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The Development and Design of the Common Core State Standards for Mathematics

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As one of the lead writers of the Common Core State Standards for Mathematics, I begin by explaining what the standards are, what they are not, and how they were developed. Then I detail some ways in which the standards differ from previous state standards. Finally, I describe some of the developments I have seen in the implementation of the standards and the key developments I would like to see in the future.

What Are Academic Standards?

I should begin by saying what academic standards are not. Standards are not textbooks. Standards are not tests. They are, fundamentally, lists. A standards document is a list of learning goals. For the past twenty years or so, states have used these lists to coordinate among various functions of education, most notably to coordinate among curriculum, assessment, and instruction.[1]

A list sounds like a humble thing, but lists are important. Making a list forces decisions about what to include and exclude. A list can be exhausting and unrealistic about the time it takes to get things done, or it can concentrate everyone's efforts on what is essential (in math, this is called *focus*). A list can be fragmentary or it can show how things fit together (in math, this is called *coherence*). Standards thus serve an intellectual function by construing or defining an academic discipline as it is to exist within the schools.

In addition to these traditional purposes, the governors and state superintendents who joined together in 2009 to create the Common Core State Standards wanted to adopt standards that could serve new purposes, principally, fostering college and career readiness and setting a globally competitive standard.

These two goals have implications for the overall shape of the standards. It turns out that qualifying for credit-bearing courses in two-year and four-year postsecondary institutions requires a thorough knowledge of algebra. And in countries like Singapore, large fractions of students learn a great deal of algebra. Altogether then, once you set yourself the goal of globally competitive expectations plus college and career readiness, you have signed on for a substantial amount of algebra. In what follows, therefore, the reader will notice a persistent theme of arithmetic, algebra, and the connections between them.

Jason Zimba, formerly a mathematical physicist, was a lead author of the Common Core State Standards for Mathematics and is a founding partner of Student Achievement Partners, a nonprofit organization.

How Were the Standards Developed?

Preparation for Developing the Common Core State Standards

In 2007, state superintendents attending a meeting of the Council of Chief State School Officers discussed the possibility of developing common standards at the state level. By 2009, the National Governors Association had joined the effort and the governors and superintendents of forty-eight states and territories had signed a memorandum to develop math and English language arts/literacy standards in common.

In April of that year, a preliminary drafting committee was assembled, drawing primarily on experts affiliated with ACT, the College Board, and Achieve, because all three organizations had conducted and published research about college readiness or career readiness, all had produced standards or standards-like documents for college or career readiness based on this research, and all had already worked across state lines.[2] For example, Achieve's American Diploma Project had already brought thirty-five states together to begin developing work shared math and literacy standards.

In September 2009, the preliminary committee (of which I was a member) presented a document for review and feedback by states and members of the public. This document, entitled "College and Career Readiness Standards," was not a set of grade-by-grade standards; it was a draft list of mathematical knowledge and skills required for college and career readiness. At this point the preliminary committee had served its purpose and was dissolved. A new, much larger group was then assembled to develop the Common Core State Standards.

Development of the Common Core State Standards

In September 2009, the Council of Chief State School Officers and the National Governors Association assembled several committees to develop the Common Core State Standards: two working groups, two feedback committees, and a validation committee. There were fifty-one people in the Mathematics Work Team, twenty-two people in the Mathematics Feedback Group, and twenty-nine people on the Validation Committee.[3] The project lead for mathematics was William McCallum, a mathematician and University Distinguished Professor at the University of Arizona.

Of the seventy-three math committee members, two were affiliated with testing organizations—one with ACT, and one with the College Board. Both national teachers unions were represented on the committees. The committee members included university mathematicians, current and former math teachers, math education researchers, among whom were experts in early childhood education, and state education leaders.

At the center of the working group for mathematics was the three-member writing team, of which I was a part, along with Phil Daro, a longtime math educator, and William McCallum. My fellow writers are leading experts in math education: William McCallum was honored by the American Mathematical Society in 2010 for his contributions to math education, and Phil Daro received the 2014 Taylor/Gilbert award from the National Council of Supervisors of Mathematics.

Elsewhere, I have described the writing team's role as "certainly about writing and taking first cuts at things. But it was even more about reading, listening, revising, and finding ways to problem solve and reconcile all the different signals. During this process, we went back to the evidence continually."[4]

The reason for the final remark about evidence is that the working group for the Common Core was charged with using research and evidence in developing the standards. Some of the works consulted can be seen on pages 91–93 of the standards document.[5] These works include standards documents from high-performing countries, previous state standards documents, major national reports, such as *Foundations for Success* and *Mathematics Learning in Early Childhood*, published research about math education, and research about college and career readiness.[6]

Under the leadership of Chris Minnich, Dane Linn, and especially Gene Wilhoit, two organizations, the Council of Chief State School Officers and the National Governors Association, represented the states and territories that were participating in the development process. The two organizations delivered drafts periodically to state education agencies and collected feedback for revisions. They also solicited comments on the evolving drafts from experts within educator organizations, such as the National Council of Teachers of Mathematics.

On March 10, 2010, the public draft of the standards was released. Thousands of comments were gathered during the ensuing feedback period from members of the public as well as from teachers, researchers, other experts, and educator organizations, such as the National Association for the Education for Young Children. These comments led to significant improvements, and the Common Core State Standards were released on June 2, 2010.[7]

Today, politicians fight over the standards but educators and experts broadly support them. The presidents of every major mathematical society in the United States strongly support the Common Core.[8] Closer to the classroom, the most recent survey I have seen is a nonrepresentative survey of eighteen hundred district superintendents conducted by Gallup in May 2014.[9] Two-thirds said that the standards would improve the quality of education in their community.

Some additional things to know about the standards:

- **The standards do not determine graduation policies**. Some of the states that have adopted the standards require Algebra II for graduation; other states that have adopted the standards require less.
- **The standards do not attempt to specify all four years of high school math**. Each state chooses whether to add its own standards for calculus (as California has done) or leave it to local districts (as Massachusetts always did and still does). Either way, high school students do not have to stop at Algebra II.[10]

How Are the Standards Different from Previous State Standards?

The most important difference between the Common Core and previous state standards is that the Common Core State Standards rededicate the elementary grades to arithmetic.

Before the Common Core, arithmetic was one among many subjects in the elementary math curriculum. Teachers and students were also required to spend a lot of time on other things: shapes, probability, statistics—all of it portrayed as being equally important, despite the fact that arithmetic is much higher stakes for children and leads directly to algebra. In states that have adopted the standards, arithmetic has taken its rightful place as the essential task of elementary school math.

- Grades K–2 are a master class in addition and subtraction—concepts, skills and problem solving.

- Grades 3–5 turn to multiplication and division of whole numbers and fractions—concepts, skills, and problems solving.
- There is very little data work in grades K–5, and what is there is tightly coordinated with arithmetic developments in number systems, the number line, and problem solving using the four operations.
- Statistics (distributions, outliers, measures of center and variation) waits until grade 6.
- "Advanced" geometry (congruence and transformations) also waits until middle school.

The standards' rededication of the elementary years to arithmetic is consistent with the practice in high-performing countries. (For example, a 2005 textbook study notes, "The only statistics content in the Singapore grade 5 textbooks and workbooks is a single lesson on line graphs.")[11]

In a world overflowing with data, statistics is more important than ever. But there is an order of operations to be followed. Students who cannot do arithmetic cannot do statistics either. Conversely, arithmetic empowers those who master it—they can then use ratios and percentages flexibly, do back-of-the-envelope estimates, and be quantitative in matters affecting their work, life, and citizenship. Arithmetic is like the handle of a wrench: grasped firmly, it gives you leverage.

The standards' strong emphasis on arithmetic in the elementary grades follows longstanding domestic recommendations and conclusions drawn from Trends in International Mathematics and Science Study (TIMSS) and other international studies. See Figure 1.

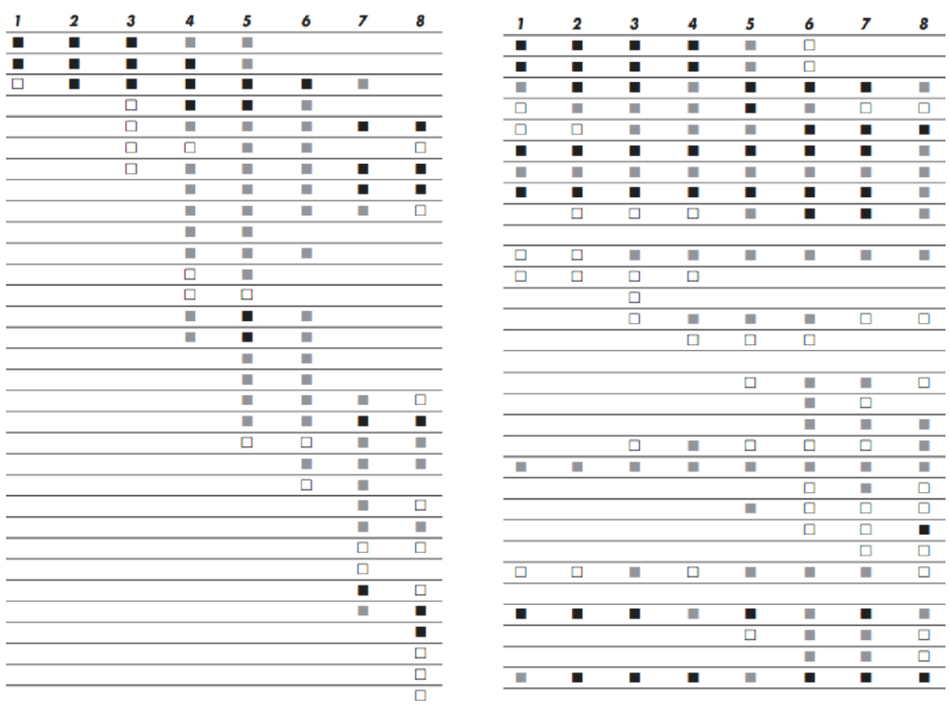


Figure 1. The shape of math in A+ countries compared with the United States before the Common Core. In both diagrams, grade levels 1–8 run horizontally and math topics (not named) run vertically, with elementary topics, such as whole numbers, at the top and advanced topics, such as functions, at the bottom. **Left diagram**: Mathematics topics intended at each grade by at least two-thirds of A+ countries. **Right diagram**: Mathematics topics intended at each grade by at least two-thirds of 21 U.S. states. Open squares denote two-thirds of countries or states; gray squares denote 83% of countries or states; and black

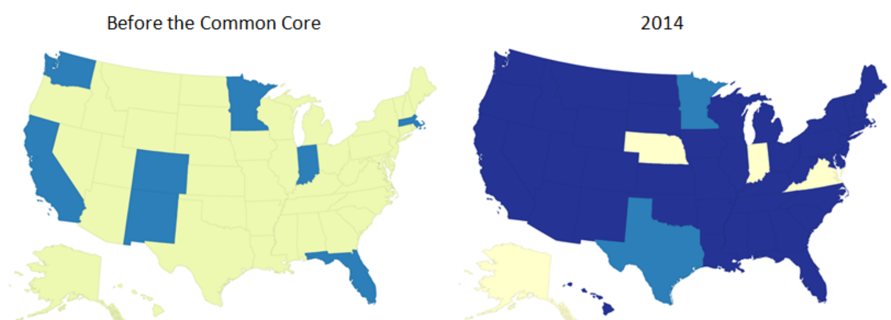
squares denote 100% of countries or states. (William Schmidt, Richard Houang, and Leland Cogan, "A Coherent Curriculum: The Case of Mathematics," *American Educator*, Summer 2002, figs. 1 and 2.)

A second difference between the Common Core and previous state standards is the agreement between the Common Core and the standards of high-performing countries. Critics of the standards make unscientific claims that the standards are "two years behind" high-performing countries. But peer-reviewed research by a leading expert on international mathematics performance has compared the grades and topics in the Common Core to high-performing countries in grades 1–8. The agreement with high-performing countries was very high. Moreover, no state's previous standards were as close a match to the high-performing countries as the Common Core.[12]

The same study also found that states whose previous standards more closely matched the Common Core tended to have higher National Assessment of Educational Progress (NAEP) scores.

A third difference between the Common Core and previous state standards is that the Common Core standards more accurately portray excellence in math as a combination of three things: mastery of procedures, understanding of math concepts, and the ability to apply math to solve problems. In what follows I discuss mastery of procedures and understanding of math concepts. (Applying math is also important because students will need to use the math they are learning to solve problems in everyday situations as well as in science or technical courses. Applications also add interest to the subject; a rigorous math curriculum for children does not have to be arid, nor should it be.)[13]

Mastery of procedures. You are not excellent in math unless you can get the right answer without hesitation. The standards require students to know the addition and multiplication facts from memory. No standards in the Common Core require students to invent algorithms. The standards require fluency with the standard algorithm for each of the four basic operations with whole numbers and decimals. No set of previous state standards ever matched the Common Core's expectations for the standard algorithms. See Figure 2.



KEY:

Dark Blue: Standard algorithm explicitly required for all four operations, and fluency with the standard algorithm explicitly expected for all four operations.

Light Blue: Standard algorithm explicitly required for all four operations, but fluency is not expected for all four operations, and/or the number of digits is limited to one, two, or three.

Green (left map), Yellow (right map): Standard algorithm not explicitly required for all four operations.

Figure 2. The standard algorithm in state standards, before and after the common core. (Author's provisional determinations based on Thomas B. Fordham Institute, *The State of State Standards (and the Common Core) in 2010* [Washington, DC: Author, 2010] and selected state standards documents.)

As Figure 2 shows, before the Common Core, no state in the United States explicitly required fluency with the standard algorithm for all four basic operations. (See map on the left. The seven states shown in light blue approached this condition more closely than did the other forty-three.) Today, because of the Common Core, most states explicitly require fluency with the standard algorithm for all four basic operations. (See map on the right, dark blue states.) Note Indiana's backward trajectory: when the state rescinded its adoption of the Common Core and revised the standards, it made relatively few changes—but one of them was to weaken the expectations for standard algorithms.

Understanding of concepts. The concepts of arithmetic are a training ground for algebra. Interestingly, it turns out that high-performing countries do not teach very much algebra to elementary students. That might seem counterintuitive; are those countries not more advanced than we are? Yet algebra accounts for 0 percent of the content in Hong Kong's primary 1–3 and 4–6 levels.[14] What these countries do instead is teach arithmetic in such a way as to prepare students for algebra. That means thinking about how numbers work, so that you have a base to build on when numbers get replaced by letters in middle school. The need to produce students who can fluently compute 5,644 + 1,878 lives alongside the mathematical reality that when you add *xy* to *yz*, you do not carry the 1.

Concepts matter because students who cannot think mathematically will typically sooner or later forget how to solve problems they once knew how to solve. So it is important for the sake of math achievement to address concepts in adequate depth.

It is important to have the right balance and interplay between concepts and procedures. Consider how both are present in this summary of Singapore textbooks:

- The first Singapore lesson [on addition], "Making Addition Stories," uses pictures and number stories to show that addition means "putting together." The lesson asks the student to create stories to go along with pictures that illustrate addition facts and write corresponding number sentences.
- The second lesson deepens understanding of addition facts, introducing the concept of "number bonds" (fact families) and illustrating the commutative property of addition.
- The third lesson, "Other Methods of Addition," illustrates other ways of thinking about addition, such as counting on and making 10, while also reinforcing addition facts.
- Exercises in the workbook use games and pictures to reiterate the different ways of thinking about addition illustrated in the lessons and to provide plenty of practice in solving problems using basic addition facts.[15]

In this country, the Left often disdains workbooks and repetitive practice, while the Right often disdains picture drawing and diverse approaches to problems. The evident loveliness of this passage out of high-performing Singapore contains some interesting food for thought for both sides.

The role of memory is another divide between Left and Right. Generally, conservative education thinkers celebrate memory, while progressives denigrate it. I side with the conservatives. Too often, progressives seem to want something for nothing: all the glories of critical thinking but without strong investment in the machinery it runs on. Progressives also appear terrified of making the students sweat through varieties of learning they might not like.

I would also argue, however, that cultural conservatives are misinterpreting what teachers say. When a math teacher says, “I like the Common Core because it means that my students don’t have to memorize everything anymore,” the key word is not “memorize”—it’s “everything.” Memorizing *everything* is a prescription for failure in mathematics. But that is pretty much the prescription we are following today. Teachers are right that students will perform better if they think about mathematics. And, in addition, we have to hold on tightly to the part of math education that serves at the command of memory.

Rather than a replacement for answer-getting or skill, concepts are a strategy for raising achievement. The intentionally close match of the standards to the topic-grade matrix of high-performing countries is another such strategy. So is the standards’ strong focus on arithmetic in the elementary grades. If I had to collect under a single heading the various ways in which the Common Core differs from previous state standards, I would summarize matters this way: the Common Core State Standards were designed not just to raise the bar but to raise achievement.

Hearing What Parents Have to Say

Reading what I have written so far, you might be getting the idea that the standards are pretty reasonable. If the standards are so reasonable, why are they so controversial?

The answer clearly has a political dimension, but here I would rather bypass politics and consider what parents have to say and why I think we should listen to them.

It is not hard to find examples on social media of parents expressing confusion, exasperation, or anger about changes to their children’s schoolwork effected in the name of the standards. I respect these concerns. Since concepts and applications are less familiar in U.S. schools, I believe there is a tendency for districts to focus attention on them and set aside procedural demands. When that happens, parents may see only longer problems, rather than what they can recognize as productive work.

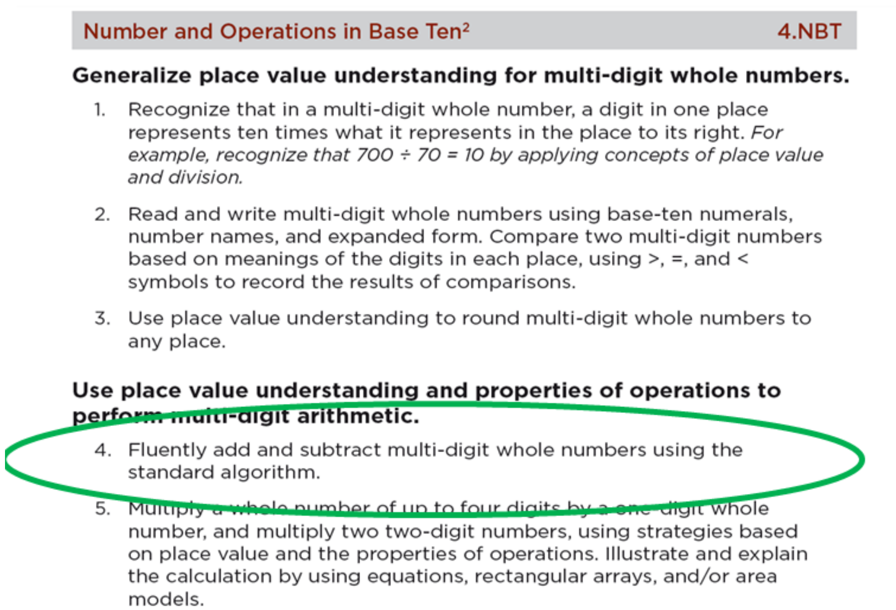
I think parents typically want their children to become skilled in written computation, and I do not think parents are wrong to want this. What is wrong is the idea parents are hearing that the standards are “a move away from procedure.” There is no excellence in mathematics without mastery of procedure, and the standards rightly demand it.

In the concerns of parents there is a wisdom we ought not dismiss. Especially in this time of transition, principals should go out of their way to solicit parents’ views about curriculum options being considered for the school. If parents have concerns, the principal should respond to them in a substantial way. For example, if a curriculum is strong on concepts and applications but weak on fluency and computation, then the curriculum should be supplemented. Computation and fluency should be a regular part of classroom work and should be regularly assessed by the teacher—not outsourced to parents or to video games, as if they were secondary goals or matters lying outside the classroom teacher’s true responsibility.

As parents look over their children’s schoolwork in math, some problems will look unfamiliar or different from what they grew up with. Because some problems will look different, it is all the more important that many others look the same.

Teachers and principals should help parents understand the curriculum in use. For their part, parents should speak face-to-face with teachers and express any concerns that arise. Trust that the teacher is creating an intentional learning environment; verify that your child is actually learning to compute in this environment.

Parents who need to take a constructive, immediate step with their children's school should use the standards themselves to advocate for their children: refer the teacher to the fluency expectations appropriate to your child's grade (see, e.g., Figure 3). If your child is far behind or well ahead of these expectations, ask the teacher how your child's individual needs can be met. Bear in mind that the previous state standards might not have required fluency with the standard algorithms (Figure 2); the teacher might not even realize that this is part of the new expectations. I bring my own copy of the standards with me to my children's parent-teacher conferences.



Number and Operations in Base Ten[2] 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Figure 3. In grade 4, the standards require fluency with the standard algorithm for adding and subtracting multi-digit numbers. (*Common Core State Standards for Mathematics*, 29.)

Determining Whether the Standards Are Improving Math Education

My main concern remains the basic need to get topics into the grades where they belong. We must be especially vigilant about grades K–5 so that our youngest students can spend the vast majority of their time learning arithmetic and preparing adequately for algebra. The anecdotal evidence I have seen suggests that too many classrooms are still pursuing a mile-wide, inch-deep curriculum and consequently falling behind year by year.

Some very large wheels, however, have begun to turn. In a recent interview, I noted:

> The curriculum market is in flux. When the standards came out, some publishers pushed out half-baked stuff, some other publishers put a sticker on the materials they'd been using for years and called it Common Core. I don't think that situation can last. I think we're beginning to see some movement simultaneously toward alignment and quality.[16]

The bulk of U.S. textbooks have been poor for decades. But in my work with educators who are implementing the standards, I see signs of a productive new emphasis on reviewing curriculum carefully. Recently, when my team and I trained a group of teachers to review math textbooks as part of a purchasing process, the teachers expressed surprise that they would be examining closely the mathematics in the books under review. Historically, textbook review processes have avoided serious consideration of the mathematics in the books being reviewed, privileging bureaucratic considerations over substantive ones.

One of the most popular elementary math programs in the country is being revised to address serious problems of alignment; revisions for grades K–2 are being piloted during the 2014–15 school year. Several states and districts have used the Instructional Materials Evaluation Tool, a rubric developed by Student Achievement Partners with input from educators and experts, to reject badly misaligned curriculum programs.[17]

The standards did not invent arithmetic, but they do seem to have given the subject new energy. Teachers who had always avoided math now ask me about such things as unit fractions, the distributive property, and the laws of exponents. A mathematician active in K–12 education for decades told me, “For the first time in my professional career, many teachers seem to realize they need more content knowledge.”

Typical U.S. teachers, relying on typical preservice preparation, doing their best with typical U.S. textbooks, in typical U.S. classrooms have been teaching mathematics badly in absolute terms. Most are eager to do better and need support in doing so. The standards will invite them to try some moves they are not yet experienced with, making for some discomfort. But each year, better textbooks will displace worse ones in more districts, better tests will replace worse ones in more states, and teachers will discover a little more about the most important elements of mathematics and the means by which young humans acquire them.

Revising to the Standards at the State Level

Florida and Indiana recently revised their standards; eventually, every state will. The good news about this trend is that it demonstrates states’ authority over their own school systems. State revisions also provide an opportunity to address principled criticisms of the Common Core. But as states revise their standards, it is important for the revisions to maintain the key shifts I describe in this article. To be clear, the point is not that these shifts are important because the Common Core was designed this way. The situation is actually the reverse. The Common Core was designed this way because these shifts are so important.

The standards adopted by states should agree with high-performing countries as well as, or better than, the Common Core in grades 1–8—as measured by the rigorous TIMSS methodology and published in a peer-reviewed scholarly journal. They should match or exceed the Common Core’s ability to retrodict state-level NAEP scores—as measured by the rigorous TIMSS methodology and published in a peer-reviewed scholarly journal.

Above all, they should preserve absolutely the Common Core’s intense focus on arithmetic in grades K–5. Topics from outside arithmetic, such as statistics and congruence, must wait until middle school, as in high-performing countries.

A more robust education system than the one we have today would revise standards based on the evidence of how students actually perform. Elsewhere I have said, “Standards shouldn’t change frequently, but over a prudent timeframe they should evolve based on what we learn from research and from educators in the field during implementation. For example, after many years of that kind of process, Singapore now considers their math topics and grade placements to be fairly well settled. This means that for the time being, they can focus entirely on improving the depth and delivery of the content."[18]

If states move far away from the Common Core's blueprint, they not only move away from its design for higher achievement, they also lose the benefits of commonality that accrue to each individual state and to the states as a body. For example, as I testified in Indiana:

> Whatever the virtues or flaws of Indiana's previous standards, it is fair to ask whether publishers ever designed around them. Indiana accounts for about 2% of the country's students. Publishers in search of market share might not be eager to develop a whole textbook series deeply aligned to the previous Indiana standards. A publisher might well write to the standards of a larger state, or develop a general purpose program, and then use a crude crosswalk technique to assemble an Indiana version using a content management system. This is not a recipe for quality. By adopting the Common Core, Indiana has now joined with other states to create a market large enough to drive the publishing market toward quality and innovation over time. Quality will take time to develop. But the Common Core gives Indiana educators access to the best of what mathematics experts across the country have to offer.[19]

More generally, as I have noted elsewhere,

> Widely shared goals give the fragmented education sector a shared agenda for strengthening practitioners' mathematical knowledge; for improving the tests we rely on to know how we are doing; for accumulating the research to resolve important questions about teaching and learning; and for achieving a rational materials marketplace in which schools more reliably choose to purchase the tools that actually work best.[20]

In 2008, two years before the release of the Common Core, a national education panel convened by the George W. Bush administration characterized mathematics education in the United States as "broken." That is a harsh judgment, and it does not adequately honor the hard work and dedication of teachers or do justice to all that teachers provide for their students every day. But objectively, it is a difficult judgment to dispute. Repairing our system will be a long-term project. The Common Core is not the end of that process, but I believe it has been a promising and necessary beginning.

Notes

[1] Laura S. Hamilton, Brian M. Stecher, and Kun Yuan, *Standards-Based Reform in the United States: History, Research, and Future Directions* (Washington, DC: Center on Education Policy, 2009), http://www.rand.org/content/dam/rand/pubs/reprints/2009/RAND_RP1384.pdf.

[2] "Common Core State Standards Development Work Group and Feedback Group Announced," National Governor's Association, July 1, 2009, http://www.nga.org/cms/home/news-room/news-releases/page_2009/col2-content/main-content-list/title_common-core-state-standards-development-work-group-and-feedback-group-announced.html.

[3] For a list of the members of the Development Teams and their affiliations, see National Governor's Association, and the Council of Chief State School Officers, *Common Core State Standards Initiative: K–12 Standards Development Teams*, accessed August 4, 2014, http://www.nctm.org/uploadedFiles/Standards_and_Focal_Points/mathcommoncoreorg/2010COMMONCOREK12TEAM-math.pdf.

[4] Rick Hess, "Straight Up Conversation: Common Core Guru Jason Zimba," *Education Week*, February 11, 2013, http://blogs.edweek.org/edweek/rick_hess_straight_up/2013/02/rhsu_straight_up_conversation_sap_honcho_jason_zimba.html.
[5] Common Core State Standards Initiative, *Common Core State Standards for Mathematics*, accessed September 9, 2014, http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf.
[6] U.S. Department of Education, *Foundations for Success: The Final Report of the National Mathematics Panel* (Washington, DC: Author, 2008); National Research Council, *Mathematics Learning in Early Childhood: Paths Toward Equity and Excellence*. (Washington, DC: National Academies Press, 2009).
[7] For additional information about the process, and an interactive timeline, see "Development Process," Common Core State Standards Initiative, accessed September 9, 2014, www.corestandards.org/about-the-standards/development-process.
[8] Conference Board of the Mathematical Sciences, *Common Core State Standards for Mathematics: Statement by Presidents of CBMS Member Professional Societies*, accessed August 4, 2014, http://www.nctm.org/uploadedFiles/New_and_Noteworthy/CBMS%20Support% , 20Statement%20for%20CCSSM.pdf.
[9] *Understanding Perspectives on American Public Education: Results of a Gallup–"Education Week" Survey of K–12 School District Superintendents*, July 1, 2014, available at http://www.gallup.com/strategicconsulting/172025/understanding-perspectives-american-public-education.aspx.
[10] Since 2010, the number of students taking AP Calculus has increased by 19 percent; see College Board, *AP Exam Volume Changes (2003–2013)*, 2013, http://media.collegeboard.com/digitalServices/pdf/research/2013/2013-Exam-Volume-Change.pdf. (I do not argue that this increase is due to the standards, but neither does the increase seem consistent with critics' claims that the standards will prevent students from taking calculus in high school.)
[11] Alan Ginsburg, Steven Leinwand, Terry Anstrom, and Elizabeth Pollock *What the United States Can Learn from Singapore's World-Class Mathematics System (and What Singapore Can Learn from the United States): An Exploratory Study* (Washington, DC: American Institutes for Research, 2005), 49.
[12] William H. Schmidt and Richard T. Houang, "Curricular Coherence and the Common Core State Standards for Mathematics," *Educational Researcher* 41, no. 8 (2012): 294–308. (See also Achieve, *Comparing the Common Core State Standards and Singapore's Mathematics Syllabus*, August 2010, accessed August 4, 2014, http://www.achieve.org/files/CCSSandSingapore.pdf; Achieve, *Comparing the Common Core State Standards in Mathematics with Japan's Mathematics' Curriculum in the Course of Study*, August 2010, http://www.achieve.org/files/CCSSandJapan.pdf. The conclusions were both favorable. Note that the Achieve studies are not peer-reviewed research in scholarly journals. That said, Achieve has a track record in international benchmarking. Still, the Achieve studies should not be considered in the same league as Schmidt and Houang, "Curricular Coherence." Finally, note that college readiness in the United States means something much different from college readiness in other countries. Simplistic comparisons must therefore be avoided.)
[13] For more on applications, see *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics* and *High School Publishers' Criteria for the Common Core State Standards for Mathematics*, both available at http://achievethecore.org/page/686/publishers-criteria, and see "PISA and the U.S. Common Core State Standards for Mathematics," chap. 4 in *Strong Performers and Successful Reformers in Education: Lessons from PISA 2012 for the United States* (Paris: OECD, 2013),.http://www.oecd.org/pisa/keyfindings/PISA2012-US-CHAP4.pdf.
[14] Alan Ginsburg, Steven Leinwand, and Katie Decker, *Informing Grades 1–6 Standards Development: What Can Be Learned from High-Performing Hong Kong, Korea, and Singapore?* (Washington, DC: American Institutes for Research, 2009), table 5, p. 7.
[15] Ginsburg et al., *What the United States Can Learn from Singapore's World-Class Mathematics System*, 49.
[16] "Inside the Common Core: Math," American RadioWorks, recorded July 1, 2014, podcast available at https://soundcloud.com/americanradioworks/american-radioworks-inside.
[17] *Instructional Materials Evaluation Toolkit (IMET): Mathematics, Grades K–8*, 2014, http://achievethecore.org/content/upload/IMET_Version2_Math_K-8.pdf.
[18] Hess, "Straight Up Conversation."
[19] *Expanded Testimony of Jason Zimba before the Interim Study Committee on Common Core Educational Standards*, August 5, 2013, www.tinyurl.com/ZimbaIndiana2013.
[20] Jason Zimba, "A New Course for K–12 Mathematics Education," posted January 29, 2014, http://jhupressblog.com/category/math/.