Human-Sized Number Lines: Let's Compare Fractions!

This lesson was adapted from Lesson 7 in Cole, S., Heilbronner, N., Gubbins, E. J., Corbishley, J., Savino, J., & McAnallen, R. (2009). *Geometry & measurement for all shapes & sizes* (2nd Ed.). Storrs, CT: The National Research Center on the Gifted and Talented, University of Connecticut. Annotation by Student Achievement Partners.

GRADE LEVEL Fourth

IN THE STANDARDS 4.NF.A.2 (4.MD.A, 4.MD.A.2)

WHAT WE LIKE ABOUT THIS Lesson

Mathematically:

- Allows students to see fraction equivalence with a large visual model.
- Promotes coherence by highlighting prior knowledge and pointing to the mathematics that will be built from these ideas.
- Encourages students to use appropriate tools strategically (MP5) (see additional thoughts below).
- Requires students' use of precise grade-appropriate mathematical language (MP6).
- Reinforces that fractional comparisons are valid only when referring to the same whole.
- Uses fractions greater than 1 to support students' understanding of fractions as numbers (e.g., 3/2, 5/3).

In the classroom:

- Captures student attention by using an engaging context.
- Gives formal and informal opportunities for teachers to check for understanding.
- Provides robust opportunities for students to discuss mathematical concepts; includes guiding questions for teachers to use to facilitate discussion.

For English Language Learners:
This lesson was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practices in the following ways:

- Discussions allow for the opportunity to point out metacognitive strategies.
- Instruction and tasks allow for multimodal representations.
- Lesson provides opportunities for students to practice and refine their use of mathematical language.
- Lesson provides opportunities for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.

MAKING THE SHIFTS[1]

Focus	Belongs to the Major Work[2] of fourth grade
Coherence	Builds on key understandings of fractions as numbers (3.NF.A), equivalence and comparing fractions with same numerators and denominators from grade 3.
Rigor[3]	Conceptual Understanding: Primary in this lesson
	Procedural Skill and Fluency: Not addressed in this lesson
	Application: Not addressed in this lesson

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Four in the Supplemental Resources below.
[3]Lessons may target one or more aspect(s) of rigor.

ADDITIONAL THOUGHTS

This lesson would best fit early in the fraction unit of fourth grade. It is not intended for students to meet the full expectations of the targeted grade-level standards through only this selected lesson. The content in the lesson builds on grade 3 work of comparing fractions with same numerators and denominators and extends to comparing fractions with unlike numerators and denominators. The lesson lays a strong foundation for students to extend fraction equivalence and ordering from grade 3 and builds on students' understanding of fractions as numbers. The use of the number line supports students in their continued development of fraction understanding. Solving problems posed involving measurement and representing length by using the number line supports coherence within the lesson by connecting supporting work to the Major Work of the lesson.

The format of the lesson has some interesting aspects to highlight. It is highly engaging and active, and is meant to spark conversation between teacher and students as well as students with one another. It is important to note that within the individual lesson the selection of lengths are deliberate and purposeful. Not only are grade level content constraints adhered to, but also careful attention is paid to the number line length (8 feet for length of 4/2 , 48 inches for length of 2/2) and denominators (2, 3, 4, 6, 8, 12). It is important that the teacher checks student partitions to ensure student number lines are precise enough to move forward with the lesson.

This lesson allows students to use their understanding of fraction equivalence to compare fractions with unlike numerators and unlike denominators, and students can rewrite fractions to show equivalence using visual models. This individual lesson encourages students to use appropriate tools strategically in order to partition their number lines as closely as possible to the exact location (MP5). Teacher questions allow for student responses that attend to the precise language of fraction comparison and equivalence (MP 6). For more insight on the grade-level concepts addressed in this lesson, read pages 6-7 of the progression document, *Number and Operations- Fractions, 3-5*, available at http://www.achievethecore.org/progressions.

For a direct link, go to: http://achievethecore.org/page/2850/human-sized-number-lines-let-s-compare-fractions

SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Prior to this lesson, ensure students have had multiple opportunities to read, write, speak, listen for, and understand the mathematical concepts that are represented by these key vocabulary words:

- Fraction
- Numerator
- Denominator
- Number line
- Halves
- Equal
- Whole
- Greater than
- Less than
- Partition

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work (for example, through engagement in mathematical routines).

For a direct link, go to: http://achievethecore.org/page/2850/human-sized-number-lines-let-s-compare-fractions

Human-Sized Number Lines: Let's Compare Fractions!

Big Mathematical Ideas

Reasoning about fractions can be difficult because meanings are highly context-dependent. For example, half of an inch conjures up a different image than half of a cookie, yet both represent some whole that has been partitioned into equal parts. Deep understanding of fractions begins in grade 3 with the overarching goal that students understand that fractions are numbers. Grade 3 work of comparing fractions with the same numerator and denominator extends in grade 4 to comparing fractions with unlike numerators and denominators.

Lesson Objectives

- Students will partition number lines to represent a given denominator.
- Students will be able to compare fractions with different numerators and denominators using differently partitioned number lines to justify their comparisons.

Lesson Preview

Students partition human-sized number lines into equal parts based on a given denominator. They use the number lines to justify comparisons of fractions with different numerators and denominators.

Standards Addressed

Focal Standard(s):

4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Supporting Standard(s):

4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Materials

- Student Page—Fractional Paths [Student Mathematician Journal, page 65]
- Colored painter's tape or masking tape (1/4 inch wide or less works best)
- Measuring tape
- Sticky notes
- Rulers
- Large counters or small figurines

Lesson Set Up

On the classroom floor (or outdoors on a blacktop or in a hallway) set up six "number line segments." Each number line segment will be 8 feet long. Use the following instructions to create the six segments.

Lesson Set Up Flexibility Suggestions

- Depending on space constraints, adjust lengths of number line segments as necessary [4 feet, 8 feet, and 12 feet work best for keeping the partitioning within the grade level standards]. Longer segments allow students to physically walk on the number lines, which can increase student engagement with the actual lesson.
- For classes with more than 20 students, teachers may want to set up two sets of number line segments to make the group sizes more manageable.

Instructions:

1. Lay out a piece of blue painter's tape 8 feet in length (the total length will represent the distance from 0 to 2 on the number line).
2. Put a piece of tape on each end perpendicular to the first as shown in Figure 1, measuring from the center of each piece.
3. Put another piece of tape to mark at 4 feet.
4. Repeat steps 1-3 FIVE more times, so that you have 6 equal length segments. The 0, 1, and 2 should be lined up to show the same starting point on each number line. Leave enough distance in between number lines to give students room to work.
5. Label the mark on the left "0," the mark in the center "1," and the mark on the right "2.

Initiate

Gather students around the six number lines. Ask what they notice and wonder about the number line segments.

For English Language Learners

Have students use a Notice and Wonder protocol to dive into the set-up of the number lines. Allow students to independently think about or write what they notice and wonder regarding the six number lines. Then in groups of 3 or 4, have students share their observations while one student creates a record:

I NOTICE....	I WONDER....

In the full group, have one student from each group (preferably not the recorder) read aloud one observation that has not been shared by another group. Each observation should begin with the phrase 'I notice...' or "I wonder..." Continue until there are no new observations.

Sample Responses: They all show 0, 1 & 2; they are the same length from 0 to 2; 0, 1, and 2 are all in the same position (lined up). Students may wonder why there are 6 numbers lines, why all 6 are lined up, how long each number line is, or what is going on in the space between 0 and 1 and 1and 2.

Tell students that today you will be partitioning the number lines to create equal length markings that will help locate some other numbers between 0 and 2. Ask students to name some numbers between 0 and 2 other than 1. If possible, record the student responses and pause on each one so all students can connect the name of the number to its numeral.

> Sample responses: Any fraction or decimal greater than 0 and less than 2. Some examples might include 1/2, 0.7, or 5/4.

Tell students that the class will work together to partition the first number line into halves, or to show fractions with denominator 2. Then small groups of students will be assigned a denominator and asked to partition the other five number lines to show fractions with a given denominator.

Ask: If we want to show fractions on the first number line with denominator 2 that from 0 to 2, what fractions might we show? Allow for individual think time and pair-and-share so all students have an opportunity to process and name the fractions between 0 and 2 with denominator 2.

> Sample responses: 0/2, 1/2, 2/2, 3/2, and 4/2

Ask students to stand on the number line to show where they think the fractions 1/2, 2/2, 3/2, and 4/2 are [you might do this in a slightly different order]. Then ask: How can we find the location of 1/2 or 3/2 more precisely or more closely than estimating?

Let students make suggestions about a process that can be used. If they make suggestions that do not include measuring tools, encourage them to think of ways that will get them to the exact location as closely as possible. Then ask some students to implement the process to mark off and label 1/2.

Use similar discussion to help students locate and label 2/2, 3/2, and 4/2 on the number line.

Key discussion points:

- The total distance from 0 to 1 is 4 feet so 1/2 should be at 2 feet OR the distance from 0 to 1 is 48 inches so the number 1/2 should be halfway to 48 inches, or 24 inches.
- The distance from 1 to 3/2 should also be 2 feet, or 48 inches.
- The number 2/2 is located at 1 and the number 4/2 is located at 2. [Note: This is an understanding first developed in grade 3.]

Investigate

Partitioning Number Lines
Assign groups of students to the remaining five number lines. The differently partitioned number lines should go in order by increasing denominator starting with the one partitioned into halves already.

Denominators: 3, 4, 6, 8, 12

Students should partition the number line to show the correct number of equal length segments between 0 and 1 and between 1 and 2.

> A misconception that may arise is thinking that they are partitioning the distance from 0 to 2 instead of from 0 to 1 and 1 to 2. It also may be helpful to remind students of the fractional representation of 1. For example, since 6/6 is equal to 1, students will need to show 1/6, 2/6, 3/6, 4/6 and 5/6 between 0 and 1. Other good probing questions to ask the small groups is “What will

be the first fraction after 0 with your denominator? What will be the first fraction after 1 with your denominator?"

Check the number lines for accuracy before proceeding.

Exploring
Ask one member of each group to place their group's counter (or other figure) on the first mark after 0. Be sure somebody does this for the line partitioned into halves also. Ask students what they notice about the location of the counters across all of the number lines.

For English Language Learners: Repeat the Notice and Wonder protocol from above for what students notice and wonder about the location of the counters.

Students should notice that the counter on the line partitioned into halves is much farther from 0 than the one on the line partitioned into 12ths. Pretend to be shocked and tell students how surprised you are to see that the line partitioned into 12 equal parts from 0 to 1 has a counter that is so much closer to zero than on the one partitioned into only 2 equal parts. Ask students to explain this and then have each group use a sticky note or index card to label the first mark to show the unit fraction for their group's number line. Do a couple of simple comparisons of unit fractions, such as 1/6 and 1/12.

Ask students to use the sticky notes to label all the remaining marks on their number lines, including those already labeled with 0 and 1.

Give students one of the two "Fractional Number Line Segments" assignments to complete. You may choose to assign them to the page that includes the specific path they were partitioning or to mix it up since this part of the lesson requires students to look across different paths to make comparisons.

After the Lesson

There is an important statement in grade 3, standard 3.NF.A.3d: "Recognize that comparisons are valid only when the two fractions refer to the same whole." To ensure that this point is not lost in this grade 4 lesson, close with the following task:

On the board, draw 2 segments: one that is 1/2 foot long and one that is 1/2 yard long. Label the segments with the lengths 2/4 foot and 4/8 yard.

Step back and look confused.

- Say, "I'm wondering a few things as I look at these two segments. I'm wondering if I drew them incorrectly because I know 2/4 is equal to 4/8, so the segments should be the same length."
- Pause and think some more.
- Say, "Or maybe these are drawn correctly. Maybe since 4/8 is greater than 2/4, that one is supposed to be longer."
- Pause and think some more.
- Say "Or maybe something else is happening here."

Pose this question to the students: What is the reason the 4/8 segment is longer than the 2/4 segment? Explain.

For English Language Learners: Use the Stronger, Clearer Each Time Mathematical Language Routine.

- Before talking to anyone, students write down their initial ideas
- Students go to first partner (this could be done in a random way or in a structured way, for example, with the class divided into 2 groups and each group having one circle inside another circle facing each other.)
- Students share their ideas with their first partner. Ideally this is done orally. Students ask each other questions and give feedback.
- Students then go to a new partner and repeat the process.
- Students return to their seat and revise their explanation.
- If time permits, students can share with a new partner, focusing on getting the explanation stronger and clearer.

Fractional Number Line Segments

Student Mathematician: _____

Use the number line segments that show denominators 2, 3, 4, and 6 to complete this work.

1.

a. Write a fraction shown on another number line segment that is equal to the given fraction.

1/3

3/6

6/4

b. Explain with words or pictures how you used the number line segments to find a fraction equal to 6/4.

2.

a. Decide which number from each pair is greater. Circle the greater number. Write a comparison statement, such as $\frac{1}{2} > \frac{2}{6}$.

		Comparison Statement
$\frac{1}{2}$	$\frac{3}{4}$	
$\frac{2}{3}$	$\frac{5}{6}$	
$\frac{7}{4}$	$\frac{3}{2}$	
$\frac{7}{6}$	$\frac{3}{2}$	

b. Choose one of the comparison statements you wrote. Explain how you used the number line to tell which number in the pair is greater.

Fractional Number Line Segments

Student Mathematician: _____

Use the number line segments that show denominators 6, 8, and 12 to complete this work.

1.

a. Name a fraction shown on another number line segment that is equal to the given fraction.

1/6

8/6

$18/12$

b. Explain with words or pictures how you used the number line segment to find a fraction equal to 8/6.

2.

a. Decide which number from each pair is greater. Circle the greater number. Write a comparison statement, such as $\frac{1}{2} > \frac{2}{6}$.

		Comparison Statement
$\frac{3}{6}$	$\frac{7}{12}$	
$\frac{10}{12}$	$\frac{5}{8}$	
$\frac{10}{6}$	$\frac{16}{12}$	

b. Choose one of the comparison statements you wrote. Explain how you used the number line to tell which number in the pair is greater.