# Front Matter for Progressions for the Common Core State Standards in Mathematics (draft)

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[*]This list includes Progressions that currently exist in draft form as well as planned Progressions.

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# Preface for the Draft Progressions

The Common Core State Standards in mathematics began with progressions: narrative documents describing the progression of a topic across a number of grade levels, informed both by educational research and the structure of mathematics. These documents were then sliced into grade level standards. From that point on the work focused on refining and revising the grade level standards, thus, the early drafts of the progressions documents do not correspond to the 2010 Standards.

The Progressions for the Common Core State Standards are updated versions of those early progressions drafts, revised and edited to correspond with the Standards by members of the original Progressions work team, together with other mathematicians and education researchers not involved in the initial writing. They note key connections among standards, point out cognitive difficulties and pedagogical solutions, and give more detail on particularly knotty areas of the mathematics.

**Audience**   The Progressions are intended to inform teacher preparation and professional development, curriculum organization, and textbook content. Thus, their audience includes teachers and anyone involved with schools, teacher education, test development, or curriculum development. Members of this audience may require some guidance in working their way through parts of the mathematics in the draft Progressions (and perhaps also in the final version of the Progressions). As with any written mathematics, understanding the Progressions may take time and discussion with others.

Revision of the draft Progressions will be informed by comments and discussion at **http://commoncoretools.me**, The Tools for the Common Core blog. This blog is a venue for discussion of the Standards as well as the draft Progressions and is maintained by lead Standards writer Bill McCallum.

**Scope**   Because they note key connections among standards and topics, the Progressions offer some guidance but not complete guidance about how topics might be sequenced and approached across

and within grades. In this respect, the Progressions are an inter-mediate step between the Standards and a teachers manual for a grade-level textbook—a type of document that is uncommon in the United States.

**Other sources of information** Another important source of infor-mation about the Standards and their implications for curriculum is the *Publishers' Criteria for the Common Core State Standards for Mathematics*, available at `www.corestandards.org`. In addition to giving criteria for evaluating K–12 curriculum materials, this docu-ment gives a brief and very useful orientation to the Standards in its short essay "The structure is the Standards."

Illustrative Mathematics illustrates the range and types of math-ematical work that students experience in a faithful implementa-tion of the Common Core State Standards. This and other ongoing projects that involve the Standards writers and support the Common Core are listed at `http://ime.math.arizona.edu/commoncore`.

Understanding Language aims to heighten awareness of the crit-ical role that language plays in the new Common Core State Stan-dards and Next Generation Science Standards, to synthesize knowl-edge, and to develop resources that help ensure teachers can meet their students' evolving linguistic needs as the new Standards are implemented. See `http://ell.stanford.edu`.

Teachers' needs for mathematical preparation and professional development in the context of the Common Core are often sub-stantial. The Conference Board of the Mathematical Sciences re-port *The Mathematical Education of Teachers II* gives recommenda-tions for preparation and professional development, and for math-ematicians' involvement in teachers' mathematical education. See `www.cbmsweb.org/MET2/index.htm`.

# Introduction

The college- and career-readiness goals of the Common Core State Standards of the Standards were informed by surveys of college faculty, studies of college readiness, studies of workplace needs, and reports and recommendations that summarize such studies.• Created to achieve these goals, the Standards are informed by the structure of mathematics as well as three areas of educational research: large-scale comparative studies, research on children's learning trajectories, and other research on cognition and learning.

References to work in these four areas are included in the "works consulted" section of the Standards document. This introduction outlines how the Standards have been shaped by each of these influences, describes the organization of the Standards, discusses how traditional topics have been reconceptualized to fit that organization, and mentions aspects of terms and usage in the Standards and the Progressions.

**The structure of mathematics**    One aspect of the structure of mathematics is reliance on a small collection of general properties rather than a large collection of specialized properties. For example, addition of fractions in the Standards extends the meanings and properties of addition of whole numbers, applying and extending key ideas used in addition of whole numbers to addition of unit fractions, then to addition of all fractions.• As number systems expand from whole numbers to fractions in Grades 3–5, to rational numbers in Grades 6–8, to real numbers in high school, the same key ideas are used to define operations within each system.

Another aspect of mathematics is the practice of defining concepts in terms of a small collection of fundamental concepts rather than treating concepts as unrelated. A small collection of fundamental concepts underlies the organization of the Standards. Definitions made in terms of these concepts become more explicit over the grades.• For example, subtraction can mean "take from," "find the unknown addend," or "find how much more (or less)," depending on context. However, as a mathematical operation subtraction can be defined in terms of the fundamental relation of addends and sum. Students acquire an informal understanding of this definition in Grade 1• and use it in solving problems throughout their mathematical work. The number line is another fundamental concept. In

• These include the reports from Achieve, ACT, College Board, and American Diploma Project listed in the references for the Common Core State Standards as well as sections of reports such as the American Statistical Association's *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A PreK–12 Curriculum Framework* and the National Council on Education and the Disciplines' *Mathematics and Democracy, The Case for Quantitative Literacy.*

• In elementary grades, "whole number" is used with the meaning "non-negative integer" and "fraction" is used with the meaning "non-negative rational number."

• Note Standard for Mathematical Practice 6: "Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. . . . By the time they reach high school they have learned to examine claims and make explicit use of definitions."

• Note 1.OA.4: "Understand subtraction as an unknown-addend problem." Similarly, 3.OA.6: "Understand division as an unknown-factor problem."

elementary grades, students represent whole numbers (2.MD.6), then fractions (3.NF.2) on number line diagrams. Later, they understand integers and rational numbers (6.NS.6), then real numbers (8.NS.2), as points on the number line.•

**Large–scale comparative studies**   One area of research compares aspects of educational systems in different countries. Compared to those of high–achieving countries, U.S. standards and curricula of recent decades were "a mile wide and an inch deep."•

In contrast, the organization of topics in high–achieving countries is more focused and more coherent. Focus refers to the number of topics taught at each grade and coherence is related to the way in which topics are organized. Curricula and standards that are focused have few topics in each grade. They are coherent if they are:

> articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential and hierarchical nature of the disciplinary content from which the subject matter derives.•

Textbooks and curriculum documents from high–achieving countries give examples of such sequences of topics and performances.•

**Research on children's learning trajectories**   Within the United States, researchers who study children's learning have identified developmental sequences associated with constructs such as "teaching–learning paths," "learning progressions," or "learning trajectories." For example,

> A learning trajectory has three parts: a specific mathematical goal, a developmental path along which children develop to reach that goal, and a set of instructional activities that help children move along that path.•

Findings from this line of research illuminate those of the large–scale comparative studies by giving details about how particular instructional activities help children develop specific mathematical abilities, identifying behavioral milestones along these paths.

The Progressions for the Common Core State Standards are not "learning progressions" in the sense described above. Well–documented learning progressions for all of K–12 mathematics do not exist. However, the Progressions for Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base Ten, Geometry, and Geometric Measurement are informed by such learning progressions and are thus able to outline central instructional sequences and activities which have informed the Standards.•

• For further discussion, see "Overview of School Algebra" in U.S. Department of Education, 2008, "Report of the Task Group on Conceptual Knowledge and Skills" in Foundations for Success: The Final Report of the National Mathematics Advisory Panel.

• See Schmidt, Houang, & Cogan, 2002, "A Coherent Curriculum," *American Educator*, `http://aft.org/pdfs/americaneducator/summer2002/curriculum.pdf`.

• Schmidt & Houang, 2012, "Curricular Coherence and the Common Core State Standards for Mathematics," *Educational Researcher*, `http://edr.sagepub.com/content/41/8/294`, p. 295.

• For examples of "course of study" documents from other countries, see `http://bit.ly/eb60lT`. Some textbooks from other countries are readily available. The University of Chicago School Mathematics Project (`http://bit.ly/18tEN7R`) has translations of Japanese textbooks for grades 7–9 and Russian grades 1–3. Singapore Math (`www.singaporemath.com`) has textbooks from Singapore. Global Education Resources (GER, `http://www.globaledresources.com`) has translations of Japanese textbooks for grades 1–6 and translations of the teaching guides for grades 1–6 and 7–9. Portions of the teachers manuals for the Japanese textbooks have been translated and can be downloaded at Lesson Study Group at Mills College (`http://bit.ly/12bZ1KQ`). The first page of a two-page diagram showing connections of topics for Grades 1–6 in Japan can be seen at `http://bit.ly/12EOjfN`.

• Clements & Sarama, 2009, *Learning and Teaching Early Math: The Learning Trajectories Approach*, Routledge, p. viii.

• For more about research in this area, see the National Research Council's reports *Adding It Up: Helping Children Learn Mathematics*, 2001, and *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, 2009 (online at www.nap.edu); Sarama & Clements, 2009, *Early Childhood Mathematics Education Research: Learning Trajectories for Young Children*, Routledge.

**Other research on cognition and learning**  Other research on cognition, learning, and learning mathematics has informed the development of the Standards and Progressions in several ways. Fine-grained studies have identified cognitive features of learning and instruction for topics such as the equal sign in elementary and middle grades, proportional relationships, or connections among different representations of a linear function. Such studies have informed the development of standards in areas where learning progressions do not exist.[•] For example, it is possible for students in early grades to have a "relational" meaning for the equal sign, e.g., understanding $6 = 6$ and $7 = 8 - 1$ as correct equations (1.OA.7), rather than an "operational" meaning in which the right side of the equal sign is restricted to indicating the outcome of a computation. A relational understanding of the equal sign is associated with fewer obstacles in middle grades, and is consistent with its standard meaning in mathematics. Another example: Studies of students' interpretations of functions and graphs indicate specific features of desirable knowledge, e.g., that part of understanding is being able to identify and use the same properties of the same object in different representations. For instance, students identify the constant of proportionality (also known as the unit rate) in a graph, table, diagram, or equation of a proportional relationship (7.RP.2b) and can explain correspondences between its different representations (MP.1).

Studies in cognitive science have examined experts' knowledge, showing what the results of successful learning look like. Rather than being a collection of isolated facts, experts' knowledge is connected and organized according to underlying disciplinary principles.[•] So, for example, an expert's knowledge of multiplying whole numbers and mixed numbers, expanding binomials, and multiplying complex numbers is connected by common underlying principles rather than four separately memorized and unrelated special-purpose procedures. These findings from studies of experts are consistent with those of comparative research on curriculum. Both suggest that standards and curricula should attend to "key ideas that determine how knowledge is organized and generated within that discipline."[•]

The ways in which content knowledge is deployed (or not) are intertwined with mathematical dispositions and attitudes.[•] For example, in calculating $30 \times 9$, a third grade might use the simpler form of the original problem (MP.1): calculating $3 \times 9 = 27$, then multiplying the result by 10 to get 270 (3.NBT.3). Formulation of the Standards for Mathematical Practice drew on the process standards of the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics*, the strands of mathematical proficiency in the National Research Council's *Adding It Up*, and other distillations.[•]

- For reports which summarize some research in these areas, see National Research Council, 2001, *Adding It Up: Helping Children Learn Mathematics*; National Council of Teachers of Mathematics, 2003, *A Research Companion to Principles and Standards for School Mathematics*; U.S. Department of Education, 2008, "Report of the Task Group on Learning Processes" in *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. For recommendations that reflect research in these areas, see the National Council of Teachers of Mathematics reports *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*, 2006 and *Focus in High School Mathematics: Reasoning and Sense Making*, 2009.

- See the chapter on how experts differ from novices in the National Research Council's *How People Learn: Brain, Mind, Experience, and School* (online at `http://www.nap.edu/catalog.php?record_id=9853`).

- Schmidt & Houang, 2007, "Lack of Focus in the Intended Mathematics Curriculum: Symptom or Cause?" in *Lessons Learned: What International Assessments Tell Us About Math Achievement, Brookings Institution Press*.

- See the discussions of self-monitoring, metacognition, and heuristics in How People Learn and the Problem Solving Standard of Principles and Standards for School Mathematics.

- See Harel, 2008, "What is Mathematics? A Pedagogical Answer to a Philosophical Question" in Gold & Simons (eds.), *Proof and Other Dilemmas*, Mathematical Association of America; Cuoco, Goldenberg, & Mark, 1996, "Habits of Mind: An Organizing Principle for a Mathematics Curriculum," *Journal of Mathematical Behavior* or Cuoco, 1998, "Mathematics as a Way of Thinking about Things," in *High School Mathematics at Work*, National Academies Press, which can be read online at `http://bit.ly/12Fa27m`. *Adding It Up* can be read online at `http://bit.ly/mbeQs1`.

# Organization of the Common Core State Standards for Mathematics

An important feature of the Standards for Mathematical Content is their organization in groups of related standards. In K–8, these groups are called domains and in high school, they are called conceptual categories. The diagram below shows K–8 domains which are important precursors of the conceptual category of algebra.• In contrast, many standards and frameworks in the United States are presented as parallel K–12 "strands." Unlike the diagram in the margin, a strands type of presentation has the disadvantage of deemphasizing relationships of topics in different strands.

Other aspects of the structure of the Standards are less obvious. The Progressions elaborate some features of this structure•, in particular:

- Grade–level coordination of standards across domains.

- Connections between standards for content and for mathematical practice.

- Key ideas that develop within one domain over the grades.

- Key ideas that change domains as they develop over the grades.

- Key ideas that recur in different domains and conceptual categories.

**Grade-level coordination of standards across domains or conceptual categories**   One example of how standards are coordinated is the following. In Grade 4 measurement and data, students solve problems involving conversion of measurements from a larger unit to a smaller unit.[4.MD.1] In Grade 5, this extends to conversion from smaller units to larger ones.[5.MD.1]

These standards are coordinated with the standards for operations on fractions. In Grade 4, expectations for multiplication are limited to multiplication of a fraction by a whole number (e.g., $3 \div 2/5$) and its representation by number line diagrams, other visual models, and equations. In Grade 5, fraction multiplication extends to multiplication of two non–whole number fractions.[5.NF.6, 5.MD.1]

**Connections between content and practice standards**   The Progressions provide examples of "points of intersection" between content and practice standards. For instance, standard algorithms for operations with multi–digit numbers can be viewed as expressions of regularity in repeated reasoning (MP.8). Such examples can be found by searching the Progressions electronically for "MP."

• For a more detailed diagram of relationships among the standards, see `http://commoncoretools.me/2012/06/09/jason-zimbas-wiring-diagram`.
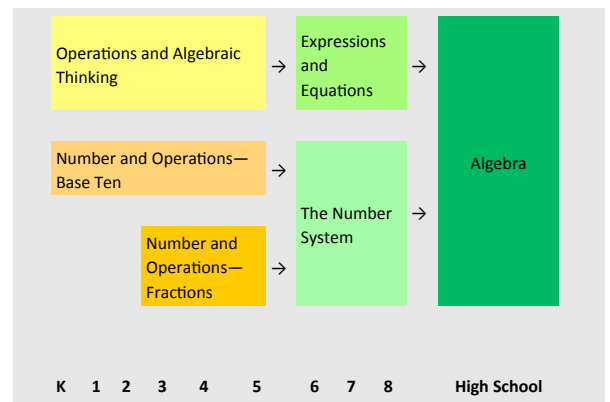


• Because the Progressions focus on key ideas and the Standards have different levels of grain-size, not every standard is included in some Progression.

[4.MD.1] Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.

[5.MD.1] Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

[5.NF.6] Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

[5.MD.1] Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

**Key ideas within domains** Within the domain of Number and Operations Base Ten, place value begins with the concept of ten ones in Kindergarten and extends through Grade 6, developing further in the context of whole number and decimal representations and computations.

**Key ideas that change domains** Some key concepts develop across domains and grades. For example, understanding number line diagrams begins in geometric measurement, then develops further in the context of fractions in Grade 3 and beyond.

Coordinated with the development of multiplication of fractions, measuring area begins in Grade 3 geometric measurement for rectangles with whole-number side lengths, extending to rectangles with fractional side lengths in Grade 5. Measuring volume begins in Grade 5 geometric measurement with right rectangular prisms with whole-number side lengths, extending to such prisms with fractional edge lengths in Grade 6 geometry.

**Key recurrent ideas** Among key ideas that occur in more than one domain or conceptual category are those of:

- composing and decomposing

- unit (including derived and subordinate unit).

These begin in elementary grades and continue through high school. Students develop tacit knowledge of these ideas by using them, which later becomes more explicit, particularly in algebra.

A group of objects can be decomposed without changing its cardinality, and this can be represented in equations. For example, a group of 4 objects can be decomposed into a group of 1 and a group of 3, and represented with various equations, e.g., $4 = 1 + 3$ or $1 + 3 = 4$. Properties of operations allow numerical expressions to be decomposed and rearranged without changing their value. For example, the 3 in $1 + 3$ can be decomposed as $1 + 2$ and, using the associative property, the expression can be rearranged as $2 + 2$. Variants of this idea (often expressed as "transforming" or "rewriting" an expression) occur throughout K–8, extending to algebra and other categories in high school.

One-, two-, and three-dimensional geometric figures can be decomposed and rearranged without changing—respectively—their length, area, or volume. For example, two copies of a square can be put edge-to-edge and be seen as composing a rectangle. A rectangle can be decomposed to form two triangles of the same shape. Variants of this idea (often expressed as "dissecting" and "rearranging") occur throughout K–8, extending to geometry and other categories in high school.

In K–8, an important occurrence of units is in the base-ten system for numbers. A whole number can be viewed as a collection of

ones, or organized in terms of its base-ten units. Ten ones compose a unit called a ten. That unit can be decomposed as ten ones. Understanding place value involves understanding that each place of a base-ten numeral represents an amount of a base-ten unit: ones, tens, hundreds, …and tenths, hundredths, etc. The regularity in composing and decomposing of base-ten units is a major feature used and highlighted by algorithms for computing operations on whole numbers and decimals.

Units occur as units of measurement for length, area, and volume in geometric measurement and geometry. Students iterate these units in measurement, first phys‑ically and later mentally, e.g., placing copies of a length unit side‑by‑side to measure length, tiling a region with copies of an area unit to measure area, or packing a con‑tainer with copies of a volume unit to measure volume. They understand that a length unit determines derived units for area and volume, e.g., a meter gives rise to a square meter and cubic meter.

Students learn to decompose a one ("a whole") into subordinate units: unit fractions of equal size. The whole is a length (possibly represented by an endpoint) on the number line or is a single shape or object. When possible, students are able to write a number in terms of those units in different ways, as a fraction, decimal, or mixed number. They expand their conception of unit by learning to view a group of objects as a unit and partition that unit into unit fractions of equal size.

Students learn early that groups of objects or numbers can be decomposed and reassembled without changing their cardinality. Later, students learn that specific length, area, or volume units can be decomposed into subordinate units of equal size, e.g., a meter can be decomposed into decimeters, centimeters, or millimeters.

These ideas are extended in high school. For example, derived units may be created from two or more different units, e.g., miles per hour or vehicle‑mile traveled. Shapes are decomposed and reassem‑bled in order to determine certain attributes. For example, areas can be decomposed and reassembled as in the proof of the Pythagorean Theorem or angles can be decomposed and reassembled to yield trigonometric formulas.

**Representing amounts in terms of units**

| | two units | notation | one unit | notation |
|---|---|---|---|---|
| Base-ten units | 1 ten, 3 ones | 13 | 13 ones | – |
| Fractional units | 1 one, 3 fifths | $1\frac{3}{5}$ | 8 fifths | $\frac{8}{5}$ |
| Measurement units | 1 foot, 3 inches | 1 ft, 3 in | 15 inches | 15 in |
| Base-ten units | 1 one, 3 tenths | 1.3 | 13 tenths | – |

*An amount may be represented in terms of one unit or in terms of two units, where one unit is a composition of the other.*

## Reconceptualized topics; changed notation and terminology

This section mentions some topics, terms, and notation that have been frequent in U.S. school mathematics, but do not occur in the Standards or Progressions.

**"Number sentence" in elementary grades**   "Equation" is used in‑stead of "number sentence," allowing the same term to be used

throughout K–12.

**Notation for remainders in division of whole numbers**    One aspect of attending to logical structure is attending to consistency. This has sometimes been neglected in U.S. school mathematics as illustrated by a common practice. The result of division within the system of whole numbers is frequently written like this:

$$84 \div 10 = 8 \text{ R } 4 \text{ and } 44 \div 5 = 8 \text{ R } 4.$$

Because the two expressions on the right are the same, students should conclude that $84 \div 10$ is equal to $44 \div 5$, but this is not the case. (Because the equal sign is not used appropriately, this usage is a non-example of Standard for Mathematical Practice 6.) Moreover, the notation 8 R 4 does not indicate a number.

Rather than writing the result of division in terms of a whole-number quotient and remainder, the relationship of whole-number quotient and remainder can be written like this:

$$84 = 8 \times 10 + 4 \text{ and } 44 = 8 \times 5 + 4.$$

**Conversion and simplification**    To achieve the expectations of the Standards, students need to be able to transform and use numerical and symbolic expressions. The skills traditionally labeled "conversion" and "simplification" are a part of these expectations. As noted in the statement of Standard for Mathematical Practice 1, students transform a numerical or symbolic expression in order to get the information they need, using conversion, simplification, or other types of transformations. To understand correspondences between different approaches to the same problem or different representations for the same situation, students draw on their understanding of different representations for a given numerical or symbolic expression as well as their understanding of correspondences between equations, tables, graphs, diagrams, and verbal descriptions.

**Fraction simplification, fraction-decimal-percent conversion**    In Grade 3, students recognize and generate equivalences between fractions in simple cases (3.NF.3). Two important building blocks for understanding relationships between fraction and decimal notation occur in Grades 4 and 5. In Grade 4, students' understanding of decimal notation for fractions includes using decimal notation for fractions with denominators 10 and 100 (4.NF.5; 4.NF.6). In Grade 5, students' understanding of fraction notation for decimals includes using fraction notation for decimals to thousandths (5.NBT.3a).

Students identify correspondences between different approaches to the same problem (MP.1). In Grade 4, when solving word problems that involve computations with simple fractions or decimals (e.g.,

4.MD.2), one student might compute

$$\frac{1}{5} + \frac{12}{10}$$

as

$$.2 + 1.2 = 1.4,$$

another as

$$\frac{1}{5} + \frac{6}{5} = \frac{7}{5};$$

and yet another as

$$\frac{2}{10} + \frac{12}{10} = \frac{14}{10}.$$

Explanations of correspondences between

$$\frac{1}{5} + \frac{12}{10}, \quad .2 + 1.2, \quad \frac{1}{5} + \frac{6}{5}, \quad \text{and} \quad \frac{2}{10} + \frac{12}{10}$$

draw on understanding of equivalent fractions (3.NF.3 is one build-ing block) and conversion from fractions to decimals (4.NF.5; 4.NF.6). This is revisited and augmented in Grade 7 when students use nu-merical and algebraic expressions to solve problems posed with rational numbers expressed in different forms, converting between forms as appropriate (7.EE.3).

In Grade 6, percents occur as rates per 100 in the context of finding parts of quantities (6.PR.3c). In Grade 7, students unify their understanding of numbers, viewing percents together with fractions and decimals as representations of rational numbers. Solving a wide variety of percent problems (7.RP.3) provides one source of opportu-nities to build this understanding.

**Simplification of algebraic expressions** In Grade 6, students apply properties of operations to generate equivalent expressions (6.EE.3). For example, they apply the distributive property to $3(2 + x)$ to generate $6 + 3x$. Traditionally, $6 + 3x$ is called the "simplification" of $3(2+x)$, however, students are not required to learn this terminology. Although the term "simplification" may suggest that the simplified form of an expression is always the most useful or always leads to a simpler form of a problem, this is not always the case. Thus, the use of this term may be misleading for students.

In Grade 7, students again apply properties of operations to gen-erate equivalent expressions, this time to linear expressions with ra-tional number coefficients (7.EE.1). Together with their understand-ing of fractions and decimals, students draw on their understanding of equivalent forms of an expression to identify and explain corre-spondences between different approaches to the same problem. For example, in Grade 7, this can occur in solving multi-step problems posed in terms of a mixture of fractions, decimals, and whole numbers (7.EE.4).

In high school, students apply properties of operations to solve problems, e.g., by choosing and producing an equivalent form of an expression for a quadratic or exponential function (A–SSE.3). As in earlier grades, the simplified form of an expression is one of its equivalent forms.

## Terms and usage in the Standards and Progressions

In some cases, the Standards give choices or suggest a range of options. For example, standards like K.NBT.1, 4.NF.3c, and G–CO.12 give lists such as: "using objects or drawings"; "replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction"; "dynamic geometric software, compass and straightedge, reflective devices, and paper folding." Such lists are intended to suggest various possibilities rather than being comprehensive lists of requirements. The abbreviation "e.g." in a standard is frequently used as an indication that what follows is an example, not a specific requirement.

On the other hand, the Standards do impose some very important constraints. The structure of the Standards uses a particular definition of "fraction" for definitions and development of operations on fractions (see the Number and Operations—Fractions Progression). Likewise, the standards that concern ratio and rate rely on particular definitions of those terms. These are described in the Ratios and Proportional Relationships Progression.

Terms used in the Standards and Progressions are not intended as prescriptions for terms that teachers must use in the classroom. For example, students do not need to know the names of different types of addition situations, such as "put-together" or "compare," although these can be useful for classroom discourse. Likewise, Grade 2 students might use the term "line plot," its synonym "dot plot," or describe this type of diagram in some other way.