Quadratic Equations Mini-Assessment

A-REI.B.4 and A-CED.A.1 Conceptual Understanding, Procedural Skill and Fluency, and Application Mini-Assessment by Student Achievement Partners

OVERVIEW

This mini-assessment is designed to illustrate standards A-REI.B.4 and A-CED.A.1, which set an expectation for students to create, use, and solve quadratic equations. This mini-assessment is designed for teachers to use in the classroom, for self-learning, or in professional development settings to:

- **Evaluate students' understanding** of A-REI.B.4 and A-CED.A.1 in order to prepare to teach this material or to check for student ability to demonstrate understanding and apply these concepts;
- **Gain knowledge about** assessing quadratic equations;
- **Use in professional development** as an illustration of CCR-aligned assessment problems;
- **Illustrate best practices** for writing tasks that allow access for all learners; and
- **Support mathematical language** acquisition by offering specific guidance.

MAKING THE SHIFTS

This mini-assessment attends to **focus** as it addresses quadratic equations, which are Widely Applicable Prerequisites for college and careers.[1] This mini-assessment highlights **coherence** across grades as these questions build on similar work with linear equations. The mini-assessment targets all three aspects of **rigor**: conceptual understanding, procedural skill/fluency and application.

Problem	Primary Aspect(s) of Rigor
1	Conceptual Understanding
2	Procedural Skill/Fluency
3	Procedural Skill/Fluency
4	Procedural Skill/Fluency and Conceptual Understanding
5	Application

A CLOSER LOOK

A-REI.B.4: Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

The first problem in the mini-assessment consists of a series of quick conceptual questions to assess understanding of how quadratic equations work. The second problem contains a series of equations for students to solve using any process they choose, while problem #3 explicitly requires completing the square (as is expected in the standard). The next series of problems targets conceptual understanding and procedural skill as students must first define (implicitly or explicitly) and solve for variables, and then answer the question. The last problem highlights application and addresses the modeling domain (pp. 72 and 73 of the CCSSM).

If students struggle with the longer application problem, teachers may differentiate it by giving the problem as group work, providing leading questions to struggling students, and/or leading a whole class discussion about ways to start the problem. As students gain familiarity with these items, these scaffolds can be removed so that students can engage with the mathematical practices.

A-CED.A.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

[1] For more on the Widely Applicable Prerequisites for a range of college majors, postsecondary programs, and careers, see https://achievethecore.org/content/upload/Widely%20Applicable%20Prerequisites.pdf

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO GRADE-LEVEL CONTENT

The conceptual questions in #1 can be answered efficiently if students see structure (MP7 – Look for and make use of structure). Students must reason quantitatively and abstractly in problem #4 to define variables and interpret the solutions (MP2 – Reason abstractly and quantitatively). Problem #5 requires MP1 (Make sense of problems and persevere in solving them). Students will need to evaluate different entry points into the problem and continue to evaluate their progress as they persevere toward a solution. Students should look at the situation and begin to create variables and/or diagrams that help represent the givens and relationships. Students will have to engage in these processes at multiple times as they get to a certain step of the problem solving process and need to re-evaluate their progress to the ultimate goal.

SUPPORT FOR ENGLISH LANGUAGE LEARNERS

This lesson was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction and assessment. Go here to learn more about the research behind these supports. Features that support access in this mini-assessment include:

- Tasks that allow for multi-modal representations, which can deepen understanding of the mathematics and make it easier for students, especially ELLs, to give mathematical explanations.
- Tasks that avoid unnecessarily complex language to allow students, especially ELLs, to access and demonstrate what they know about the mathematics of the assessment.

Prior to this mini-assessment, ensure students have had ample opportunities in instruction to read, write, speak, listen for, and understand the mathematical concepts that are represented by the following terms and concepts:

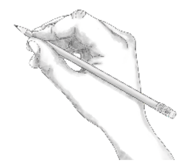
- equation
- real solution
- variable
- product
- integers
- diagonal
- ratio
- area
- height
- width
- length
- function
- positive
- square inches

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work (for example, through engagement in mathematical routines).

ELLs may need support with the following Tier 2 words found in this mini-assessment:

- check
- consecutive

In preparation for giving this mini-assessment, teachers should strive to use these words in context so they become familiar to students. It will be important to offer synonyms, rephrasing, visual cues, and modeling of what these words mean in the specific contexts represented in the items in this mini-assessment. Additionally, teachers may offer students the use of a student-friendly dictionary, or visual glossary (example below) to ensure they understand what is being asked of them in each item.

Sketch	
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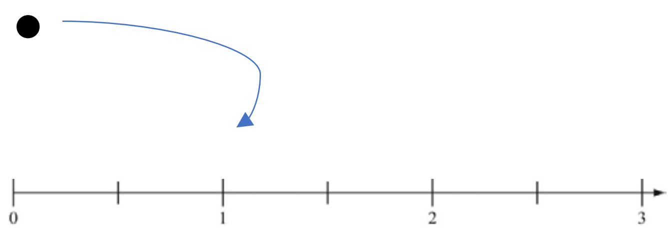
Locate	

An example of a visual glossary for student use.

Name:____Date:____________________

1. The table below contains single equations in a single variable. Decide whether there are no real solutions, exactly 1 real solution, or exactly 2 real solutions. Check the appropriate box for each row.

		No Real Solutions	**Exactly 1 Real Solution**	**Exactly 2 Real Solutions**
a.	$(a+5)^2 = 25$			
b.	$(n-5)^2 = 25$			
c.	$(z+5)^2 = -25$			
d.	$(x-5)^2 = 0$			
e.	$16-(l+5)^2 = 25$			
f.	$(f+1)^2 = (f+2)^2$			
g.	$5b^2 = 5b^2 + 1$			

2. Solve each equation.

a. $13g = \frac{1}{2}g^2 + 12\frac{1}{2}$

b. $7h^2 + 6 + 2h = h^2 + 4h + 26 + 5h$

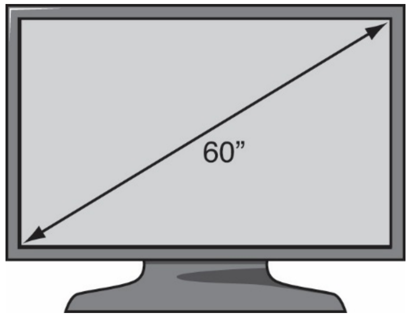
c. $(1.6J - 0.2)^2 = 1$	d. $42K + 112 = 7K^2$

3. Rewrite the equation $4x^2 + 16x = 65$ in the form $(x + p)^2 = q$

$$(x + \_\_\_\_\_\_\_\_)^2 = \_\_\_\_\_\_\_\_$$

4. The product of two consecutive positive integers is 380. What are the two integers? Show your work.

5. TV screens are measured on the diagonal.



The 60” TV shown above has a width to height ratio of 16 to 9.

a. What is the area, in square inches, of this TV screen? Show your work.

b. The width to height ratio of 16:9 is common for TVs today. Write a function *A* that gives the screen area *A(d)*, in square inches, when the length of the diagonal is *d inches* .

1.

		No Real Solutions	Exactly 1 Real Solution	Exactly 2 Real Solutions
a.	$(a+5)^2 = 25$			✓
b.	$(n-5)^2 = 25$			✓
c.	$(z+5)^2 = -25$	✓		
d.	$(x-5)^2 = 0$		✓	
e.	$16-(l+5)^2 = 25$	✓		
f.	$(f+1)^2 = (f+2)^2$		✓	
g.	$5b^2 = 5b^2 + 1$	✓		

2a. $g = 1$ or 25

2b. $h = -\frac{4}{3}$ or $\frac{5}{2}$

2c. $J = -0.5$ or 0.75

2d. $K = -2$ or 8

3.

$$(x + \underline{\quad \mathbf{2} \quad})^2 = \frac{\mathbf{81}}{\mathbf{4}}$$

4. 19 and 20 [Students could solve using an equation like: $(x)(x+1) = 380$]

5. Sample Answer:

a. I can name the height of the TV to be h inches. Then, the width of the TV is $\frac{16}{9}h$. So,

$$h^2 + \left(\frac{16}{9}h\right)^2 = 60^2$$
$$h^2 + \frac{256}{81}h^2 = 3600$$
$$\frac{337}{81}h^2 = 3600$$
$$h^2 \approx 865.282$$

$$h \approx \sqrt{865.282} \approx 29.42\ inches$$

So, since the height of the TV is approximately 29.42 inches, the width of the TV will be $\frac{16}{9}$ $(29.42) \approx 52.30\ inches$. This will yield an area of 29.42 × 52.30 = **1538.14 square inches.**

b. To write a function A that gives the area A(d) for any tv with width to height ratio 16:9 given its diagonal length, I can substitute *d* into the equation used from part (a) to get *h* in terms of *d.*

$$d^2 = h^2 + \left(\frac{16}{9}h\right)^2$$

$$d^2 = \frac{337}{81}h^2$$

$$\sqrt{\frac{81}{337}d^2 = h}$$

$$0.49d \approx h$$

Since the width is 16/9 times the height, the area in terms of the diagonal length is given by

$$A(d) \approx \frac{16}{9}(0.49d)(0.49d)$$

$$A(d) \approx 0.4263d^2$$