasks require students to analyze both written and symbolic statements and select a number line. The third task requires students to analyze written statements, construct a symbolic statement, and select diagrams from several options.
Thus, the stimulus material for all three tasks is complex.

Response Mode

The response mode for the first two tasks is multiple choice (select all) and so is not complex. The third task requires students to use a drop-down menu and to drag pictures on the screen, which gives this task a medium level of response mode complexity. Note that they can try different things without penalty and there is a reset option.

Solution: 1

a. Ben is correct that 2/3 is greater than 2/5.

b. $\frac{2}{3} > \frac{2}{5}$

c. See video for correct solution.

This is a 2-point item

d. One point is for correctly determining that 2/3 is greater and selecting $\frac{2}{3} > \frac{2}{5}$.

e. One point is for selecting diagrams that show why $\frac{2}{3} > \frac{2}{5}$.