Sharing Chocolate

Sample task from achievethecore.org
Task by NCTM, annotation by Student Achievement Partners

GRADE LEVEL Fifth

IN THE STANDARDS 5.NF.B.3

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Provides opportunity to use concrete representations (pictures or sketches of chocolate bars or actual bars of chocolate) to model the mathematics.
- Allows for varied solution methods that lead to correct answers.
- Connects to adding fractions with unlike denominators (5.NF.A).
- Expands on comparing and adding fractions with like denominators (4.NF.A, 4.NF.B.3d).

In the classroom:

- Includes discussion questions that prompt students to share their developing thinking and assist teacher in checking for understanding.
- Facilitates using a variety of solution methods and representations to strengthen the understanding of all students.
- Allows for small group, partner, or individual work.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Elicits evidence of student thinking both verbally and in written form.
- Includes a mathematical routine that reflects best practices to supporting English Language Learners in accessing mathematical concepts.
- Offers the opportunity for students to act out the problem when the task features complex real-world situations.

MAKING THE SHIFTS[1]

Focus	Belongs to the Major Work[2] of grade five
Coherence	Revisits students' understanding of fraction equivalence and comparison, and builds on adding fractions with like denominators
Rigor[3]	Conceptual Understanding: primary in this task Procedural Skill and Fluency: not targeted in this task Application: primary in this task

[1] For more information read Shifts for Mathematics.
[2] For more information, see Focus in Grade Five.
[3] Tasks will often target only one aspect of rigor.

INSTRUCTIONAL ROUTINE

Engage students in the Capturing Quantities Instructional Routine. In this routine, students identify quantities and relationships, create a diagram to capture them, share and discuss their diagrams and finally reflect on what they learned about reasoning quantitatively.

Project the Sharing Chocolate problem and scenario for question 1: "In the first group of friends, four students receive three chocolate bars." Using a think, pair, share structure, have students identify important quantities and relationships in the scenario and create a public record for the class. You may want to delay sharing the specific question ("How much chocolate did each person get in the first group?") until you are discussing the diagrams students have created with the full group in order to allow students to make sense of the quantities and the problem.

Have students independently think about how they might represent important information from the problem, then work with partners to create a diagram. While students work, circulate and choose 1-3 diagrams to be shared with the full group. When discussing the diagrams, have students identify where quantities from the problem are represented in the diagram (e.g., the number of chocolate bars, the number of friends in the group, the number of pieces each friend receives). See additional discussion and follow-up questions below. Ask students how they would use the diagrams to answer the question. Encourage precision around units when discussing the chocolate bar pieces (e.g., ¼ of (what?))

Facilitate a reflection process that allows students to identify new mathematical understandings about representing quantities and relationships with visual diagrams. Create a public record of the reflections generated in the room for future reference.

Students may be ready to dive into question 2 and 3 independently. If not, consider using Capturing Quantities again.

LANGUAGE DEVELOPMENT

ELLs may need support with the following vocabulary words during the classroom discussion:

- Receive
- Given
- More
- Fairly
- Remains
- Sharing
- Same amount

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work.

DISCUSSION & FOLLOW-UP QUESTIONS

Teachers and those who support teachers may find the Instructional Practice Guide: Coaching tool useful in implementing best practices in the classroom that allow all students to master the content of the lesson.

- To promote student thinking:
  - Which group are you thinking about?
  - How many chocolate bars does this group have? How many students?
  - How could you share the chocolate equally?
  - Do you think each person will get more or less than one chocolate bar?
  - What does the "3" mean: three what?
  - So how much chocolate does each person get?
  - How could you find out which group of students got more chocolate?
- To prompt discussion:
  - Do you have any questions about this solution?
  - Which answer is correct?
  - Is one-half and one-fourth of a chocolate bar the same as three-fourths of a chocolate bar? How do you know?
  - Which is right, three-fourths or three-twelfths?
- For follow up:
  - Who thinks group 2 got more chocolate? Why?
  - Who disagrees? Why?
  - If group 2 got six pieces, isn't that more than group 1, because it got three pieces?
  - How can three-fourths be the same as six-eighths?

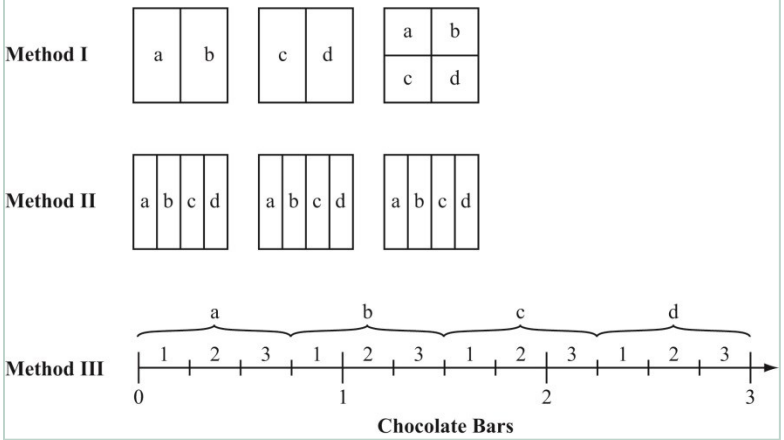
SOLUTIONS

The solutions to the three parts are:

1. Fractions equivalent to $\frac{3}{4}$ *
2. Fractions equivalent to $\frac{6}{8}$ *
3. A statement that the groups received the same amount of chocolate for each person in the group.

* If the student considers all the chocolate to be the whole, then fractions equivalent to $\frac{3}{12}$ would be correct for the first part and fractions equivalent to $\frac{6}{48}$ would be correct for the second part (See Where's the Math for more information.). But these students would need to do more reasoning to compare the amount of chocolate each person received.

Figure 1: Sample Solution Methods for Question 1



Sample work may include:

1. For the first part:
   - Using Method I: Each person gets $\frac{1}{2}+\frac{1}{4}$ (or $\frac{3}{4}$) chocolate bar.
   - Using Method II: Each person gets $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ (or $\frac{3}{4}$) chocolate bar.

- Using Method III: Each person gets 3 parts of a whole bar broken into 4 parts (or $\frac{3}{4}$).

2. For the second part:
   - Using Method I: Each person gets $\frac{1}{2}$ chocolate bar which leaves 2 whole chocolate bars remaining. Then the 8 friends divide those chocolate bars into quarters so that they each get an equal share. So, each person gets $\frac{1}{2}+\frac{1}{4}$ (or $\frac{3}{4}$) chocolate bar.
   - Using Method II: Each of the 6 chocolate bars is divided into eighths, with each of the 8 friends taking one piece from each bar. So, each person gets $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ (or $\frac{6}{8}$) chocolate bar.
   - Using Method III: Each person gets 6 parts of a whole bar broken into 8 parts (or $\frac{6}{8}$).
3. Correct reasoning comparing the two amounts:
   - "I know that $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent because $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$."
   - "I made a model using one chocolate bar as a whole. Then, I created an area model of $\frac{3}{4}$ and $\frac{6}{8}$. The two models showed the same amount."
   - "I made a number line and put both numbers on it. They were at the same location on the number line so I knew they were the same amounts."

ADDITIONAL THOUGHTS

This task offers students an opportunity to think about dividing whole numbers in a variety of ways. Whichever method a student chooses, the key is that students understand and interpret a fraction as division of the numerator (the number of chocolate bars) by the denominator (the number of people). In order to make this connection explicit, students need time to think about how the solutions for problems 1 and 2 relate back to the division required (e.g., how $\frac{3}{4}$ relates back to the original 3 chocolate bars shared equally by 4 students). Teachers could then show students another fraction (e.g., $\frac{2}{5}$) and ask, "If 5 students were sharing chocolate bars equally and each received 2/5 of a bar, how many chocolate bars were there to begin?"

For more insight on fractions as division in grade 5, read pages 12 and 13 of the progression document, *Number and Operations – Fractions, 3–5*.

Sharing chocolate

BANANASTOCK/THINKSTOCK

People enjoy sharing with their friends. What could be better for sharing than a sweet treat like a chocolate bar? In this problem scenario, students must determine how to create fair shares of a candy bar.

Problem scenario

Two groups of friends are sharing chocolate bars. Each group wants to share the chocolate bars fairly so that every person gets the same amount and no chocolate remains.

In the first group of friends, four students receive three chocolate bars. How much chocolate did each person get? In the second group of friends, eight students are given six chocolate bars. How much chocolate did each person get? Which group of students got more chocolate?

See the student **activity sheet** on page **139**.

Classroom setup

Before presenting the problem to your students, you will need to gather some materials:

- A large piece of paper for each pair or group of students to record their solutions
- Pens or markers
- Pictures or sketches of the chocolate bars and students for both situations (three chocolate bars and four friends, six chocolate bars and eight friends)
- Copies of a blackline master with equal-size rectangles (eight rectangles to a page)
- A digital camera, a smartphone, or a tablet with a camera

Present the problem scenario to students, using pictures or sketches as props that can be left at the front of the classroom for reference. Make sure the pictures of the chocolate bars are plain rectangles without any interior lines that partition the rectangle. Have your students turn to an "elbow partner" to discuss today's problem. Ask one student to explain today's problem to the rest of the class to ensure understanding of the task. Make students aware of the premade pages of rectangles in case they

would like to use them to help solve the problem. Students could also choose to draw their own. Organize students in pairs or triads to solve the problem.

As students are working, walk around the classroom and observe the strategies that students are using to solve the problem. You may want to take some pictures with a digital camera, smartphone, or tablet to help gather evidence of student thinking during the solution process. Try not to tell the students how to do the math, but use such questions as the following to promote their thinking:

- Which group are you thinking about?
- How many chocolate bars does this group have? How many students?
- How could you share the chocolate equally?
- Do you think each person will get more or less than one chocolate bar?
- What does the 3 mean; three what?
- So how much chocolate does each person get?
- How could you fi nd out which group of students got more chocolate?

When all groups have solved the problem, select a few solutions to share with the whole class. Begin by focusing on different solutions for the problem scenario with three chocolate bars and four students. For example, one solution may have determined that each person received one-half and one-fourth of a chocolate bar. A second solution might show that each person received three-fourths of a chocolate bar. A third solution suggests that students each received three-twelfths of the chocolate. As you display the solutions, you could ask questions of the class to prompt discussion:

Where's the math?

In this problem, to share the chocolate bars equally in a situation where there are fewer bars than people, students will need to divide a "whole" into fractional amounts. This problem scenario encourages students to think of fractions to represent the division of the chocolate into equal amounts and could lead students to generalize that the fraction would represent the number of chocolate bars divided by the number of people. It also may cause students to think about defi ning the "whole" in situations involving fractions. Because the question asks how much chocolate each person will get, some students may provide the answer with the fractional amount three-fourths, using one chocolate bar as the whole. Another student may consider all three chocolate bars as the whole and provide an answer of three-twelfths. Both answers could be correct, depending on the defi nition of *the whole*.

The problem also provides opportunities for discussing the meaning and creation of *equivalent fractions*, such as one-half plus one-fourth being equal to three-fourths or by determining that three-fourths can be changed into six-eighths by dividing each fourth in half.

- Do you have any questions about this solution?
- Which answer is correct?
- Is one-half and one-fourth of a chocolate bar the same as three-fourths of a chocolate bar? How do you know?
- Which is right, three-fourths or three-twelfths?

Use similar questions to facilitate a class discussion about some of the solutions from the second problem scenario with six chocolate bars and eight students. When the class has discussed how much chocolate each person received in both groups, invite students to compare the amounts from each group by asking which group got more chocolate. Follow-up questions might include these:

- Who thinks group 2 got more chocolate? Why?
- Who disagrees? Why?
- If group 2 got six pieces, isn't that more than group 1, because it got three pieces?
- How can three-fourths be the same as six-eighths?

As class members share ideas, create a list on the board or on chart paper as a record of student thinking.

Extensions and modifi cations

For students who fi nish early, provide a third scenario: a group of fi ve friends who have four chocolate bars. Ask students to determine how much chocolate this group would get and to fi nd out if it is equal to the other groups.

Because this activity contains three questions, it has built-in opportunities for differentiation.


RVLSOFT/THINKSTOCK

RVLSOFT/THINKSTOCK

Some students may be able to provide an answer for only the first question. Others may be able to solve the first two questions but not the third, which asks them to compare the amounts. These students can still participate in solving the problems and in the resulting discussions.

Share your students' work

consultant with the Waterloo Region District School Try this problem in your interested in how your students responded to the problem, which problem-solving strategies they used, and how they explained or justified their reasoning. Send your thoughts and reflections—including information your problem solvers in action—by **December 1, 2015**—to Problem Solvers department editor **Ed Enns**, Waterloo Region District School Board, Learning Services, 51 Ardelt Avenue, Kitchener, Ontario, Canada, N2C 2R5; or e-mail him at **ed_enns@wrdsb.on.ca** Selected submissions will be published in a subsequent issue of *TCM* and acknowledged by name, grade level, and school name unless you indicate otherwise.

section of the Problem Solvers department features a new challenge for students. Readers are encouraged to submit problems to be considered for future columns. Receipt of problems will not be acknowledged; however, those selected for publication will be

classroom. We are Board in Kitchener, Ontario, Canada. Each month, this about how you posed the problem, samples credited to the author. Find submission guidelines for of students' work, and photographs showing all departments at **www.nctm.org/tcmdepartments**.

problem solvers activity sheet

Name_____

Sharing Chocolate

Two groups of friends are sharing chocolate bars. Each group wants to share the chocolate bars fairly so every person gets the same amount and no chocolate remains.

1. In the fi rst group of friends, four students receive three chocolate bars. How much chocolate did each person get in the fi rst group?

2. In the second group of friends, eight students are given six chocolate bars. How much chocolate did each person get?

3. Which group of students got more chocolate?

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