Profit of a Company, Assessment Variation

Sample task from achievethecore.org
By Illustrative Mathematics and Student Achievement Partners

GRADE LEVEL High School

IN THE STANDARDS A-SSE.A.1, A-SSE.B.3

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Prompts students to analyze three equivalent quadratic expressions.
- Allows students to focus on the structure of the expressions without focusing on the procedural skill of factoring or expanding the expressions.
- Gives a real-world context for students to interpret the properties of quadratic expressions.
- Requires students to look for and make use of structure (MP7).

In the classroom:

- Offers students and teachers an opportunity to see an assessment-type task with two response types.
- Allows teachers to target specific student misunderstandings for reteaching.
- With follow-up questions, teachers can prompt students to share their thinking about the concepts in this task.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Elicits evidence of student thinking both verbally and in written form.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.
- Provides opportunities to support students in connecting mathematical language with mathematical representations.

MAKING THE SHIFTS[1]

Focus	Belongs to the Widely Applicable Prerequisites for College and Careers[2]
Coherence	Builds on work with algebraic expressions begun in grades 6–8, with added complexity appropriate to high school
Rigor[3]	Conceptual Understanding: primary in this task Procedural Skill and Fluency: not targeted in this task Application: Secondary in this task

[1] For more information read Shifts for Mathematics.
[2] For more information, see Widely Applicable Prerequisites.
[3] Tasks will often target only one aspect of rigor.

For a direct link, go to: http://www.achievethecore.org/page/883/profit-of-a-company-assessment-variation

INSTRUCTIONAL ROUTINE

Engage students in the Connecting Representations Instructional Routine. This routine positions students to think structurally as they connect varying forms of an expression to the context of the problem.
Display all three forms of the function and two of the three task questions rephrased as statements:

$P(x) = -2x^2 + 24x - 54$

$P(x) = -2(x - 3)(x - 9)$

$P(x) = -2(x - 6)^2 + 18$

This form of P(x) reveals the price which gives a profit of zero.

This form of P(x) reveals the price which produces the highest possible profit.

Ask students to think about the form of each expression and the two statements and then share with a partner any connections they are beginning to make. Encourage partners to build off one another's thinking as they continue to solidify connections between the expressions and their meaning.

Have partners justify their connections first with each other, then assign one student the role of pointer and one student the role of speaker to share and study the connections in the full class. Annotate the expressions and the statements to highlight the structural connections between the two forms (for example, "P(x)" and "profit" can be underlined with the same color marker). This process encourages the student who is speaking to be precise with language, the student who is pointing to gesture accurately, and the whole class to understand the meaning of the share-out.

After students accurately connect the two expression forms to the two statements they represent, there should be one expression remaining. Prompt students to create their own statement to describe what the remaining expression reveals in the context of the problem (think, pair, share). Select one representation to share, discuss, and annotate with the full class.

Finally, facilitate a reflection process that allows students to identify new mathematical understandings (e.g., looking for terms that are isolated in an expression or connecting factors to intercepts are strategies that can help make meaning of an expression). Create a public record of the reflections generated in the room for future reference.

LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Expressions
- Factor
- Price
- Profit
- Substitute
- Evaluate
- P(x)
- Parabola
- Vertex
- Squared
- Exponent
- Origin
- Maximum
- Coordinates

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Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work.

ADDITIONAL THOUGHTS

The Standards avoid the term "simplify" as there isn't always an obvious simplest form. The emphasis instead is on "purposeful transformation of expressions into equivalent forms that are suitable for the purpose at hand." This task shows three equivalent expressions and requires that students understand the structure and purpose of each one. This is a critical aspect of Seeing Structure in Expressions. For more information on the expectations of this domain, read pages 4–6 of the progression document, *High School, Algebra*, available at www.achievethecore.org/progressions.

For further analysis on this task from an assessment perspective, read the Cognitive Complexity section on the Illustrative Mathematics site.

A-SSE Profit of a company, assessment variation

Task

The profit, P (in thousands of dollars), that a company makes selling an item is a quadratic function of the price, x (in dollars), that they charge for the item. The following expressions for $P(x)$ are equivalent:

$$P(x) = -2x^2 + 24x - 54$$
$$P(x) = -2(x-3)(x-9)$$
$$P(x) = -2(x-6)^2 + 18$$

1. Which of the equivalent expressions for $P(x)$ reveals *the price which gives a profit of zero* without changing the form of the expression?

2. Find a price which gives a profit of zero.

3. Which of the equivalent expressions for $P(x)$ reveals *the profit when the price is zero* without changing the form of the expression?

4. Find the profit when the price is zero.

5. Which of the equivalent expressions for $P(x)$ reveals *the price which produces the highest possible profit* without changing the form of the expression?

6. Find the price which gives the highest possible profit.

For each of parts (a), (c), and (e), students choose which form of the function is displayed, and then use that to answer the corresponding questions in parts (b), (d), and (f) respectively. Students are assessed both on selecting the correct form of the function and on their numerical answer for each part.

Commentary

This task is part of a joint project between Student Achievement Partners and Illustrative Mathematics to develop prototype machine-scorable assessment items that test a range of mathematical knowledge and skills described in the CCSSM and begin to signal the focus and coherence of the standards.

Task Purpose

The primary purpose of this task is to assess students' knowledge of certain aspects of the mathematics described in the High School domain A-SSE: Seeing Structure in Expressions. Specifically, standard A-SSE.3 reads

> *A-SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
> *a. Factor a quadratic expression to reveal the zeros of the function it defines.*
> *b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines*

This tasks emphasizes the *choice* aspect of the full standard, giving students the option of several forms of a quadratic expression to choose from when answering questions about a real-world scenario governed by that expression. The task avoids having the students do the algebraic manipulations themselves, focusing instead on using the structure of the expressions.

Cognitive Complexity

Mathematical Content

Students will likely have encountered several "complete the square" problems which focus on the algebraic manipulation of quadratic expressions. By stripping this procedural aspect of the content away, students are assessed on a more conceptual understanding of quadratic expressions.

Mathematical Practices: The task, and the standard A-SSE.3 in general, ties closely to MP 7, Look for and make use of structure.

At the high school level, structure of the type mentioned in MP 7 manifests itself in the form of algebraic expressions, and the current task concerns itself quite explicitly with exploiting the structure to solve problems. For example, students need to recognize that the structure of the factored form (along with the zero-product property for real numbers) allows them to much more easily find a price that leads to a profit of zero than would, say, the standard form of the quadratic.

Linguistic Demand:

The language used to describe the business context is moderate for use in high school, though not entirely trivial for nonnative speakers.

Stimulus Material:

Students are presented with a block of text to read and process.

Response Mode:

For each of three questions, students choose one of the three forms from a drop-down menu, and enter their final answer into a text box.

Solutions

Solution: Solution

a. $P(x) = -2(x-3)(x-9$ shows the values of) x that make the profit zero without any computation because a quadratic function in factored form $P(x) = a(x-r_1)(x-r_2)$ has roots r_1 and r_2 and the profit is zero when $P(x) = 0$.

b. The profit is zero when $-2(x-3)(x-9) = 0$, which occurs when $x = 3$ or $x = 9$. The company breaks even

if the price charged for the product is \$3 or \$9.

c. $P(x) = -2x^2 + 24x - 54$ shows the profit when the price is zero without any computation because the value of a quadratic function in standard form $P(x) = ax^2 + bx + c$ when $x = 0$ is c.

d. In this case $c = -54$, so the profit is -54 (in thousands of dollars) when the price is zero. If the company gives the product away for free, it loses \$54,000.

e. $P(x) = -2(x-6)^2 + 18$ gives us the price which maximizes profit without any computation because the maximum value of a quadratic function in vertex from $P(x) = a(x-h)^2 + k$ is k and occurs at $x = h$.

f. From $P(x) = -2(x-6)^2 + 18$, we see that the maximum profit is 18 thousand dollars, and it occurs when $x = 6$. The company should charge a price of \$6 for this product in order to maximize its profit.

This is a 3-point item.
