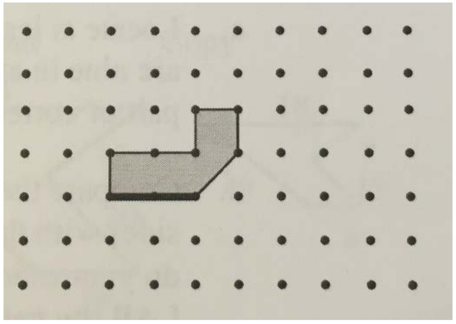
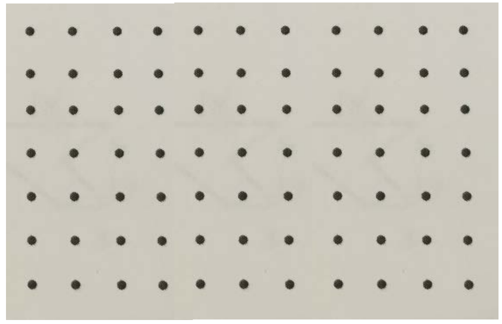
Math IMET 201 Resource Packet

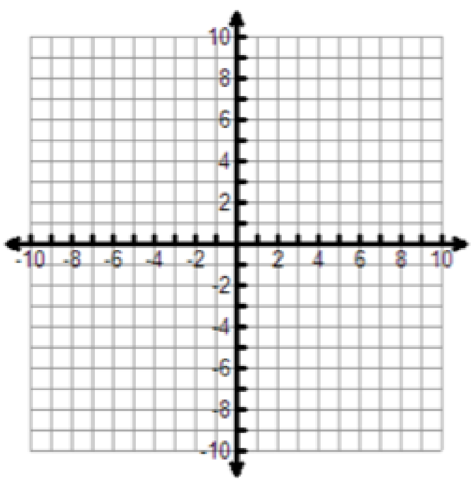
Similarity question that is appropriate for 7th grade:

Create a copy of the figure scaled by a factor of 1.5.

Similarity question that is NOT appropriate for 7th grade:

You are helping to design a new practice field at your school. As you sketch out plans for this project you, you plot the design on a coordinate grid with the coordinates (-2, 3), (2, 3), (-2, -2), and (2, -2).

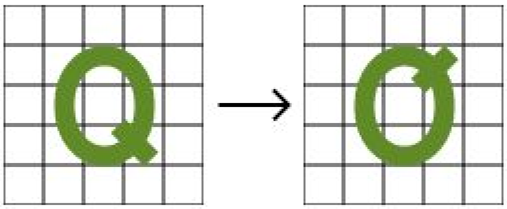


Part A: Plot the points on the coordinate grid below to create your design.

Part B: Which of the following transformations would change the size of your practice field? Select **all** that apply.

☐ A. $(3x, 3y)$
☐ B. $(x + 2, y - 1)$
☐ C. $(x, 2y)$
☐ D. $(½x - 1, ½y)$
☐ E. $(-x, y)$

What has been done to this letter?



flip

turn

slide

Engaging with the Content:

Focus on the Major Work of Each Grade

- How would you summarize the major work of grade band K – 2, 3 – 5, and/or 6 – 8?

- What would you have expected to be a part of the major work of K - 2, 3 – 5, and/or 6 - 8 that is not?

- Give an example of how you would approach something differently in your teaching if you thought of it as supporting the major work, instead of being a separate discrete topic.

33 An organization that raises funds for charity raised \$2.45 million last year and \$1.96 million this year. Find the percent change in the amount raised.

34 A company suffered a loss of \$5.4 million in its first year. It lost another \$3.1 million in the second year. It made a profit of \$4.9 million in the third year.

a) Find the average profit or loss for the first three years.

b) After its fourth year of business, the company's combined profit for all four years was \$0. Find the company's profit or loss during the fourth year.

Brain @ Work

1 The average of seven rational numbers is −5.16.

a) Find the sum of the rational numbers.

b) If six of the numbers each equal −5.48, find the seventh number.

2 The temperature of dry air decreases by about 0.98°F for every 100 meters increase in altitude.

a) The temperature above a town is 16°F on a dry day. Find the temperature of an aircraft 3.6 kilometers above the town.

b) The temperature outside an aircraft 1.8 kilometers above the town is −5.64°F. Find the temperature in the town.

3 Julie finds a way to use mental math to find the averages of these numbers:

15, 19, 18, 12, 20

She guesses that the mean is about 17, and uses mental math to find how far above or below this value each data item is:

−2, 2, 1, −5, 3

She uses mental math to add these amounts:

$-2 + 2 + 1 + (-5) + 3 = -1$

She then divides −1 by 5 to get an average of −0.2. She says this means that the average of the numbers is 0.2 less than 17, the number she estimated. Check that Julie's method gives the correct average. Then use it to find the average of these 4 numbers:

32, 35, 38, 36

Examples of Linking Supporting Clusters to the Major Work of the Grade

KINDERGARTEN

- Even within mathematics itself, understanding, for example, that 18 is ten ones and eight more ones (K.NBT.A.1) requires, but also supports, understanding what it means to combine 10 and 8 or to take apart 18 (K.OA).
- K.MD.B.3 offers a context in which to decompose numbers less than or equal to 10 in more than one way (see K.OA.A.3).

GRADE 1

- When students work with organizing, representing, and interpreting data, the process includes practicing using numbers and adding and subtracting to answer questions about the data (see the part of 1.MD.C.4 after the semicolon (". . . ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another"), and see the K–5 progressions document for Measurement and Data (data part), especially Table 1 on page 4 and the discussion of categorical data on pages 5 and 6).20
- Telling and writing time on digital clocks (1.MD.B.3) is a context in which one can practice reading numbers (1.NBT.A.1), a kind of "application," but no more. Relating those times to meanings— such as events during a day—is not part of 1.MD.B.3, but making sense of what one is doing (MP.1) and contextualizing (MP.2) are essential elements of good mathematical practice and should be part of the instructional foreground at all times.

GRADE 2

- When students work with time and money (cluster 2.MD.C), their work with dollars, dimes, and pennies should support their understanding and skill in place value (2.NBT). Their work with nickels, with telling time to the nearest five minutes on analog clocks, with counting by 5s (2.NBT.A.2), and with arrays of five rows and/or five columns (cluster 2.OA.C) should be related.
- In cluster 2.MD.D ("Represent and interpret data"), standard 2.MD.D.10 represents an opportunity to link to the major work of grade 2. Picture graphs and bar graphs can add variety as contexts for posing and solving addition and subtraction problems. The language in 2.MD.D.10 mentions word problems (2.OA) explicitly. See the K–5 progressions document for Measurement and Data (data part).

GRADE 3

- Represent and interpret data: Students multiply and divide to solve problems using information presented in scaled bar graphs (3.MD.B.3). Pictographs and scaled bar graphs are a visually appealing context for one- and two-step word problems.
- Reason with shapes and their attributes: Work toward meeting 3.G.A.2 should be positioned in support of area measurement and understanding of fractions.

GRADE 4

- Gain familiarity with factors and multiples: Work in this cluster supports students' work with multi-digit arithmetic as well as their work with fraction equivalence.
- Represent and interpret data: The standard in this cluster requires students to use a line plot to display measurements in fractions of a unit and to solve problems involving addition and subtraction of fractions, connecting it directly to the Number and Operations — Fractions clusters.

GRADE 5

- Convert like measurement units within a given measurement system: Work in these standards supports computation with decimals. For example, converting 5 cm to 0.05 m involves computation with decimals to hundredths.
- Represent and interpret data: The standard in this cluster provides an opportunity for solving real world problems with operations on fractions, connecting directly to both Number and Operations — Fractions clusters.

GRADE 6

- Solve real-world and mathematical problems involving area, surface area, and volume: In this cluster, students work on problems with areas of triangles and volumes of right rectangular prisms, which connects to work in the Expressions and Equations domain. In addition, another standard within this cluster asks students to draw polygons in the coordinate plane, which supports other work with the coordinate plane in The Number System domain.

GRADE 7

- Use random sampling to draw inferences about a population: The standards in this cluster represent opportunities to apply percentages and proportional reasoning. To make inferences about a population, one needs to apply such reasoning to the sample and the entire population.
- Investigate chance processes and develop, use, and evaluate probability models: Probability models draw on proportional reasoning and should be connected to the major work in those standards.

GRADE 8

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

7 Multiplication

When the standard algorithm is the only algorithm taught

Jason Zimba

Editor's note: This post originally appeared in a slightly different form on the Tools for the Common Core Standards *blog.*

Standards shouldn't dictate curriculum or pedagogy. But there has been some criticismrecently that the implementation of the Common Core State Standards may be effectively forcing a particular pedagogy on teachers. Even if that isn't happening, one can still be concerned if everybody's pedagogical interpretation of the standards turns out to be exactly the same. Fortunately, one can already see different approaches in various post-CCSS curricular efforts. And looking to the future, the revisions I'm aware of that are underway to existing programs aren't likely to erase those programs' mutual pedagogical differences, either.

Of course, standards do have to have meaningful implications for curriculum, or else they aren't standards at all. The Instructional Materials Evaluation Tool (IMET) is a rubric that helps educators judge high-level alignment of comprehensive instructional materials to the standards. Some states and districts have used the IMET to inform their curriculum evaluations, and it would help if more states and districts did the same.

The criticism that I referred to earlier comes from math educator Barry Garelick, who has written a series of blog posts that aims to sketch a picture of good, traditional pedagogy consistent with the Common Core. The concrete proposals in his series are a welcome addition to the conversation math educators are having about implementing the standards. Reading these posts led me to consider the following question:

If the only computation algorithm we teach is the standard algorithm, then can we still say we are following the standards?

Provided the standards as a whole are being met, I would say that the answer to this question is yes. The basic reason for this is that the standard algorithm is "based on place value [and] properties of operations." That means it qualifies. In short, the Common Core requires the standard algorithm; additional algorithms aren't named, and they aren't required.

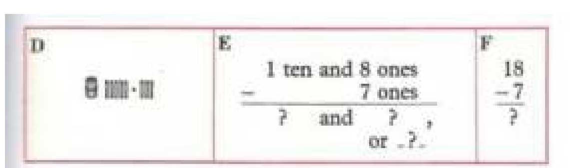
Additional mathematics, however, is required. Consistent with high-performing countries, the elementary grades' standards also require algebraic thinking, including an understanding of the properties of operations and some use of this understanding of mathematics to make sense of problems and do mental mathematics.

The section of the standards that has generated the most public discussion is probably the progression leading to fluency with the standard algorithms for addition and subtraction. So in a little more detail (but still highly simplified!), the accompanying table sketches a picture of how one might envision a progression in the early grades with the property that the only algorithm being taught is the standard algorithm.

The approach sketched in the table is something I could imagine trying if I were left to myself as an elementary teacher. There are certainly those who would do it differently! But the ability to teach differently under the standards is exactly my point today. I drew this sketch to indicate one possible picture that is consistent with the standards—not to argue against other pictures that are also consistent with the standards.

Whatever one thinks of the details in the table, I would think that, if the culminating standard in grade four is realistically to be met, one likely wants to introduce the standard algorithm pretty early in the addition-and-subtraction progression.

Writing about algorithms is very difficult. I ask for the reader's patience, not only because passions run high on this subject, but also because the topic itself is bedeviled with subtleties and apparent contradictions. For example, consider that even the teaching of a mechanical algorithm still has to look "conceptual" at times—or else it isn't actually teaching. Even the traditional textbook that Garelick points to as a model attends to concepts briefly, after introducing the algorithm itself:



Brownell et al., 1955

This screenshot of a fifties-era textbook is as old-school as it gets, yet somebody on the Internet could probably turn it into a viral Common-Core scare if they wanted to. What I would conclude from this example is that it might prove difficult for the average person even to decide how many algorithms are being presented in a given textbook.

Standards can't settle every disagreement—nor should they. As this discussion of just a single slice of the math curriculum illustrates, teachers and curriculum authors following the standards still may, and still must, make an enormous range of decisions.

This isn't to say that the standards are consistent with every conceivable pedagogy. It is likely that some pedagogies just don't do the job we need them to do. The conflict of such outliers with CCSS isn't best revealed by closely reading any individual standard; it arises instead from the more general fact that CCSS sets an expectation of a college- and career-ready level of achievement. At one extreme, this challenges pedagogies that neglect the key math concepts that are essential foundations for algebra and higher mathematics. On the other hand, routinely delaying skill development until a fully mature understanding of concepts develops is also a problem because it slows the pace of learning below the level that the college- and career-ready endpoint imposes on even the elementary years. Sometimes these two extremes are described using the labels of political ideology, but I have declined to use these shorthand labels. That's because I believe that achievement, not ideology, ought to decide questions of pedagogy in mathematics.

Jason Zimba was a member of the writing team for the Common Core State Standards for Mathematics and is a founding partner of Student Achievement Partners, a nonprofit organization.

Appendix

The Structure is the Standards

Essay by Phil Daro, William McCallum, and Jason Zimba, February 16, 2012[19]

You have just purchased an expensive Grecian urn and asked the dealer to ship it to your house. He picks up a hammer, shatters it into pieces, and explains that he will send one piece a day in an envelope for the next year. You object; he says "don't worry, I'll make sure that you get every single piece, and the markings are clear, so you'll be able to glue them all back together. I've got it covered." Absurd, no? But this is the way many school systems require teachers to deliver mathematics to their students; one piece (i.e. one standard) at a time. They promise their customers (the taxpayers) that by the end of the year they will have "covered" the standards.

In the Common Core State Standards, individual statements of what students are expected to understand and be able to do are embedded within domain headings and cluster headings designed to convey the structure of the subject. "The Standards" refers to all elements of the design—the wording of domain headings, cluster headings, and individual statements; the text of the grade level introductions and high school category descriptions; the placement of the standards for mathematical practice at each grade level.

The pieces are designed to fit together, and the standards document fits them together, presenting a coherent whole where the connections within grades and the flows of ideas across grades are as visible as the story depicted on the urn.

The analogy with the urn only goes so far; the Standards are a policy document, after all, not a work of art. In common with the urn, however, the Standards were crafted to reward study on multiple levels: from close inspection of details, to a coherent grasp of the whole. Specific phrases in specific standards are worth study and can carry important meaning; yet this meaning is also importantly shaped by the cluster heading in which the standard is found. At higher levels, domain headings give structure to the subject matter of the discipline, and the practices' yearly refrain communicates the varieties of expertise which study of the discipline develops in an educated person.

Fragmenting the Standards into individual standards, or individual bits of standards, erases all these relationships and produces a sum of parts that is decidedly less than the whole. Arranging the Standards into new categories also breaks their structure. It constitutes a remixing of the Standards. There is meaning in the cluster headings and domain names that is not contained in the numbered statements beneath them. Remove or reword those headings and you have changed the meaning of the Standards; you now have different Standards; you have not adopted the Common Core.

Sometimes a remix is as good as or better than the original. Maybe there are 50 remixes, adapted to the preferences of each individual state (although we doubt there are 50 good ones). Be that as it may, a remix of a work is not the same as the original work, and with 50 remixes we would not have common standards; we would have the same situation we had before the Common Core.

Why is paying attention to the structure important? Here is why: The single most important flaw in United States mathematics instruction is that the curriculum is "a mile wide and an inch deep." This finding comes from research comparing the U.S. curriculum to high performing countries, surveys of

[19] http://commoncoretools.me/2012/02/16/the-structure-is-the-standards/.

college faculty and teachers, the National Math Panel, the Early Childhood Learning Report, and all the testimony the CCSS writers heard. The standards are meant to be a blueprint for math instruction that is more focused and coherent. The focus and coherence in this blueprint is largely in the way the standards progress from each other, coordinate with each other and most importantly cluster together into coherent bodies of knowledge. Crosswalks and alignments and pacing plans and such cannot be allowed to throw away the focus and coherence and regress to the mile-wide curriculum.

Another consequence of fragmenting the Standards is that it obscures the progressions in the standards. The standards were not so much assembled out of topics as woven out of progressions. Maintaining these progressions in the implementation of the standards will be important for helping all students learn mathematics at a higher level. Standards are a bit like the growth chart in a doctor's office: they provide a reference point, but no child follows the chart exactly. By the same token, standards provide a chart against which to measure growth in children's knowledge. Just as the growth chart moves ever upward, so standards are written as though students learned 100% of prior standards. In fact, all classrooms exhibit a wide variety of prior learning each day. For example, the properties of operations, learned first for simple whole numbers, then in later grades extended to fractions, play a central role in understanding operations with negative numbers, expressions with letters and later still the study of polynomials. As the application of the properties is extended over the grades, an understanding of how the properties of operations work together should deepen and develop into one of the most fundamental insights into algebra. The natural distribution of prior knowledge in classrooms should not prompt abandoning instruction in grade level content, but should prompt explicit attention to connecting grade level content to content from prior learning. To do this, instruction should reflect the progressions on which the CCSSM are built. For example, the development of fluency with division using the standard algorithm in grade 6 is the occasion to surface and deal with unfinished learning with respect to place value. Much unfinished learning from earlier grades can be managed best inside grade level work when the progressions are used to understand student thinking.

This is a basic condition of teaching and should not be ignored in the name of standards. Nearly every student has more to learn about the mathematics referenced by standards from earlier grades. Indeed, it is the nature of mathematics that much new learning is about extending knowledge from prior learning to new situations. For this reason, teachers need to understand the progressions in the standards so they can see where individual students and groups of students are coming from, and where they are heading. But progressions disappear when standards are torn out of context and taught as isolated events.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

KINDERGARTEN

- In addition to laying the groundwork for place value in grade 1, working with numbers 11–19 (K.NBT.A.1) provides opportunities for cardinal counting beyond 10 (see K.CC.B.5) and for writing two-digit numbers (see K.CC.A.3). Ten frames, strips with ten ones and some loose ones, and math drawings can be helpful for this work. 11 Material adapted from National Research Council. Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity (Washington, DC: The National Academies Press, 2009), p. 138. PARCC K–2 Model Content Frameworks for Mathematics14 September 2014
- K.MD.B.3 provides opportunities for cardinal counting (see K.CC.B.5) and for comparing numbers (see K.CC.C.6). K.MD.B.3 also offers a context in which to decompose 10 in more than one way (see K.OA.A.3).
- K.G.A.2 and K.G.B.4 offer some opportunities for counting and comparing numbers.

GRADE 1

- A thorough understanding of how place-value language and notation represent number (cluster 1.NBT.A) is needed for meaningful calculation (cluster 1.NBT.B) in many ways—not just pencil and-paper calculation, but mental calculation as well. For purposes of calculation, it is valuable to use the tens and ones in two-digit numbers, single-digit knowledge, and properties of the operations (1.OA.B.4). In Grade 1, calculation ranges from simple mental adding, such as 40 + 20 (add the 4 tens and 2 tens) and 58 + 6 (6 gives 2 to 58 to make 60, then 60 plus the 4 left in 6 equals 64), to the more complex cases that require composing ten ones to make a ten, such as 37 + 56. PARCC K–2 Model Content Frameworks for Mathematics20 September 2014
- The study of word problems in grade 1 (1.OA.A.1, 1.OA.A.2) can be coordinated with students' growing proficiency with addition and subtraction within 20 (1.OA.C.6) and their growing proficiency with multi-digit addition and subtraction (1.NBT) and can involve easier and more accurate forward methods. 15
- Word problems can also be linked to students' growing understanding of properties of addition and the relationship between addition and subtraction. For example, put-together/take-apart problems with unknown addends can show subtraction as finding an unknown addend (see the "Problem Types" section). 16
- Units are a connection between place value (1.NBT) and measurement (1.MD). Working with place value depends on having a sense of the sizes of the base-ten units and being able to see a larger unit as composed of smaller units within the system. As measurement develops through the grades, measurement also depends on having a sense of the sizes of units and being able to see a larger unit as composed of smaller units within the system. In later grades, unit thinking will become important throughout arithmetic, including in the development of multi-digit multiplication and division algorithms and the development of fraction concepts and operations.17
- Measurement standards 1.MD.A.1 and 1.MD.A.2 together support and provide a context for the 1.OA.A.1 goal of solving problems that involve comparing. To meet 1.MD.A.1, students compare the lengths of two objects by means of a third object, e.g., a length of string, that allows a "copy" of the length of an immovable object to be moved to another location to compare with the length of a movable object. When students cannot find the exact difference because of the magnitude of the numbers that arise from measurement—as may occur in comparing two students' heights—they may still compare the measurements to know which is greater (1.NBT.B.3). (Grade 2 standard 2.MD.B.6 formalizes this idea on a number-line diagram.)

GRADE 1 (cont.)

- While students are dealing with the limited precision of only whole hours and half-hours, they must distinguish the position of the hour hand and connect it to the geometry standard 1.G.A.3, partitioning circles into halves and quarters.
- Composing shapes to create a new shape (1.G.A.2) is the spatial analogue of composing numbers to create new numbers. This concept is also connected to length measurement (1.MD.A.2) since students must visualize an object that is to be measured as being built up out of equal-sized units (see also 1.G.A.3). Though assembling two congruent right triangles into a rectangle does not use the same facts or reasoning that assembling two fives into a ten uses, the 15 See page 13 of the K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking, available at http://math.arizona.edu/~ime/progressions/ (2014). 16 See page 13 of the K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking, available at http://math.arizona.edu/~ime/progressions/ (2014). 17 See Units, a Unifying Idea in Measurement, Fractions, and Base Ten, available at http://commoncoretools.me/2013/04/19/units-a-unifying-idea/ (2013). Measuring a hallway using students as length units (1.MD.A.2). (The students are posited as having equal heights.) PARCC K–2 Model Content Frameworks for Mathematics21 September 2014 idea of looking at how objects in some domain (numbers or shapes) can be combined to make other objects in that domain, and looking for new true statements one can make about these combinations, is a big idea that is common across mathematics.

GRADE 2

- Problems involving dollars, dimes, and pennies (2.MD.C.8) should be connected with the place value learning of hundreds, tens, and ones (2.NBT.A.1), though the notation is different. A dollar is 100 cents, or a "bundle" of 10 dimes, each of which is a "bundle" of 10 pennies. Work with dollars, dimes, and pennies (without the notation) can support methods of whole-number addition (e.g., dimes are added to dimes). Addition that is appropriate with whole numbers can be explored in the new notation of money contexts (though fluency with that notation is not a standard at this grade).
- Students' work with addition and subtraction word problems (2.OA.A.1) can be coordinated with their growing skill in multi-digit addition and subtraction (2.OA.B.2; cluster 2.NBT.B).
- Work with nickels (2.MD.C.8) and with telling time to the nearest five minutes on analog clocks (2.MD.C.7) should be taken together with counting by 5s (2.NBT.A.2) as contexts for gaining familiarity with repeating groups of 5 (2.OA.C.4). Recognizing time by seeing the minute hand at 3 and knowing that that signifies 15 minutes; recognizing three nickels as 15 cents; and seeing the 15-ness of a 3-by-5 rectangular array held in any position (including with neither base horizontal) will prepare for understanding, in grade 3, what the new operation of multiplication means.
- •A number line (2.MD.B.6) connects numbers, lengths, and units. Number lines are first used in grade 2. A number line shows units of length; the numbers at the end points of the lengths tell how many lengths so far. Bar-graph scales (2.MD.D.10) and rulers (2.MD.A.1, 2, 3, 4) are number lines. Length units can be added and subtracted using rulers or number-line diagrams (2.MD.B.5, 6); adding lengths is an extension of adding and subtracting numbers of things, which has been a PARCC K–2 Model Content Frameworks for Mathematics27 September 2014 focus in kindergarten and grade 1 and will be a focus in grade 2 OA and NBT standards. The purpose of number lines is to represent numbers, sums, and differences as lengths, rather than using lengths to solve all addition and subtraction problems.

GRADE 3

- Represent and interpret data: Students multiply and divide to solve problems using information presented in scaled bar graphs (3.MD.B.3). Pictographs and scaled bar graphs are a visually appealing context for one- and two-step word problems.
- Reason with shapes and their attributes: Work toward meeting 3.G.A.2 should be positioned in support of area measurement and understanding of fractions.

GRADE 4

- Gain familiarity with factors and multiples: Work in this cluster supports students' work with multi-digit arithmetic as well as their work with fraction equivalence.
- Represent and interpret data: The standard in this cluster requires students to use a line plot to display measurements in fractions of a unit and to solve problems involving addition and subtraction of fractions, connecting it directly to the Number and Operations — Fractions clusters.

GRADE 5

- The work that students do in multiplying fractions extends their understanding of the operation of multiplication. For example, to multiply $a/b \times q$ (where q is a whole number or a fraction), students can interpret $a/b \times q$ as meaning a parts of a partition of q into b equal parts (5.NF.B.4a). This interpretation of the product leads to a product that is less than, equal to or greater than q depending on whether $a/b < 1$, $a/b = 1$ or $a/b > 1$, respectively (5.NF.B.5). 13 Students able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. But the division of a fraction by a fraction is not a requirement in this grade. 24 PARCC Model Content Frameworks for Mathematics Version 4.0— December 2014
- Conversions within the metric system represent an important practical application of the place value system. Students' work with these units (5.MD.A.1) can be connected to their work with place value (5.NBT.A.1).

GRADE 6

- Students' work with ratios and proportional relationships (6.RP) can be combined with their work in representing quantitative relationships between dependent and independent variables (6.EE.C.9). 28 PARCC Model Content Frameworks for Mathematics Version 4.0— December 2014
- Plotting rational numbers in the coordinate plane (6.NS.C.8) is part of analyzing proportional relationships (6.RP.A.3a, 7.RP.A.2) and will become important for studying linear equations (8.EE.C.8) and graphs of functions (8.F).15
- Students use their skill in recognizing common factors (6.NS.B.4) to rewrite expressions (6.EE.A.3).
- Writing, reading, evaluating, and transforming variable expressions (6.EE.A.1–4) and solving equations and inequalities (6.EE.B.7–8) can be combined with use of the volume formulas $V = lwh$ and $V = Bh$ (6.G.A.2).
- Working with data sets can connect to estimation and mental computation. For example, in a situation where there are 20 different numbers that are all between 8 and 10, one might quickly estimate the sum of the numbers as $9 \times 20 = 180$.

GRADE 7

- Students use proportional reasoning when they analyze scale drawings (7.G.A.1).
- Students use proportional reasoning and percentages when they extrapolate from random samples and use probability (7.SP.C.6, 8).

GRADE 8

- Students' work with proportional relationships, lines, linear equations, and linear functions can be enhanced by working with scatter plots and linear models of association in bivariate measurement data (8.SP.A.1–3).

Name ____________________

Writing Equivalent Expressions

Practice
1-11

Use the Distributive Property to write an equivalent expression by filling in the missing numbers.

1. $10(x + 3) =$ ____$x +$ ____

2. $7(8x + 2) =$ ____$x +$ ____

3. $6(7x - 8) =$ ____$x -$ ____

4. $24x - 3 = 3($____$x -$ ____$)$

5. $20x + 4 = 4($____$x +$ ____$)$

6. $9x + 27 = 9($____$x +$ ____$)$

Find the missing number(s) so that the expressions are equivalent.

7. $8(2x - 3)$ and ____$x - 24$

8. $5(3x - 9)$ and ____$x -$ ____

9. $6(2x + 9)$ and ____$x +$ ____

10. $22x - 11$ and ____$(2x -$ ____$)$

11. $18x +$ ____ and ____$(3x + 1)$

12. $36x +$ ____ and ____$(12x + 7)$

Use the Distributive Property to write an equivalent expression.

13. $3(6x - 7)$

14. $4(9x - 2)$

15. $6(8x + 1)$

16. $35x + 30$

17. $70x - 10$

18. $18x - 36$

19. **Geometry** The formula for the perimeter of a rectangle is $2l + 2w$, where l is the length and w is the width. How can you use the Distributive Property to write an equivalent expression for $2l + 2w$? Explain.

____

____

____

20. **Writing to Explain** Eliot uses the Distributive Property to write an equivalent expression for $7(3x + 4)$. He writes $21x + 4$. Emma notices that Eliot made a mistake. How could Emma explain to Eliot where he went wrong? What is the correct equivalent expression?

____

____

____

____

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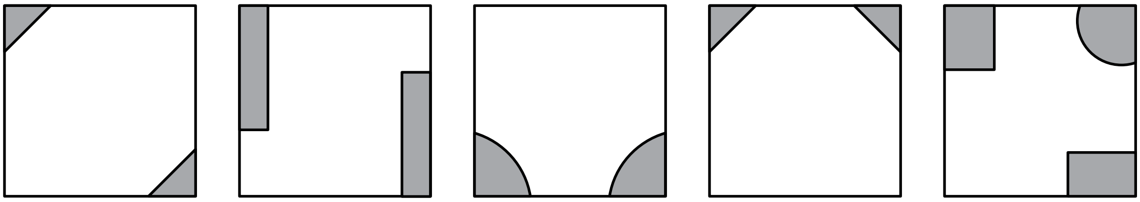
Name ____________________

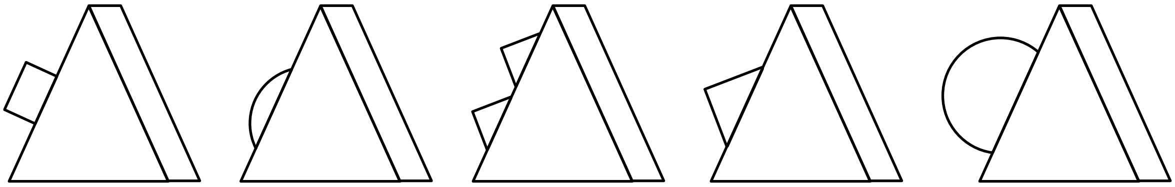
Go Figure

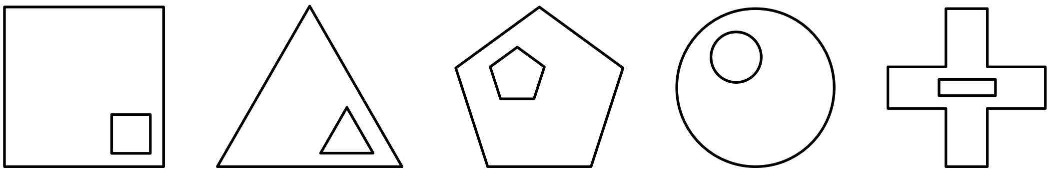
Enrichment
1-11

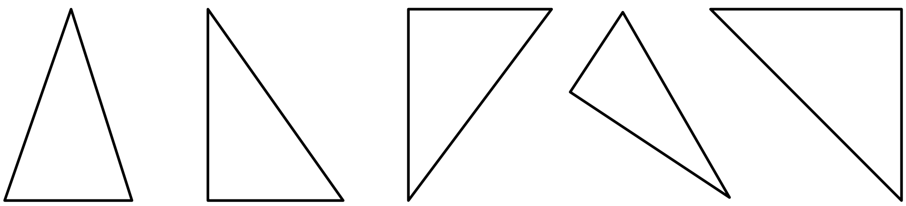
Visual Thinking

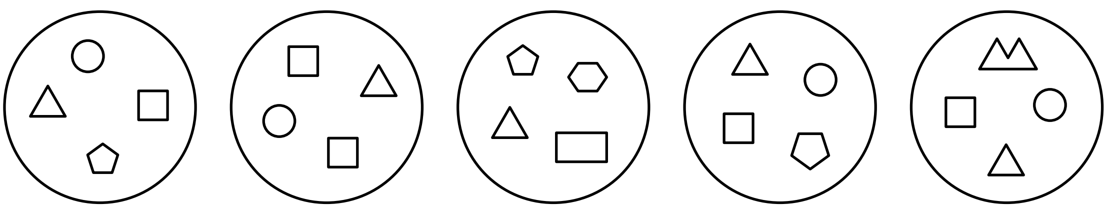
Circle the figure in each set that does NOT belong.

1. 

2. 

3. 

4. 

5. 

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Name ____________________

Reteaching
1-11

Writing Equivalent Expressions

You can use the properties of operations to write equivalent expressions. Two algebraic expressions are equivalent if they have the same value when any number is substituted for the variable.

How can you use the properties of operations to write an equivalent expression for the expression below?

$2(5x + 7)$

Use the Distributive Property to expand the expression. Then use Associative Property of Multiplication to regroup the first term and multiply 2×5.

$$\begin{aligned} 2(5x + 7) &= 2(5x) + 2(7) \\ &= (2 \times 5)x + 14 \\ &= 10x + 14 \end{aligned}$$

How can you use the Distributive Property in reverse order to write an equivalent expression for the expression below?

$9x + 3$

Look for a common factor of both terms that is greater than 1. In this expression, the common factor is 3.

$$\begin{aligned} 9x + 3 &= 3(3x) + 3(1) \\ &= 3(3x + 1) \end{aligned}$$

Use the Distributive Property to write an equivalent expression by filling in the missing numbers.

1. $4(x - 2) =$ ____$x -$ ____

2. $15x - 5 = 5($____$x -$ ____$)$

3. $3(6x + 1) =$ ____$x +$ ____

4. $21x + 6 = 3($____$x +$ ____$)$

Find the missing number(s) so that the expressions are equivalent.

5. $2(4x + 6)$ and ____$x + 12$

6. $16x - 14$ and ____$(8x -$ ____$)$

7. $3(8x - 5)$ and ____$x - 15$

8. $10x + 25$ and $5($____$x +$ ____$)$

Use the Distributive Property to write an equivalent expression.

9. $3(2x - 1)$

10. $10x - 5$

11. $7(3x + 4)$

12. $22x - 8$

13. Reasoning Jun writes the expression $5(x + 2)$. Then he uses the Distributive Property to write the equivalent expression $5x + 10$. How can he substitute a value for the variable to check to see if expressions are equivalent?

____

____

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Smart Balances in Smart Blends

By: Jason Zimba

It can be a difficult challenge for teachers to meet students where they are and also teach grade-level mathematics. Negotiating that tension is the subject of this post, in which I'll offer some thoughts on striking smart balances in rotation models. I've also included thoughts from two mathematics teachers, Allyson ("Ryan") Redd of North Carolina and Peter Tang of Tennessee.

One condition for striking a smart balance comes from the *K-8 Publishers' Criteria for the Common Core State Standards for Mathematics* . There it is stressed that the mathematics curriculum should give all students extensive work with grade-level problems. This should be true for blended classrooms as well as traditional classrooms.

Why is this important? Instructional coaches are familiar with the phenomenon of "fractions forever." Students who leave elementary school without a solid grasp of fractions can end up "getting the fraction treatment" from sixth grade onwards. The treatment continues year after year, despite its lack of success, and in the meantime the student has lost the opportunity to learn algebra.

As teachers know, unfinished learning from earlier grades is normal and prevalent. It shouldn't be ignored, but it also shouldn't be used as a basis for canceling grade-level work. Fortunately, middle-grades mathematics offers opportunities to handle unfinished learning in the context of the grade level. For example, instead of turning an eighth-grader around and frog-marching him back to fractions (or, worse, airbrushing fractions out of the algebra textbook), algebra problems could become occasions for some students to practice weak fraction skills and revisit fraction concepts in a new light (such as the idea that fractions are quotients, for example). Ideally, rotation models could provide individualized help in the context of grade-level content.

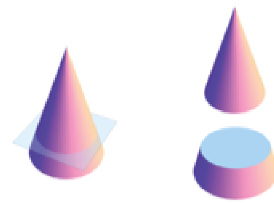
A related boundary condition is that rotation models should regularly allow time for the whole class to concentrate together on the same well-designed, carefully sequenced problem. This is a mode of classroom work in high-performing Japan. Inevitably, some students in the class will use clumsier methods to solve the problem–perhaps concrete approaches based on much earlier mathematics. Other students will use more abstract or powerful methods, closer to the new methods that the problem is aiming toward. The teacher then helps students in the class to understand each other's methods. Thus, the teacher doesn't see it as his job to give each student "just the right problem" for him or her at that point in time; all of the students are working on the same problem. Nor, however, does the teacher ignore the very real variability in his classroom. Indeed, the teacher sees it as his job to *decrease the variability* in students' approaches, in such a way that every student moves toward the mathematical goals of the lesson. That is roughly what it means, in such a system, to say that you are teaching the class the mathematics of the grade.

Ryan Redd, an instructional coach and eighth-grade mathematics teacher, sees the value of this approach. She says, "Students benefit from seeing multiple ways to solve a problem. Sometimes with rotation models there is only one view of a problem, and the student might not be able to access the problem in that particular way."

What happens when individual student needs become extreme? Peter Tang teaches seventh-grade mathematics in a school where a significant number of the students are behind by three or more grade levels. He says, “I work hard to scaffold grade-level work for these students, but it’s a struggle.” Peter uses a rotation model to handle the extreme differences in his classroom, stressing the benefits: “Essentially, rotation lowers my class size, which makes it easier to have conversations with my students about academic content.

A Challenging Problem in Grade 8 (See content standards 8.G.9 and 8.EE.2.)

A large cone is sliced parallel to its base producing two pieces of equal volume. The height of the resulting small cone is what fraction of the height of the original cone?



Personalization has a role for the most able students as well. One thing high-performing students need is challenging problems–and that doesn’t necessarily mean out-of-grade-level content. Problems that are challenging for highly able students can be created based on grade 8 content (see illustration), and it would be helpful for rotation models to leverage this fact.

A final boundary condition: ensure that the large majority of students’ time is devoted to major work. New technologies and digital tools offer exciting opportunities for individualized and nonlinear paths, but they should not turn the curriculum into a random walk through the content landscape. The strong focus of the Common Core on arithmetic in grades K-5, and the coherent progressions to algebra in grades K-8, are the north star for bringing students to college- and career-readiness and STEM readiness in mathematics. No blend is smart enough to compensate for a mile-wide, inch-deep curriculum.

Make true equations. Write one number in every space. Draw a picture if it helps.

1) 1 hundred + 4 tens = ______

2) 4 tens + 1 hundred = ______

3) 14 tens = __10__ tens + _____ tens

= _____ hundred + __4__ tens

= ______

4) 7 ones + 5 hundreds = ______

5) 8 hundreds = ______

6) 106 = __1__ hundred + _____ tens + _____ ones

7) 106 = _____ tens + _____ ones

8) 106 = _____ ones

9) 90 + 300 + 4 = ______

Are these comparisons true or false?

10) 2 hundreds + 3 ones > 5 tens + 9 ones ______

11) 9 tens + 2 hundreds + 4 ones < 924 ______

12) 456 < 5 hundreds ______

Make true equations. Write one number in every space. Draw a picture if it helps.

1) 1 hundred + 4 tens = ______

2) 4 tens + 1 hundred = ______

These two problems have the same answer, 140. This emphasizes that order doesn't matter in addition – yet order is everything in positional notation! In the second problem, you must really think to encode the quantity in positional notation.

3) 14 tens = __10__ tens + _____ tens

= _____ hundred + __4__ tens

= ______

In these three problems, the base-ten units in 140 are bundled in different ways. In the first line, "tens" are thought of as units: 14 *things* = 10 *things* + 4 *things.*

4) 7 ones + 5 hundreds = ______

By scrambling the usual order, the problem requires students to link *the values of the parts* with *the order of the digits* in the positional system. Also, to encode the quantity, the student will have to think: "no tens," emphasizing the role of 0.

5) 8 hundreds = ______

When the student writes "8-0-0," the zeros should come with a silent "no tens and no ones."

6) 106 = __1__hundred + _____tens + _____ones

7) 106 = _____tens + _____ones

8) 106 = _____ones

In these three problems, the base-ten units in 106 are bundled in different ways. This is helpful when learning how to subtract in a problem like 106 – 37, for example.

9) 90 + 300 + 4 = ______

If the order is always given "correctly," then all we do is teach students rote strategies without thinking about the size of the units or how to encode them in positional notation.

Are these comparisons true or false?

10) 2 hundreds + 3 ones > 5 tens + 9 ones

______

11) 9 tens + 2 hundreds + 4 ones < 924

______

12) 456 < 5 hundreds

______

At first, comparisons of numbers should hinge on the sizes of the quantities—not a rote "alphabetization" strategy of comparing digits from left to right. These problems invite the mathematical strategy of looking first for the largest base-ten unit on each side.

A more advanced problem might also involve bundling. For example, True or False: 20 tens + 30 ones > 230.

the Council first defined a set of Principles that "describe features of high-quality mathematics education." *Principles to Actions* now articulates and builds on an updated set of six Guiding Principles that reflect more than a decade of experience and new research evidence about excellent mathematics programs, as well as significant obstacles and unproductive beliefs that continue to compromise progress.

Three aspects of *Principles to Actions* are new, provocative, and important. First, *Principles to Actions* devotes the largest section to Teaching and Learning, the first Guiding Principle, and describes and illustrates eight Mathematics Teaching Practices (see fig. 1) that research indicates need to be consistent components of every mathematics lesson. Second, for each Guiding Principle, *Principles to Actions* offers commentary and a table that address productive and unproductive beliefs as part of a realistic appraisal of the obstacles that we face, as well as suggestions for overcoming these obstacles. Third, *Principles to Actions* issues a forceful call to action, asserting that all of us who are stakeholders have a role to play and important actions to take if we are finally to recognize our critical need for a world where the mathematics education of our students draws from research, is informed by common sense and good judgment, and is driven by a nonnegotiable belief that we must develop mathematical understanding and self-confidence in *all* students.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Fig. 1. Mathematics Teaching Practices

4.1 Understanding Equivalent Equations

Lesson Objective

- Identify equivalent equations.

Vocabulary
equivalent equations

Identify Equivalent Equations.

You have learned that factoring, simplifying, or expanding an expression produces an equivalent expression. Equivalent expressions have the same value for any given value of the variable.

Examples of equivalent expressions:

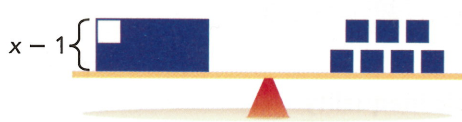
$4x + 3 + 3x = 7x + 3$	Group like terms.
$4x + 6 = 2(2x + 3)$	Factor. The common factor of $4x$ and 6 is 2.
$2(x - 5) = 2x - 10$	Use the distributive property to expand.

Equivalent equations are equations that have the same solution. Given an equation, you can use the operations of addition, subtraction, multiplication, or division to produce an equivalent equation. For example, you can subtract 2 from both sides of the equation $x - 1 = 7$ as shown.

Balance | **Algebraic Equation**

■ represents 1 counter.

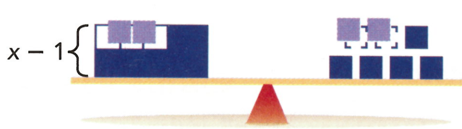
▬ represents x counters.



Left side = Right side

$x - 1 = 7$

Subtract 2 counters from both sides.



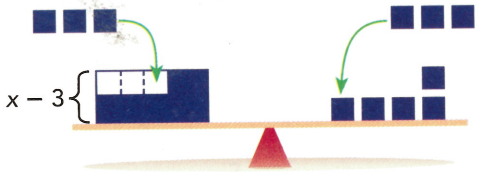
$x - 1 - 2 = 7 - 2$ Subtract both sides by 2.

$x - 3 = 5$ Simplify.

Compare the solutions of the original equation and the new equation:
$x = 8$ gives the solution of the equation $x - 1 = 7$.
$x = 8$ gives the solution of the equation $x - 3 = 5$.

Now suppose you add 3 to both sides of the equation $x - 3 = 5$.

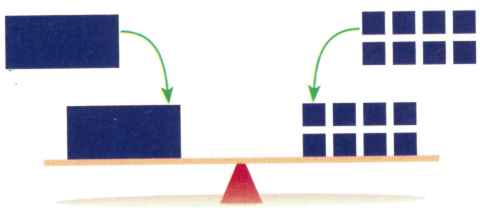
Balance	Algebraic Equation	
Add 3 counters to both sides.	$x - 3 + 3 = 5 + 3$ $x = 8$	Add 3 to both sides. Simplify.



Compare the solutions of the original equation and the new equation:
$x = 8$ gives the solution of the equation $x - 3 = 5$.
$x = 8$ gives the solution of the equation $x = 8$.

Then suppose you multiply both sides of the equation $x = 8$ by 2.

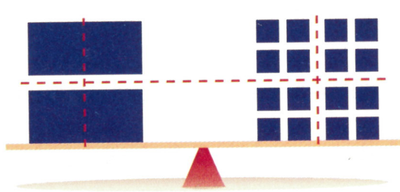
Balance	Algebraic Equation	
Multiply both sides by 2.	$x \cdot 2 = 8 \cdot 2$ $2x = 16$	Multiply both sides by 2. Simplify.



Compare the solutions of the original equation and the new equation:
$x = 8$ gives the solution of the equation $x = 8$.
$x = 8$ gives the solution of the equation $2x = 16$.

Finally, suppose you divide both sides of the equation $2x = 16$ by 4.

Balance	Algebraic Equation	
Divide into four equal groups.	$2x \div 4 = 16 \div 4$ $\frac{1}{2}x = 4$	Divide both sides by 4. Simplify.



Compare the solutions of the original equation and the new equation:
$x = 8$ gives the solution of the equation $2x = 16$.
$x = 8$ gives the solution of the equation $\frac{1}{2}x = 4$.

So, you can see that performing the same operation on both sides of an equation may produce an equivalent equation with the same solution. You can use the fact that equivalent equations have the same solution to decide whether two equations are equivalent.

Example 1 Identify equivalent equations.

State whether each pair of equations are equivalent equations. Give a reason for your answer.

a) $x + 3 + 6x = 13$ and $7x + 3 = 13$

Solution

$x + 3 + 6x = 13$

$x + 6x + 3 = 13$ — Use commutative property to group like terms.

$7x + 3 = 13$ — Add like terms.

$x + 3 + 6x = 13$ can be rewritten as $7x + 3 = 13$ using familiar number properties. So, the equations have the same solution and are equivalent.

b) $5x - 4 = 6$ and $5x = 20$

Solution

You can check to see if both equations have the same solution.

First solve $5x = 20$.

$5x \div 5 = 20 \div 5$ — Divide both sides by 5.

$x = 4$ — Simplify.

Then check to see if 4 is the solution of the equation $5x - 4 = 6$.

If $x = 4$, $5x - 4 = 5 \cdot 4 - 4$ — Substitute 4 for x.

$= 16 \quad (\neq 6)$ — 4 is not a solution.

Because the equations have different solutions, they are not equivalent equations. So, $5x - 4 = 6$ and $5x = 20$ are not equivalent equations.

c) $\frac{2}{3}x = 4$ and $x = 6$

Solution

You can check to see if both equations have the same solution.

If $x = 6$, $\frac{2}{3}x = \frac{2}{3} \cdot 6$ — Substitute 6 for x.

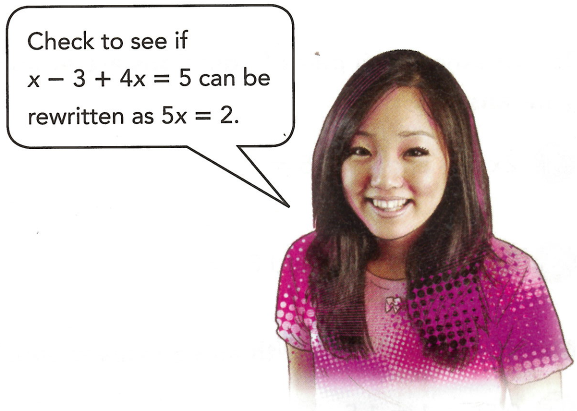
$= 4$ — 6 is a solution.

Because the equations have the same solution, 6, they are equivalent equations.

So, $\frac{2}{3}x = 4$ and $x = 6$ are equivalent equations.

Guided Practice

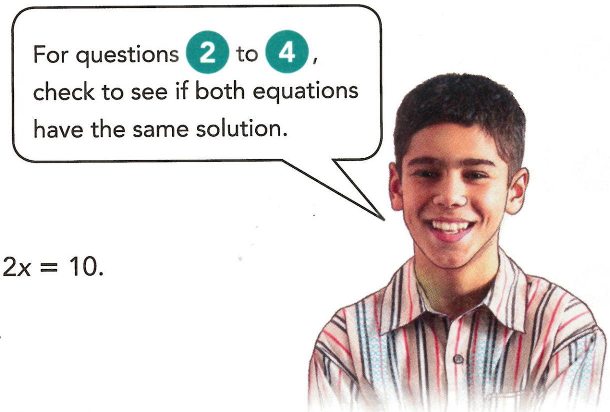
Copy and complete to state whether each pair of equations are equivalent equations. Give a reason for your answer.



1 $x - 3 + 4x = 5$ and $5x = 2$

$x - 3 + 4x = 5$	
$5x - 3 = 5$	Group like terms.
$5x - 3 + \underline{?} = 5 + \underline{?}$	Add __?__ to both sides.
$5x = \underline{?}$	Simplify.

$x - 3 + 4x = 5$ __?__ be rewritten as $5x = 2$.
So, the equations have __?__ solutions and are __?__.



2 $x + 7 = 12$ and $2x = 10$

First solve $x + 7 = 12$.

$x + 7 = 12$	
$x + 7 - \underline{?} = 12 - \underline{?}$	Subtract __?__ from both sides.
$x = \underline{?}$	Simplify.

Then check to see if __?__ is the solution of the equation $2x = 10$.

If $x = \underline{?}$, $2x = 2 \cdot \underline{?}$	Substitute __?__ for x.
$= \underline{?}$	__?__ a solution.

Because the equations have the __?__ solution, they are __?__ equations.

3 $1.2x = 2.4$ and $x - 6 = 8$

First solve $x - 6 = 8$.

$x - 6 = 8$	
$x - 6 + \underline{?} = 8 + \underline{?}$	Add __?__ to both sides.
$x = \underline{?}$	Simplify.

Then check to see if __?__ is the solution of the equation $1.2x = 2.4$.

If $x = \underline{?}$, $1.2x = 1.2 \cdot \underline{?}$	Substitute __?__ for x.
$= \underline{?}$	__?__ a solution.

Because the equations have __?__ solutions, they are __?__ equations.

4 $\frac{2}{5}x = 4$ and $x = 10$.

If $x = 10$, $\frac{2}{5}x = \frac{2}{5} \cdot 10$	Substitute 10 for x.
$= \underline{?}$	__?__ a solution.

Because the equations have the __?__ solution, they are __?__ equations.

> **Think Math**
>
> When Judy multiplies both sides of the equation $\frac{1}{4}y - 1 = 2$ by 4, she gets $y - 1 = 8$. What mistake did she make? What is the equation she should have written?

Practice 4.1

Tell whether each pair of equations are equivalent. Give a reason for your answer.

1. $2x = 4$ and $4x + 5 = 13$
2. $-2x + 9 = 7$ and $-2x = 2$
3. $5x - 4 + 3x = 8$ and $8x = 12$
4. $\frac{3}{4}x - 7 = 2$ and $x = 12$

Match each equation with an equivalent equation.

5. $0.5x + 1 = 1.5$
6. $9 + 3.5x = 16$
7. $\frac{4}{5}x = 4$
8. $2x + \frac{1}{2} = \frac{7}{2}$
9. $x - 8.3 = 1.3$
10. $13.9 = 2.5x$
11. $4x = \frac{4}{9}$

a) $6x = 9$

b) $\frac{3}{5}x = \frac{1}{15}$

c) $\frac{3}{2}x = 3$

d) $\frac{2}{3}x = \frac{2}{3}$

e) $2x = 10$

f) $1.2 + x = 6.76$

g) $\frac{1}{2}x = 4.8$

Solve.

12. ***Math Journal*** Max was asked to write an equation equivalent to $\frac{2}{3}x = 3 - x$. He wrote the following:

$$\frac{2}{3}x = 3 - x$$
$$\frac{2}{3}x \cdot 3 = 3 \cdot 3 - x$$
$$2x = 9 - x$$

He concluded that $\frac{2}{3}x = 3 - x$ and $2x = 9 - x$ are equivalent equations. Do you agree with his conclusion? Give a reason for your answer.

OA1-22 Pairs That Add to 5 or 10

Pages 88–89

Standards: 1.OA.C.5, 1.OA.C.6

Goals:
Students will recognize pairs of numbers that add to 5 or 10.
Students will complete addition sentences when one addend is unknown.

Prior Knowledge Required:
Knows that we have 5 fingers on each hand and 10 fingers in total
Understands the concept of equals and the equal sign

Vocabulary: equals (=), in all, **make 5**, **make 10**, **pair**, plus (+)

Materials:
BLM Pairs That Add to 5 (p. D-76)
BLM Five-Dot Dominoes (p. D-77)
BLM Cubes (p. H-11) to make one die per student (see details below)
playing cards
BLM Pairs That Add to 10 (p. D-78)
BLM Ten-Dot Dominoes (p. D-79)

Use your fingers to make numbers less than 10. Tell students to hold up five fingers using both hands or just one hand. Record the pairs students hold up as shown below. Encourage students to find as many pairs as possible.

left		right		total
5	+	0	=	5
3	+	2	=	5
1	+	4	=	5
⋮	+	⋮	=	⋮

Then repeat with at least two of 6, 7, 8, or 9 fingers. Challenge students to find pairs that they cannot show using their hands. Example: Students can't show 6 = 6 + 0 or 7 = 6 + 1 because they do not have six fingers on one hand.

(MP.2, MP.5) Find missing addends (to 5) using the fingers on one hand. Hold up all your fingers on one hand. ASK: How many fingers do I have up? (5) Then hold up three fingers and SAY: How many fingers do I have up? (3) How many are not up? (2) What is 3 + 2? (5) How do you know? (there are 5 fingers on one hand)

Repeat with several examples, including examples where either all or no fingers are held up. Then SAY: I want to know what number with 4 *makes 5*.

Write on the board:

4 + ___ = 5

SAY: How could I use my five fingers? How many fingers should I hold up? (4) What does the number of fingers I'm not holding up tell me? (the missing number) Add labels to the number sentence and have a volunteer fill in the blank:

4 + _1_ = 5

↑ up   ↑ not up

Complete more addition sentences this way, including sentences with 0 as an addend. Then write similar sentences on the board for students to complete individually.

Exercises: Find the missing number.
a) 3 + ___ = 5   b) 4 + ___ = 5   c) 0 + ___ = 5
Bonus:
d) 5 = 2 + ___   e) 5 = 5 + ___   f) 5 = 4 + ___   g) 5 = 0 + ___
Answers: a) 2, b) 1, c) 5, Bonus: d) 3, e) 0, f) 1, g) 5

Give students problems where the first addend is unknown. Examples: ___ + 4 = 5, ___ + 2 = 5.

Bonus: Find the missing number.
a) 5 = ___ + 3   b) 5 = ___ + 2   c) 5 = ___ + 5
Answers: a) 2, b) 3, c) 0

For extra practice, have students work on **BLM Pairs That Add to 5** (2. 4 + 1, 3. 1 + 4, 4. 2 + 3 = 5, 5. 0 + 5) and **BLM Five-Dot Dominoes** (1. 1, 2. 3, 3. 2, 4. 0, 5. 4, 6. 2, 7. 1).

Activities 1–2
1. Use **BLM Cubes** to make dice (one for each student) with top and bottom numbers adding to 5. Guide students to discover the "magic total" on the dice. (opposite sides add to five) Ask pairs of students to roll their dice at the same time. They record a win each time they roll a total of 5. Students can keep score by taking a red counter if they win and a yellow counter if they lose. Then tell students to play with only one die and to add the top and bottom numbers. What do they notice? (They win every time!) Students can use the die to practice finding what makes 5 with the number they roll. (They can check their answers by turning the die over.)

2. **A solitaire game.** Use cards 1 (ace) to 5 from a deck of cards. Shuffle the cards and turn over the first eight, putting them face up in two rows of four cards. Pull a card from the pile—if it makes 5 with any of the cards that are face up, place it on that card; otherwise discard it. Continue until you go through all the cards, then repeat with the cards in the discard pile, adding them, where possible, to any of the piles that are face up. Go through the discard pile as many times as you can, and try to use up all the cards.
(end of activities)

(MP.2, MP.5) Find missing addends to 10 the same way. Teach students to complete addition sentences such as 7 + ____ = 10 by holding up seven fingers and using the fact that they have 10 fingers altogether, so the number of fingers not up goes in the blank.

Exercises: Find the missing number.

a) 4 + __ = 10 b) __ + 5 = 10 c) 7 + __ = 10
d) 3 + __ = 10 e) __ + 2 = 10 f) 9 + __ = 10

Bonus:

g) 10 = 2 + __ h) 10 = __ + 9

Answers: a) 6, b) 5, c) 3, d) 7, e) 8, f) 1, Bonus: g) 8, h) 1

Activities 3–4

3. Repeat Activity 1 with a magic total of 10. Since there are more pairs that *make 10* than will fit on one die, you can make many different dice for students to use and share.

4. Repeat Activity 2, but use the cards from 1 (ace) to 10, and make 10 with the top number drawn instead of 5.
(end of activities)

For extra practice, give students **BLM Pairs That Add to 10** (2. 9 + 1, 3. 5 + 5, 4. 6 + 4, 5. 8 + 2) and **BLM Ten-Dot Dominoes** (1. 3, 2. 5, 3. 8, 4. 6, 5. 1, 6. 2, 7. 4).

Extensions

1. Circle the sentences that are not correct.

5 + 5 = 10 6 + 4 = 10 8 + 3 = 10 9 + 1 = 10 6 + 5 = 10

Answer: 8 + 3 = 10 and 6 + 5 = 10 are not correct.

2. Find the missing number.

a) 3 + 2 + ___ = 10 b) 4 + 2 + ___ = 10
c) 1 + 2 + ___ = 10 **Bonus:** 1 + 1 + 1 + ____ = 10

Answers: a) 5, b) 4, c) 7, Bonus: 7

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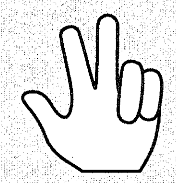
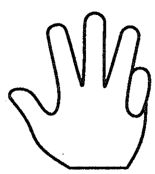
Pairs That Add to 5

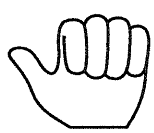
☐ Write the missing numbers.

1. 

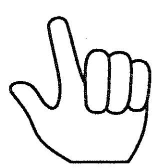
3	+	2	=	5
fingers up		fingers down		in all

2. 

☐	+	☐	=	5
fingers up		finger down		in all

3. 

☐	+	☐	=	5
finger up		fingers down		in all

4. 

☐	+	☐	=	5
fingers up		fingers down		in all

5. 

☐	+	☐	=	5
fingers up		fingers down		in all

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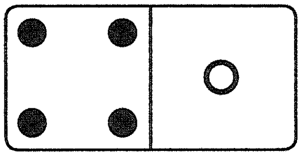
Five-Dot Dominoes

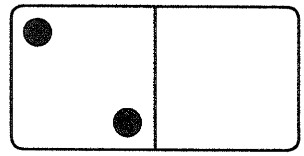
- [ ] Draw the missing dots to make 5.
- [ ] Finish the addition sentence.

1.



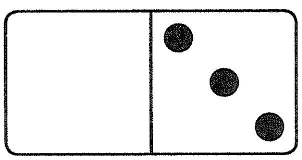
4 + □ = 5

2.



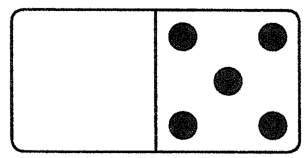
5 = 2 + □

3.



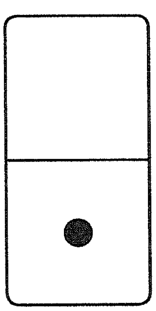
□ + 3 = 5

4.



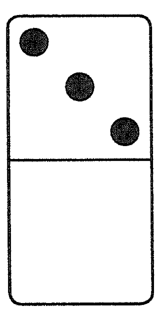
5 = □ + 5

5.



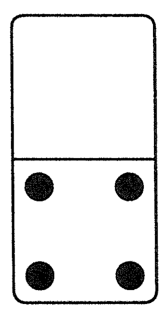
$$\begin{array}{r} \square \\ +\ 1 \\ \hline 5 \end{array}$$

6.



$$\begin{array}{r} 3 \\ +\ \square \\ \hline 5 \end{array}$$

7.



$$\begin{array}{r} \square \\ +\ 4 \\ \hline 5 \end{array}$$

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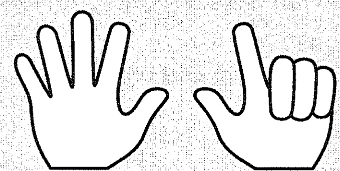
Pairs That Add to 10

☐ Write the missing numbers.

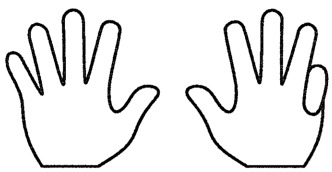
1.



7 + 3 = 10

fingers up + fingers down = in all

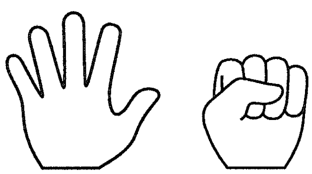
2.



☐ + ☐ = 10

fingers up + finger down = in all

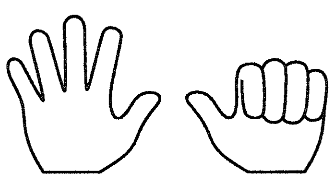
3.



☐ + ☐ = 10

fingers up + fingers down = in all

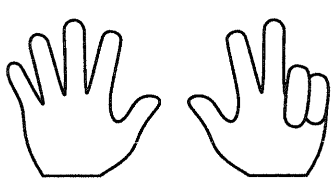
4.



☐ + ☐ = 10

fingers up + fingers down = in all

5.



☐ + ☐ = 10

fingers up + fingers down = in all

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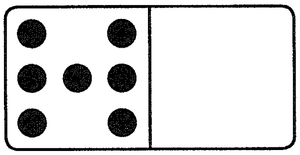
Ten-Dot Dominoes

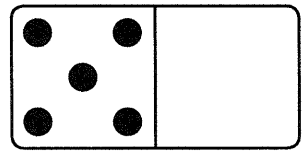
- ☐ Draw the missing dots to make 10.
- ☐ Finish the addition sentence.

1.



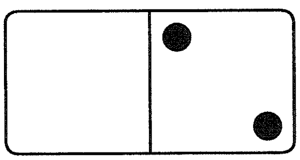
$7 + \square = 10$

2.



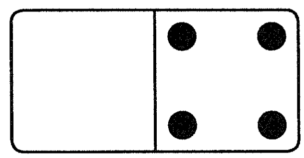
$10 = 5 + \square$

3.



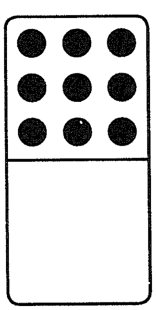
$\square + 2 = 10$

4.



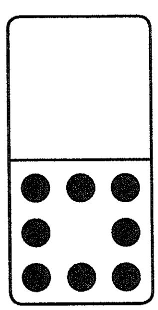
$10 = \square + 4$

5.



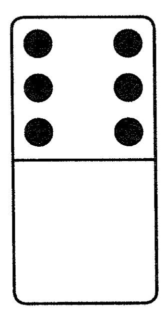
$$\begin{array}{r} 9 \\ + \ \square \\ \hline 10 \end{array}$$

6.



$$\begin{array}{r} \square \\ + \ 8 \\ \hline 10 \end{array}$$

7.



$$\begin{array}{r} 6 \\ + \ \square \\ \hline 10 \end{array}$$

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Grade 2

Grade 2 students build upon their work in Grade 1 in two major ways.[2.OA.1] They represent and solve situational problems of all three types which involve addition and subtraction within 100 rather than within 20, and they represent and solve two-step situational problems of all three types.

Diagrams used in Grade 1 to show how quantities in the situation are related continue to be useful in Grade 2, and students continue to relate the diagrams to situation equations. Such relating helps students rewrite a situation equation like $\square - 38 = 49$ as $49 + 38 = \square$ because they see that the first number in the subtraction equation is the total. Each addition and subtraction equation has seven related equations. Students can write all of these equations, continuing to connect addition and subtraction, and their experience with equations of various forms.

Because there are so many problem situation subtypes, there are many possible ways to combine such subtypes to devise two-step problems. Because some Grade 2 students are still developing proficiency with the most difficult subtypes, two-step problems should not involve these subtypes. Most work with two-step problems should involve single-digit addends.

Most two-step problems made from two easy subtypes are easy to represent with an equation, as shown in the first two examples to the right. But problems involving a comparison or two middle difficulty subtypes may be difficult to represent with a single equation and may be better represented by successive drawings or some combination of a diagram for one step and an equation for the other (see the last three examples). Students can make up any kinds of two-step problems and share them for solving.

The deep extended experiences students have with addition and subtraction in Kindergarten and Grade 1 culminate in Grade 2 with students becoming fluent in single-digit additions and the related subtractions using the mental Level 2 and 3 strategies as needed.[2.OA.2] So fluency in adding and subtracting single-digit numbers has progressed from numbers within 5 in Kindergarten to within 10 in Grade 1 to within 20 in Grade 2. The methods have also become more advanced.

The word *fluent* is used in the Standards to mean "fast and accurate." Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., "adding 0 yields the same number"), and knowing some answers from the use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students. The extensive work relating addition and subtraction means that subtraction can frequently be solved by thinking of the related addition, especially for smaller numbers. It is also important that these patterns, strategies and decomposi-

[2.OA.1] Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Related addition and subtraction equations

$87 - 38 = 49$	$87 - 49 = 38$	$38 + 49 = 87$	$49 + 38 = 87$
$49 = 87 - 38$	$38 = 87 - 49$	$87 = 38 + 49$	$87 = 49 + 38$

Examples of two-step Grade 2 word problems

Two easy subtypes with the same operation, resulting in problems represented as, for example, $9 + 5 + 7 = \square$ or $16 - 8 - 5 = \square$ and perhaps by drawings showing these steps:

> Example for $9 + 5 + 7$: There were 9 blue balls and 5 red balls in the bag. Aki put in 7 more balls. How many balls are in the bag altogether?

Two easy subtypes with opposite operations, resulting in problems represented as, for example, $9 - 5 + 7 = \square$ or $16 + 8 - 5 = \square$ and perhaps by drawings showing these steps:

> Example for $9 - 5 + 7$: There were 9 carrots on the plate. The girls ate 5 carrots. Mother put 7 more carrots on the plate. How many carrots are there now?

One easy and one middle difficulty subtype:

> For example: Maria has 9 apples. Corey has 4 fewer apples than Maria. How many apples do they have in all?

> For example: The zoo had 7 cows and some horses in the big pen. There were 15 animals in the big pen. Then 4 more horses ran into the big pen. How many horses are there now?

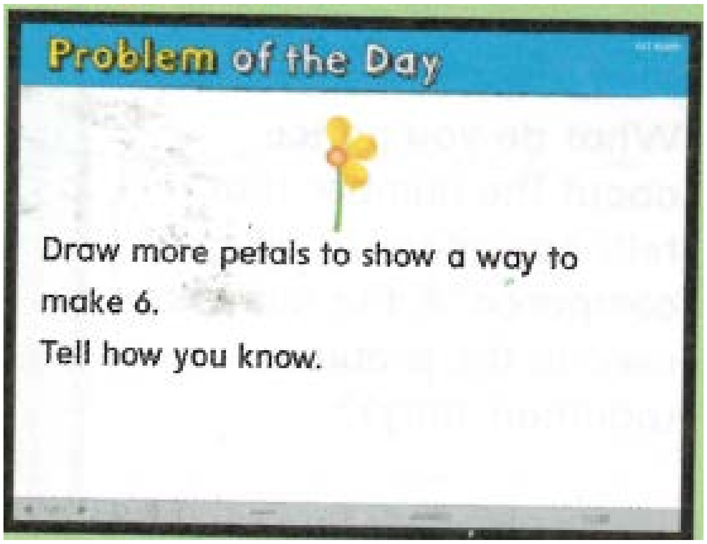
Two middle difficulty subtypes:

> For example: There were 9 boys and some girls in the park. In all, 15 children were in the park. Then some more girls came. Now there are 14 girls in the park. How many more girls came to the park?

[2.OA.2] Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

MP.7: Look for and make use of structure.

Kindergarten example:



Review

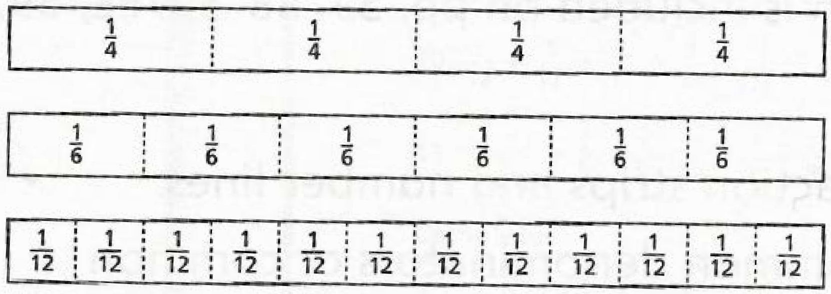
Problem of the Day

Draw more petals to show a way to make 6.
Tell how you know.
See students' work.

MP.7 **Use Structure** *e wanted to show a way to make seven petals on the flower, how many more petals would we draw on the flower. How do you know?* Sample answer: four; I know because seven is one more than six. Since drawing three petals made six, drawing one more than three would make seven.

Grade 4:

2. Use the fraction strips to compare the fractions $\frac{3}{4}$ and $\frac{5}{6}$.



$\frac{3}{4} < \frac{5}{6}$

▶ Discuss Common Denominators WHOLE CLASS

MP.7 Look for Structure Have students look at the fraction strips they used in Exercise 2. Ask them to think about how the denominators 4, 6, and 12 are related. Use the discussion below as a guide to help students start thinking about how equivalent fractions can help them compare two fractions with different numerators and different denominators.

MATH TALK in ACTION Students discuss how to use equivalent fractions to compare fractions with unlike numerators and unlike denominators.

Can you model the fractions $\frac{3}{4}$ and $\frac{5}{6}$ using the twelfths fraction strip?

Louisa: When I shade $\frac{3}{4}$, I can see that it is the same as $\frac{9}{12}$. When I shade $\frac{5}{6}$, I can see that it is the same as $\frac{10}{12}$. So, I can use the twelfths strip to model either fraction.

Leo: I didn't shade the strips. I can use multiplication to write $\frac{3}{4}$ as an equivalent fraction with 12 as the denominator: $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$. Then, I can write $\frac{5}{6}$ as an equivalent fraction with 12 as the denominator: $\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$.

Niral: When both fractions have the same denominator, I can compare the number of unit fractions to compare fractions: there are nine $\frac{1}{12}$ unit fractions in $\frac{9}{12}$ and there are ten $\frac{1}{12}$ fractions in $\frac{10}{12}$. $9 < 10$, so $\frac{9}{12} < \frac{10}{12}$.

Veasna: Then we can write the comparison: $\frac{3}{4} < \frac{5}{6}$.

Think of the other problems you solved on Student Book page 245. Can you solve them without using fraction strips or number lines?

Amadi: I can use multiplication to write equivalent fractions. As long as I can figure out what number to use as the denominator, I can write the two fractions as equivalent fractions with the same denominator.

Students can explore what denominator they would use for each pair of fractions. In the next activity, they will discuss the different situations they will encounter when finding common denominators.

Grade 7:

Problem 2.4

A For each part, choose values for a and b. Substitute those values into the three relationships below.

$$a + b = c \qquad a = c - b \qquad b = c - a$$

Then find the value of c.

1. a and b are positive rational numbers.
2. a and b are negative rational numbers.
3. a is a positive rational number, and b is a negative rational number.
4. a is a negative rational number, and b is a positive rational number.

For Parts B–F, use fact families to answer each question.

B Write a related subtraction sentence for each.

1. $-3 + (-2) = -5$
2. $25 + (-32) = -7$

C Write a related addition sentence for each.

1. $8 - (-2) = 10$
2. $-14 - (-20) = 6$

D 1. Write a related sentence for each.

a. $n - 5 = 35$ b. $n - (-5) = 35$ c. $n + 5 = 35$

2. Do your related sentences make it easier to find the value of n? Why or why not?

E 1. Write a related sentence for each.

a. $4 + n = 43$ b. $-4 + n = 43$ c. $-4 + n = -43$

2. Do your related sentences make it easier to find the value of n? Why or why not?

Practice 7: Look for and make use of structure.

In Problem 2.4, students examine the structure of fact families as they rewrite addition sentences as subtraction sentences and subtraction sentences as addition sentences. Students then determine which number sentence within a fact family makes it easiest to find the value of a missing number.

Grade 2: Non-example

5. Writing in Math If you add 2 even numbers, will the sum be odd or even? Tell how you know. Use numbers, pictures, or words.

See student samples

Exercise 5

Writing to Explain Children should tell if the sum of 2 even numbers will be odd or even and explain how they got the answer using numbers, pictures, or words.

Student Samples

3-point answer The answer and explanation are correct and complete.

4 + 4 = 8, 2 + 6 = 8, 2 + 4 = 6; The sum of 2 even numbers is even. I added 3 addition sentences. The sums are all even.

2-point answer The answer is correct, but the explanation is incomplete or incorrect.

2 + 2 = 4; even

1-point answer The answer is incorrect. The explanation is incomplete.

3 + 4 = 7; odd

Grade 2: Example

DISCUSSION

1 Adding Evens

10 MIN CLASS

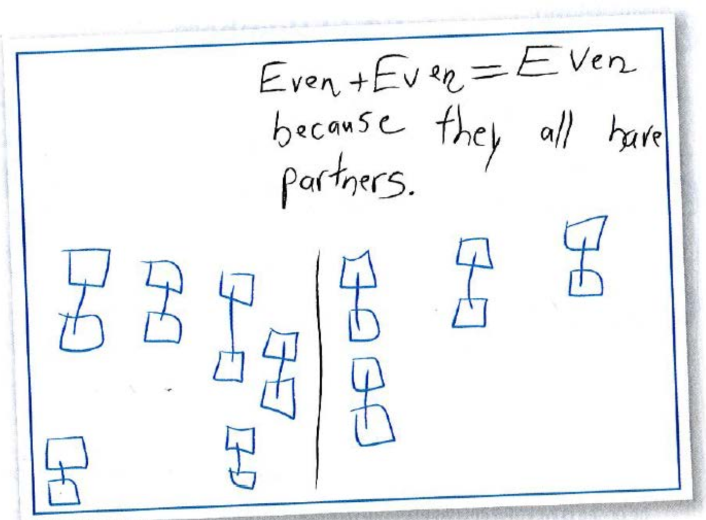
Math Focus Points for Discussion

- Making and justifying generalizations about adding even numbers

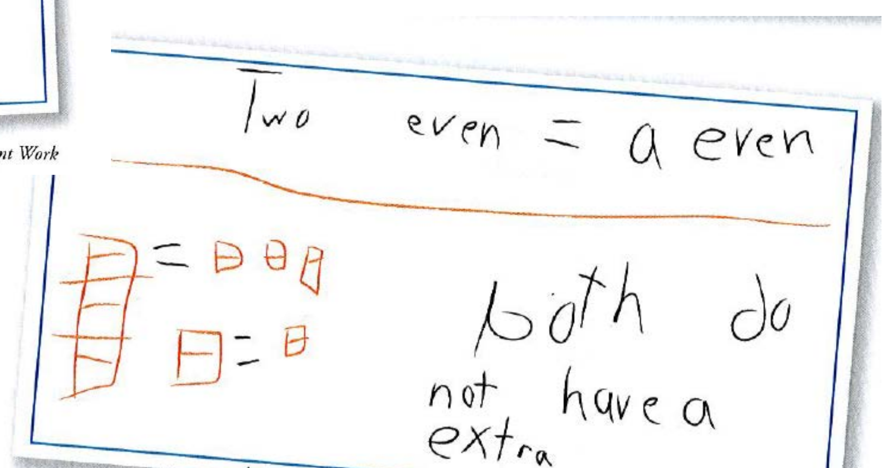
What did you find out about adding two even numbers? Why do you think that two even numbers make an even number?

Again, use two cube towers to ground the discussion in a particular example, but push students to consider whether their arguments apply to *all* even numbers and to explain why they think so.

Do you think that this is true for *all* numbers? In other words, when added together, will *any* two even numbers *always* equal an even number? Why do you think so? How would you show or prove it?



Sample Student Work



Sample Student Work