Instructional Practice Toolkit
Mathematics – Grade 6
Participant Resources

Table of Contents

Shifts at a Glance

College- and Career-Ready Shifts in Mathematics

Focus strongly where the standards focus.

Focus: The Common Core and other college- and career-ready (CCR) standards call for a greater focus in mathematics. Rather than racing to cover topics in a mile-wide, inch-deep curriculum, CCR standards require us to significantly narrow and deepen the way time and energy the way time and energy are spent in the math classroom. We focus deeply on the Major Work* of each grade so that students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom.

Coherence: Think across grades and link to major topics within grades.

Thinking across grades: College- and career-ready standards are designed around coherent progressions from grade to grade. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.

Linking to major topics: Instead of allowing additional or supporting topics to detract from the focus of the grade, these concepts serve the grade-level focus. For example, instead of data displays as an end in themselves, they are an opportunity to do grade-level word problems.

Rigor: In major topics*, pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

Conceptual understanding: CCR standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

Procedural skill and fluency: CCR standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions such as single-digit multiplication so that they have access to more complex concepts and procedures.

Application: CCR standards call for students to use math flexibly for applications in problem-solving contexts. In content areas outside of math, particularly science, students are given the opportunity to use math to make meaning of and access content.

High-level Summary of Major Work in Grades K-8

K–2   Addition and subtraction—concepts, skills, and problem solving; place value
3–5   Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving
6   Ratios and proportional relationships; early expressions and equations
7   Ratios and proportional relationships; arithmetic of rational numbers
8   Linear algebra and linear functions

*For a list of major, additional, and supporting clusters by grade, please refer to 'Focus in Math' at achievethecore.org/focus

STUDENT ACHIEVEMENT PARTNERS   achievethecore.org/shifts-mathematics

INSTRUCTIONAL PRACTICE GUIDE

MATH
SUBJECT

K−8
GRADES

Date

Teacher Name

School

Grade / Class Period / Section

Topic / Lesson / Unit

About The Instructional Practice Guide

Content-specific feedback is critical to teacher professional development. The Instructional Practice Guide (IPG) is a K−12 classroom observation rubric that prioritizes what is observable in and expected of classroom instruction when instructional content is aligned to college- and career-ready (CCR) standards, including the Common Core State Standards (CCSS), in Mathematics (corestandards.org/Math). It purposefully focuses on the limited number of classroom practices tied most closely to content of the lesson.[1]

Designed as a developmental rather than an evaluation tool, the IPG supports planning, reflection, and collaboration, in addition to coaching. The IPG encompasses the three Shifts by detailing how they appear in instruction:[2]

Focus strongly where the standards focus.

Coherence: Think across grades and link to major topics within grades.

Rigor: In major topics, pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

This rubric is divided into the Core Actions teachers should be taking. Each Core Action consists of indicators which further describe teacher and student behaviors that exemplify CCR-aligned instruction.

Using The Instructional Practice Guide

For each observation, you should make note of what you see and hear. It may be helpful to supplement what you've recorded with further evidence from artifacts such as lesson plans, tasks, or student work. Although many indicators will be observable during the course of a lesson, there may be times when a lesson is appropriately focused on a smaller set of objectives or you observe only a portion of a lesson. In those cases you should expect to not observe some of the indicators and to leave some of the tool blank. Whenever possible, share evidence you collected during the observation in a follow-up discussion.

After discussing the observed lesson, use the Beyond the Lesson Discussion Guide to put the content of the lesson in the context of the broader instructional plan. The questions in the Beyond the Lesson Discussion Guide help delineate what practices are in place, what has already occurred, and what opportunities might exist to incorporate the Shifts into the classroom during another lesson, further in the unit, or over the course of the year.

To further support content-specific planning, practice, and observation, explore the collection of free IPG companion tools, resources, and professional development modules at achievethecore.org/instructional-practice.

CORE ACTIONS AND INDICATORS

For the complete Instructional Practice Guide, go to achievethecore.org/instructional-practice.

MATH
SUBJECT

K–8
GRADES

Core Action 1

Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college- and career-ready standards in mathematics.

A.   The enacted lesson focuses on the grade-level cluster(s), grade-level content standard(s), or part(s) thereof.

Mathematical learning goal: _____

Standard(s) addressed in this lesson: _____

B.   The enacted lesson appropriately relates new content to math content within or across grades.

C.   The enacted lesson intentionally targets the aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.

Circle the aspect(s) of Rigor targeted in the standard(s) addressed in this lesson: Conceptual understanding / Procedural skill and fluency / Application

Circle the aspect(s) of Rigor targeted in this lesson: Conceptual understanding / Procedural skill and fluency / Application

Core Action 2

Employ instructional practices that allow all students to learn the content of the lesson.

A.   The teacher makes the mathematics of the lesson explicit through the use of explanations, representations, tasks, and/or examples.

B.   The teacher strengthens all students' understanding of the content by strategically sharing students' representations and/or solution methods.

C.   The teacher deliberately checks for understanding throughout the lesson to surface misconceptions and opportunities for growth, and adapts the lesson according to student understanding.

D.   The teacher facilitates the summary of the mathematics with references to student work and discussion in order to reinforce the purpose of the lesson.

Core Action 3

Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

A.   The teacher provides opportunities for all students to work with and practice grade-level problems and exercises.
Students work with and practice grade-level problems and exercises.

B.   The teacher cultivates reasoning and problem solving by allowing students to productively struggle.
Students persevere in solving problems in the face of difficulty.

C.   The teacher poses questions and problems that prompt students to explain their thinking about the content of the lesson.
Students share their thinking about the content of the lesson beyond just stating answers.

D.   The teacher creates the conditions for student conversations where students are encouraged to talk about each other's thinking.
Students talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.

E.   The teacher connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas.
Students use increasingly precise mathematical language and ideas.

If any uncorrected mathematical errors are made during the context of the lesson (instruction, materials, or classroom displays), note them here.

CORE ACTION 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college- and career-ready standards in mathematics.

INDICATORS / NOTE EVIDENCE OBSERVED OR GATHERED FOR EACH INDICATOR	RATING
A. The enacted lesson focuses on the grade-level cluster(s), grade-level content standard(s), or part(s) thereof. Mathematical learning goal: Standard(s) addressed in this lesson:	Yes- The enacted lesson focuses only on mathematics within the grade-level standards. No- The enacted lesson focuses on mathematics outside the grade-level standards.
B. The enacted lesson appropriately relates new content to math content within or across grades.	Yes- The enacted lesson builds on students' prior skills and understandings. No- The enacted lesson does not connect or has weak connections to students' prior skills and understandings.
C. The enacted lesson intentionally targets the aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed. Circle the aspect(s) of Rigor targeted in the standard(s) addressed in this lesson: Conceptual understanding / Procedural skill and fluency / Application Circle the aspect(s) of Rigor targeted in this lesson: Conceptual understanding / Procedural skill and fluency / Application	Yes- The enacted lesson explicitly targets the aspect(s) of Rigor called for by the standard(s) being addressed. No- The enacted lesson targets aspects of Rigor that are not appropriate for the standard(s) being addressed.

CORE ACTION 2: Employ instructional practices that allow all students to learn the content of the lesson.

INDICATORS[3] / NOTE EVIDENCE OBSERVED OR GATHERED FOR EACH INDICATOR	RATING
A. The teacher makes the mathematics of the lesson explicit through the use of explanations, representations, tasks, and/or examples. ☐ NOT OBSERVED	4- A variety of instructional techniques and examples are used to make the mathematics of the lesson clear. 3- Examples are used to make the mathematics of the lesson clear. 2- Instruction is limited to showing students how to get the answer. 1- Instruction is not focused on the mathematics of the lesson.
B. The teacher strengthens all students' understanding of the content by strategically sharing students' representations and/or solution methods. ☐ NOT OBSERVED	4- Student solution methods are shared, and connections to the mathematics are explicit and purposeful. If applicable, connections between the methods are examined. 3- Student solution methods are shared, and some mathematical connections are made between them. 2- Student solution methods are shared, but few connections are made to strengthen student understanding. 1- Student solution methods are not shared.
C. The teacher deliberately checks for understanding throughout the lesson to surface misconceptions and opportunities for growth, and adapts the lesson according to student understanding. ☐ NOT OBSERVED	4- There are checks for understanding used throughout the lesson to assess progress of all students, and adjustments to instruction are made in response, as needed. 3- There are checks for understanding used throughout the lesson to assess progress of some students; minimal adjustments are made to instruction, even when adjustments are appropriate. 2- There are few checks for understanding, or the progress of only a few students is assessed. Instruction is not adjusted based on students' needs. 1- There are no checks for understanding; therefore, no adjustments are made to instruction.
D. The teacher facilitates the summary of the mathematics with references to student work and discussion in order to reinforce the purpose of the lesson. ☐ NOT OBSERVED	4- The lesson includes a summary with references to student work and discussion that reinforces the mathematics. 3- The lesson includes a summary with a focus on the mathematics. 2- The lesson includes a summary with limited focus on the mathematics. 1- The lesson includes no summary of the mathematics.

3. These actions may be viewed over the course of 2–3 class periods.

CORE ACTION 3: Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.[4]

INDICATORS[5][6] / NOTE EVIDENCE OBSERVED OR GATHERED FOR EACH INDICATOR / RATING

4- Teacher provides many opportunities, and most students take them.
3- Teacher provides many opportunities, and some students take them; or teacher provides some opportunities and most students take them.
2- Teacher provides some opportunities, and some students take them.
1- Teacher provides few or no opportunities, or few or very few students take the opportunities provided.

A. The **teacher** provides opportunities for all students to work with and practice grade-level problems and exercises.

   Students work with and practice grade-level problems and exercises.

          4 3 2 1
          ☐ NOT OBSERVED

B. The **teacher** cultivates reasoning and problem solving by allowing students to productively struggle.

   Students persevere in solving problems in the face of difficulty.

          4 3 2 1
          ☐ NOT OBSERVED

C. The **teacher** poses questions and problems that prompt students to explain their thinking about the content of the lesson.

   Students share their thinking about the content of the lesson beyond just stating answers.

          4 3 2 1
          ☐ NOT OBSERVED

D. The **teacher** creates the conditions for student conversations where students are encouraged to talk about each other's thinking.

   Students talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.

          4 3 2 1
          ☐ NOT OBSERVED

E. The **teacher** connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas.

   Students use increasingly precise mathematical language and ideas.

          4 3 2 1
          ☐ NOT OBSERVED

If any uncorrected mathematical errors are made during the context of the lesson (instruction, materials, or classroom displays), note them here.

4. There is not a one-to-one correspondence between the indicators for this Core Action and the Standards for Mathematical Practice. These indicators represent the Standards for Mathematical Practice that are most easily observed during instruction.
5. Some portions adapted from 'Looking for Standards in the Mathematics Classroom' 5x8 card published by the Strategic Education Research Partnership (http://math.serpmedia.org/5x8card/).
6. Some or most of the indicators and student behaviors should be observable in every lesson, though not all will be evident in all lessons. For more information on teaching practices, see NCTM's publication Principles to Actions: Ensuring Mathematical Success for All for eight Mathematics Teaching Practices listed under the principle of Teaching and Learning (http://www.nctm.org/principlestoactions).

BEYOND THE LESSON: DISCUSSION GUIDE
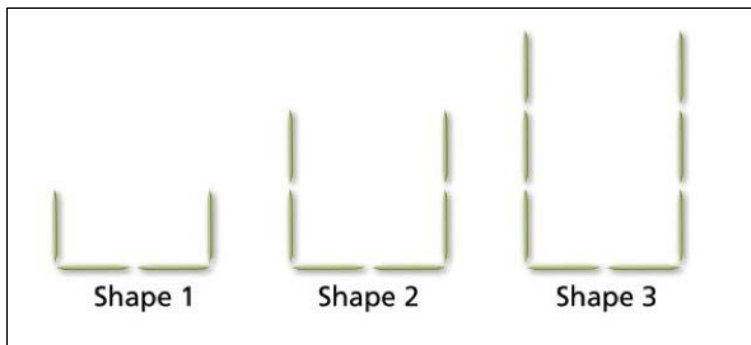## MATHEMATICS

INTRODUCTION

The Beyond the Lesson Discussion Guide is designed for the post-observation conversation using the Instructional Practice Guide (achievethecore.org/instructional-practice) or any other observation rubric. The questions put the content of the lesson in the context of the broader instructional plan for the unit or year. The conversation should first reflect on the evidence collected during the observation to consider what worked, what could improve, and what resources are available to support improvement. If any parts of the Lesson Planning Tool (achievethecore.org/lesson-planning-tool) were used in preparing for the lesson, refer to that information during the discussion. After discussing the observed lesson, use the "Beyond the Lesson" questions to help clearly delineate what practices are in place, what has already occurred, and what opportunities might exist in another lesson, further in the unit, or over the course of the year to incorporate the Shifts into the classroom.

1. **Is this unit targeting the Major Work of the Grade? Does the prior unit target Major Work? Does the next unit target Major Work? How much time would you estimate will be spent on the Major Work in this class this year? (K–8)** Focus means significantly narrowing the scope of content in each grade so that students achieve at higher levels and experience more deeply that which remains. For more information on Major Work of the Grade, see achievethecore.org/focus.

2. **Does this unit target the Supporting Work of the Grade? If so, will this unit highlight the connection to the Major Work of the Grade? Explain how. (K–8)** Supporting content enhances Focus and Coherence simultaneously by engaging students in the Major Work of the Grade. For example, materials for K–5 generally treat data displays as an occasion for solving grade-level word problems using the four operations (see 3.MD.3); materials for grade 7 take advantage of opportunities to use probability to support ratios, proportions, and percents.

3. **Summarize how this lesson fits within the unit. Describe how the other lessons and tasks in this unit are intentionally sequenced to help students develop increasingly sophisticated understanding, skills, and practices.** For more information on coherent connections across and within grades, see http://ime.math.arizona.edu/progressions/.

4. **Which of the three aspects of Rigor (conceptual understanding, procedural skill and fluency, and application) are attended to within this unit? If more than one aspect is attended to, when in the unit are they attended to individually, and when are students using them together?** Rigor is defined as pursuing conceptual understanding, procedural skill and fluency, and application with equal intensity. The standards are written using language that informs the reader as to which aspect of Rigor certain standards address. Some clusters or standards specifically require one aspect of Rigor; some require multiple aspects. All aspects of Rigor need not be addressed in every lesson.

5. **How will you meet all students' needs while working on grade/course-level content in this unit? (e.g., How will you provide scaffolding for students below grade/course level so they can reach the grade/course-level expectations? How will you create opportunities for students who are advanced to go deeper into the grade/course-level content?)** For more information, see Adapting the Lesson under Problems & Exercises in the Lesson Planning Tool: achievethecore.org/lesson-planning-tool.

6. **What off-grade/course-level standards have you taught this year and why?** There may be reasons for addressing topics in a strategic way before or after the grade in which the topic is central in the standards. However, any such purposeful discrepancies should enhance the required learning, not unduly interfere with or displace grade/course-level content, and be clearly aimed at helping students meet the standards as written.

7. **In what ways do you provide diagnostic feedback to students? Do students have opportunities to revise their thinking? Does student work include revisions of solutions, explanations, and justifications?**

8. **In what ways have your students made progress towards mastering the grade/course-level content standards? How are you monitoring and tracking their achievement of the standards? What work still needs to be done to ensure all students achieve mastery of each standard by the end of the year?** For more information on the Standards for Mathematical Content, see corestandards.org/Math.

9. **In what ways have you seen your students increase their independence in applying the Standards for Mathematical Practice in learning content this year? Which practice standards do students still need to develop and how can you support them in doing so?** For more information on the Standards for Mathematical Practice, see corestandards.org/Math/Practice.

10. **What tools are appropriate for students to independently access when solving mathematical problems in this unit? Do students frequently choose and use appropriate tools strategically in this class?** For more information on SMP5, see corestandards.org/Math/Practice.

Student Assignment: Participant Handout

Instructions:

1. Draw the next 3 stages of the toothpick pattern.
2. Complete the graphic organizer to represent the pattern.

Shape 1   Shape 2   Shape 3

Algebraic Expression	Description in Words

Table	Model

Shape Number							
Number of Toothpicks							

Target Expression: 2n+2

Directions: Determine if these expressions are equivalent to our target expression. Use mathematical evidence to support your answer and write yes or no. *BONUS:* Write what property you can use to support your answer.

1. $2n + 2$

2. $n + n + 1 + 1$

3. $n^2 + 2$

4. $2(n + 1)$

5. $2(n + 2)$

6. $2(n + 2) - 2$

7. $3n + 5 - n - 3$

8. $n + 1 - n - 1$

9. $3(n + 1) - n - 1$

Exit Slip
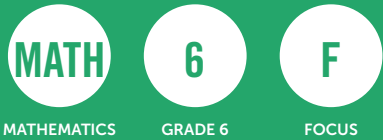
Determine which, if any, expressions are equivalent.

Regina's Expression: 2n + 2 + (n - 1)

Davere's Expression: 3(n + 1) - 2

Itzel's Expression: 4n - (n - 1)

Aniqa's Expression: 3 + 1n

CCSS
WHERE TO FOCUS
GRADE 6
MATHEMATICS

MATH · MATHEMATICS
6 · GRADE 6
F · FOCUS

This document shows where students and teachers should spend the large majority of their time in order to meet the expectations of the Standards.

Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the Standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

Students should spend the large majority[1] of their time on the major work of the grade (■). Supporting work (□) and, where appropriate, additional work (○) can engage students in the major work of the grade.[2, 3]

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 6

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters   □ Supporting Clusters   ○ Additional Clusters

6.RP.A	■	Understand ratio concepts and use ratio reasoning to solve problems.
6.NS.A	■	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
6.NS.B	○	Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.C	■	Apply and extend previous understandings of numbers to the system of rational numbers.
6.EE.A	■	Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.B	■	Reason about and solve one-variable equations and inequalities.
6.EE.C	■	Represent and analyze quantitative relationships between dependent and independent variables.
6.G.A	□	Solve real-world and mathematical problems involving area, surface area, and volume.
6.SP.A	○	Develop understanding of statistical variability.
6.SP.B	○	Summarize and describe distributions.

HIGHLIGHTS OF MAJOR WORK IN GRADES K–8

K–2	Addition and subtraction – concepts, skills, and problem solving; place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional relationships; early expressions and equations
7	Ratios and proportional relationships; arithmetic of rational numbers
8	Linear algebra and linear functions

REQUIRED FLUENCIES FOR GRADE 6

6.NS.B.2	Multi-digit division
6.NS.B.3	Multi-digit decimal operations

1 At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see Criterion #1 of the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics www.achievethecore.org/publisherscriteria.
2 Refer also to criterion #3 in the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics www.achievethecore.org/publisherscriteria.
3 Note, the critical areas are a survey of what will be taught at each grade level; the major work is the subset of topics that deserve the large majority of instructional time during a given year to best prepare students for college and careers.

CCSS
WHERE TO FOCUS
GRADE 6
MATHEMATICS

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.

** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

The Observation and Feedback Cycle: Best Practices for Low Inference Notes

Observe
The school leader visits the classroom and takes low-inference notes on teacher and student actions.

Best Practices for Observation
1. **Eliminate effects of bias.** Enter the classroom without judgment and work from evidence.
2. **Take low-inference notes.** Write down only what teacher and students say and do.
3. **Look for learning.** Seek evidence of what students know and are able to do.
4. **Remain, review, reflect.** Pause to organize your evidence before rating.

Collecting low inference evidence during an observation
Capturing high-quality notes during the observation is the first step in ensuring that ratings are accurate and feedback aligns to teachers' needed areas of improvement. **Low-inference note-taking is a skill**, not knowledge. Knowing how to do a push-up doesn't mean you can do 25 of them in 60 seconds; it comes with practice. When taking low-inference notes, the school leader describes what is taking place without drawing conclusions or making judgments about what he or she observes. When taking notes on instruction, ask:

- What do you see and hear the teacher and students saying and doing?
- What evidence can you gather of student learning?
- What will students know and be able to do at the end of the lesson?

Common mistakes/pitfalls to avoid
- Distinguish between low-inference statements and opinions. For instance, you can identify key words that give away subjectivity: e.g., *"I think,"* or *"I feel."* Be cognizant of keeping evidence separate from opinions, using this framework:

Evidence	Opinion
• Is observable	• Makes inferences
• Is not influenced by the observer's perspective	• Depends on observer's perspective
• Is free of evaluative words	• Includes evaluative words
• Does not draw conclusions	• Draws conclusions

- Replace vague quantifiers by capturing more specific evidence: e.g., *"a lot of students raised their hands"* vs. *"17 of 20 students raised their hands."*
- Swap Edu-Speak for Evidence. For example, rather than saying, *"You differentiated by scaffolding questions during the mini-lesson,"* identify the actual questions that the teacher asked, such as *"What is the name of this shape? How is it different from a square or rectangle? Where in real life have you seen this shape?"*

For electronic version, visit: https://www.weteachnyc.org/media2016/filer_public/2a d3/2ad3839feeae-42ba-8249-614b33136717/ best_practices_for_low_inference_notes.pdf

Tips for low inference note taking

Where to find the data for student outcomes during an observation:

- Sit with a table/group of students. Write down the questions asked and answers given by the students in that group.
- Copy down what each student has written on their paper VERBATIM into your observation notes (e.g., answer to #2 on handout, response to quick-write prompt). The observer can obtain a handout from the teacher, if available, and record the answers directly onto it.
- Write down the time and circulate in the room. Record the item that all students are working on in that moment. Then, go around a second time.
- Select a problem, determine the correct answer, and tally the number of students who have the correct response written on their papers.
- If recording observation notes using an iPad, use the iPad to take pictures of actual student work during the classroom observation.
- Move around the classroom and identify students performing at high, medium, low levels and strategically capture their work
- Monitor observation notes to ensure that the "student side" is not neglected.
- Ask students to tell you what they are learning/doing, why they are learning, and if they have learned anything new today.
- Collect the lesson plan and/or copies of student work prior to leaving the classroom.

How do I capture as much evidence as possible?

- Set up a coding system (T= teacher, S= student, HU= hands up)
- Time transitions, each section of the lesson, work time, etc.
- Copy objective or aim, or make a note if it is not posted
- Draw circles to represent groups of students or teacher interaction with students

Diagram of groups of students
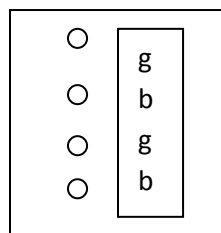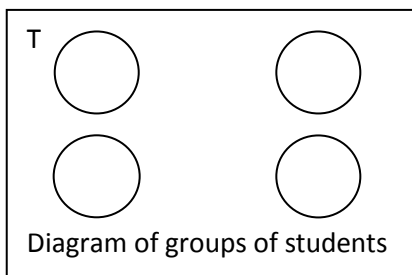
- If you notice a trend, create a tally on the side, so you can capture other evidence that may be occurring while also documenting the trend. For example, Jane is the only one responding to the teacher's questions. You may capture several instances verbatim, but you can also capture how many times it occurs if you can't capture everything Jane said.

    Use tallies or shorthand in the diagram or a chart:

Jane is called on	卌 I
Times teacher provides feedback to front table	III

- Quality over quantity: collect a full interaction.
    - *When teacher did __, student __. When student said __, teacher said__.*

Low-Inference Note-Taking Samples: Strong versus Weak

Strong example of low-inference notes:

Time	Teacher Actions	Student Actions
1:00	Teacher says to walking students, "You need to be on the rug in 3-2-1."	Twenty-four students on the carpet facing the front of the room. 3 students walking around the classroom. As teacher said "one" students joined classmates.
1:01	Teacher asked "How many days are there in the week?" Teacher repeated question and then said, "Anyone?" Teacher asked kids to stand and lead them in "The Days of the Week" song.	5-6 kids spoke to each other when teacher spoke. She called on Terrence who said "7." 16 of the 27 kids stood up for the song.
1:02	Teacher asked "What day comes after Saturday?"	Steven shouted out, "Monday!" Most students laughed – 2 boys physically rolled around and knocked over 2 girls. Steven walked away from the group, and sat in the opposite corner of the classroom.
1:03	Teacher said, "OK boys and girls if you hear my voice clap once, If you hear my voice clap twice."	After two claps, all but 2 boys were quiet and looking at her.

Weak example of low-inference notes:

Time	Teacher Actions	Student Actions
1:00		Students on carpet during mini-lesson. Lots of students walking around the classroom while the teacher tried to get their attention.
1:01	Teacher asked questions about the calendar.	Many students were not listening while the teacher reviewed the days of the week.
1:02		Steven called out over and over again when you asked the question about the days of the week.
1:03		Steven walked away from the group and the class fell apart.
1:04	Mini-lesson is not successful. Little student learning accomplished as teacher has no classroom management skills.	
1:05	Poor classroom management continues through sloppy transitions from carpet to desks.	Several students are talking to one another.
1:06	The teacher seemed to be okay with this.	A few students go to the round table. Some start reading and some don't.

Teacher-created Lesson Plan

<u>6th grade Lesson Plan</u>

Standard: *6.EE.A4* Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

Focus Question: How can we show whether expressions are equivalent?

SWBAT identify and create equivalent expressions using number properties.

Launch: 3 min

Explain directions for completing the graphic organizer. Take questions.

Present the first 3 stages of a pattern.

Explore 1: 15 minutes

Think/Pair/Share: Draw the next 3 stages of the pattern. Reveal w/student work on doc reader or model on SmartBoard.

Directions/ Expectations: Decide whether to work alone or w/table mates and what part of the graphic organizer to complete next (expression, words, table). Will have to explain how at least 2 of the quadrants are related.

Circulate/prepare for summary: Identify 2 students who correctly connected at least 2 representations in different ways. Or, if a struggling kid, how the class can help them create their expression/next part. (Use monitoring tool)

Summary 1: 5 minutes

Describe how any 2 of your quadrants are connected (represent the same information). Specifically, how does your model relates to your expression, table and/or words?

Launch 2: 2 minutes

We used our table, words, and/or model to create an expression. I collected a few expressions from your class and from other classes. NOW we have to determine if these expressions are equivalent to this one (choose one we want to represent our class and one that is not in the list)

We have these 5 expressions… our job is to figure out if they are equivalent. You can use your model, your words or your table to determine if your expression is equivalent to these expressions and are these expressions equivalent to each other.

Explore 2: 10 minutes

Directions/ Expectation of Outcome(s): Which of these expressions (any, all) are equivalent? How do you know? Is your expression equivalent to these? How do we know? Prove it! Use properties to prove it.

1. $2n + 2$ (yes) – our target expression
2. $n + n + 1 + 1$ (yes)
3. $n^2 + 2$ (no)
4. $2(n + 1)$ (yes)
5. $2(n + 2)$ (no)
6. $2(n + 2) - 2$ (yes)
7. $3n + 5 - n - 3$ (yes)
8. $n + 1 - n - 1$ (no)
9. $3(n + 1) - n - 1$ (yes)

Summarize 2: 7 minutes

Getting at Focus Question: How did you determine if the expressions were equivalent? What are the strategies we used to determine if expressions were equivalent? What properties did we use to prove this?

Exit Slip: 5 minutes

Which of these expressions are equivalent? (provide a number of expressions) Explain how you know with words and numbers.

Exit Slip Rubric for Grading:

4 - exceeds standard by correctly identifying all equivalent expressions and providing a well-developed, clearly written and accurate explanation.

3 – meets standard by correctly identifying all equivalent expressions and providing a written explanation that has a minor flaw in reasoning.

2 – approaching standard by partially identifying the equivalent expressions. Explanation is unclear, weak or incomplete.

1 – below standard by incorrectly identifying equivalent expressions. Explanation is unclear or incorrect.

Materials:

SmartBoard presentation, grid paper, graphic organizer, plain white paper, toothpicks (or some other manipulative to help students continue the pattern)

Differentiation:

Student choice; multiple points of entry: tiered questions (students have to prove that at least 3 of the expressions are equivalent or not. Other 2 are extensions); manipulatives: grid paper, toothpicks; heterogeneous grouping.

Where this lesson fits in a scope and sequence:

Throughout this unit of study students have created models, tables, algebraic expressions and used words to describe linear algebraic patterns. In these problems, students will revisit the concept of describing linear algebraic patterns that lead to 2 or more equivalent expressions to provide opportunities for deeper understanding.

Advancing and Assessing Questions:

Describe how you see the pattern growing.

So I see you drew… try… (grounding them in the geometric reasoning)

Do the expressions predict the same number of toothpicks in shape n?

Can you write another expression equivalent to these? Explain why they are equivalent. How can you prove this?

What properties can you use to prove these expressions are equivalent?

Can you use an area model to demonstrate equivalence?

How many terms does the expression have?

What are the constants? Coefficients?

Lesson Plan Analysis

Lesson: _____

Use this document to record information/evidence from the sample lesson plan. Evidence should consider the Core Actions. Evidence recorded will be integrated into the Feedback Summary worksheet.

Core Action 1:
Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by the college- and career-ready standards in mathematics.
Discussion Questions
- Which standard(s) and/or cluster(s) are targeted in this lesson? Does the lesson address a part of the standard(s) or all aspects of the standard(s)? Are they grade-level standards? Are they part of the Major Work of the grade?   - If the standard(s) targeted are Supporting Work of the grade, how will connections be made to engage students in the Major Work of the grade?   - What is the mathematical learning goal for students in this lesson?   - Which aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, and application) do the targeted standards require? What features of the lesson support the aspect(s) of Rigor present in the targeted standards?   - How does the teacher plan to make explicit connections to build on students' prior skills and understandings? What will the teacher say to students or show students to make this connection clear?

Core Action 2:
Employ instructional practices that allow all students to learn the content of the lesson.

Discussion Questions

- How does the teacher plan to use explanations, representations, tasks, and/or examples that will make the mathematics of this lesson clear to students?
- What will students produce? Are they expected to produce only answers?
- What ideas/concepts will be the focus of discussions?
- How will students share/present their mathematical work to support all students' understanding of the topic?
- When in the lesson does the teacher plan to check for understanding?
- How does the teacher plan to summarize the mathematics of the lesson? Will the summary include student work and discussion to reinforce the mathematical learning goal of the lesson?

Core Action 3:
Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

Discussion Questions

- What mathematical language will be used in this lesson? How will the teacher support students' use of increasingly precise language, including for English language learners if applicable?
- Are mathematical models, mathematical representations, mathematical arguments, and mathematical counter-arguments expected from students, as required by the standards? What problem(s) and question(s) will allow students to share their thinking and/or justify their conclusions?
- When will students be doing grade-level problems and exercises? Will all students have this opportunity?

Student Work Samples

Student A

Directions: Determine if these expressions are equivalent to our target expression. Use mathematical evidence to support your answer and write yes or no. *BONUS:* Write what property you can use to support your answer.

1. $2n + 2$
Yes

2. $n + n + 1 + 1$
Yes

3. $n^2 + 2$
NO

4. $2(n + 1)$
Yes

5. $2(n + 2)$
No

6. $2(n + 2) - 2$
Yes

7. $3n + 5 - n - 3$
Yes

8. $n + 1 - n - 1$
NO

9. $3(n + 1) - n - 1$
Yes

Student B

Directions: Determine if these expressions are equivalent to our target expression. Use mathematical evidence to support your answer and write yes or no. *BONUS:* Write what property you can use to support your answer.

1. $2n + 2$
yes

2. $n + n + 1 + 1$
$2n + 2$
yes

3. $n^2 + 2$
no

4. $2(n + 1)$
$2n + 2$
yes
distributive

5. $2(n + 2)$
no

6. $2(n + 2) - 2$
distributive property
$2n + 4 - 2$
$60 - 2$
$4n$
no

7. $3n + 5 - n - 3$
no

8. $n + 1 - n - 1$
no

9. $3(n + 1) - n - 1$
$3n + 3 - n - 1$
no
distributive

25

Student C

Directions: Determine if these expressions are equivalent to our target expression. Use mathematical evidence to support your answer and write yes or no. *BONUS:* Write what property you can use to support your answer.

1. $2n + 2$ yes ✓

2. $n + n + 1 + 1$ yes ✓
   $n+n$ $2n$

3. $n^2 + 2$ ✓
   no
   Can't multiply a variable

4. $2(n + 1)$ yes ✓
   $2n + 2$

5. $2(n + 2)$ no ✓
   $2n + 4$

6. $2(n + 2) - 2$ yes
   yes $4 = 2$

7. $(3n) + 5 - n - 3$ $15n$
   $2n + 2$ ✓

8. $(n) + 1 (-n) - 1 = $ No
   $n - n + 1 - 1$
   0 0

9. $3(n + 1) - n - 1$ yes
   yes

Student D

Directions: Determine if these expressions are equivalent to our target expression. Use mathematical evidence to support your answer and write yes or no. *BONUS:* Write what property you can use to support your answer.

1. $2n + 2$
   Yes

2. $n + n + 1 + 1$
   $2n + 2$ Yes

3. $n^2 + 2$
   No

4. $2(n + 1)$
   $2n + 2$
   Yes

5. $2(n + 2)$
   $2n + 4$
   No

6. $2(n + 2) - 2$
   $2n + 4 - 2$ Yes
   $2n + 2$

7. $3n + 5 - n - 3$
   No Yes

8. $n + 1 - n - 1$
   No

9. $3(n + 1) - n - 1$
   yes No $3n + 3 - n - 1$
   $6n - n - 1$

26

Student E

Directions: Determine if these expressions are equivalent to our target expression. Use mathematical evidence to support your answer and write yes or no. *BONUS:* Write what property you can use to support your answer.

(1.) $2n + 2$ ④
$1 + 2 \times 2 = 4$

$\boxed{2n + 2}$

(2.) $n + n + 1 + 1$
$1 + 1 + 1 + 1 = 4$

$\boxed{n + 2}$

(3.) $n^2 + 2$
$1 + 2 = 3$

$\boxed{n^2 + 2}$

(4.) $2(n + 1)$ ④
$1 + 1 = 2 \times 2 = 4$

$\boxed{2n + 2}$

(5.) $2(n + 2)$
$1 + 2 = 3 \times 2 = 6$

$\boxed{2n + 4}$

$\dfrac{2n + 4}{-2}$
$2n + 2$

(6.) $2(n + 2) - 2$
$1 + 2 = 3 \times 2 = 6 - 2 = 4$

$\boxed{2n + 2}$

(7.) $3n + 5 - n - 3$
$1 \times 3 = 3 + 5 = 8 - 4 = 4$

$3n + (5 - n) - 3$

(8.) $n + 1 - n - 1$
$1 + 1 = 2 - 1 = 1 - 1 = 0$

$\boxed{n + -1}$

(9.) $3(n + 1) - n - 1$

$\boxed{3 + 2}$
$3n + 3$
$3 + 2$
$2 + 2 = 4$

Student F

Directions: Determine if these expressions are equivalent to our target expression. Use mathematical evidence to support your answer and write yes or no. *BONUS:* Write what property you can use to support your answer.

1. $2n + 2$
Yes b/c $2n + 2$ is equivalent to $2n + 2$.

2. $n + n + 1 + 1$
Yes b/c $2n + 2$ can be expanded by adding the #'s and adding 2.

3. $n^2 + 2$
No b/c if you multiply the # of the shape twice, and add 2, you will not get the correct answer.

4. $2(n + 1)$
Yes b/c if you use the distributive property you can get an equivalent expression

5. $2(n + 2)$
No b/c by using the distributive property you will see that it is not the correct answer

6. $2(n + 2) - 2$
$2 + 4 = 6 - 2 = 4 \checkmark$
Yes

7. $3n + 5 - n - 3$
$8 - 1 = 7 - 3 = 4$
Yes

8. $n + 1 - n - 1$
NO

9. $3(n + 1) - n - 1$
Yes

Student Work Analysis

Lesson: _____

Use this document to record information/evidence from the sample student work. Evidence should consider the Core Actions. Evidence recorded will be integrated into the Feedback Summary worksheet. **Before analyzing student work, be sure to have first completed the student assignment.**

General notes and observations about the task:

1. Which standard(s) and/or cluster(s) are targeted in this assignment? Are they grade-level standards?

2. What is the mathematical purpose of the assignment?

3. What aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, and application) does the assignment address? Explain.

Analyzing individual student samples (worksheet on next page):

1. What does the student's work demonstrate about his/her understanding of the expectations of the assignment?

2. What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?

(See worksheet)

Student Work Analysis Worksheet

Student Work Sample	What does the student's work demonstrate about his/her understanding of the expectations of the assignment?	What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?
Student A		
Student B		
Student C		
Student D		

Note: For a collection of more than four samples of student work, print this page multiple times.

1. How did the directions and/or prompts for the assignment allow students to demonstrate the requirements of the targeted standard(s)?

2. How did the mathematical content of the assignment allow students to demonstrate the requirements of the targeted standard(s)?

3. What patterns do you notice in the student work?
   - What did students do consistently well?
   - Were there any common errors?

Feedback Summary

Lesson: _____

Using the completed Instructional Practice Guide, the Lesson Plan Analysis, and Student Work Analysis, consider the aggregate strengths and considerations for the lesson. Choose relevant Beyond the Lesson questions to guide longer-term reflection.

Evidence of the Shifts and standards-aligned practice	Areas where alignment to the Shifts and standards can improve
Core Action 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by the college- and career-ready standards in mathematics.	
Core Action 2: Employ instructional practices that allow all students to learn the content of the lesson.	

Evidence of the Shifts and standards-aligned practice	Areas where alignment to the Shifts and standards can improve
Core Action 3: Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.	

Beyond the Lesson
Choose relevant Beyond the Lesson questions to guide longer-term reflection.

Implications and Next Steps

Appendix
Model Responses

Instructional Practice Guide – Equivalent Expressions (Grade 6): Model Response

Indicator	Evidence
Core Action 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college-and career ready/standards in mathematics.	
A. The enacted lesson focuses on the grade-level cluster(s), grade-level content standard(s), or part(s) thereof.	Mathematical learning goal: • The lesson focuses on determining if expressions are equivalent, which is in the grade 6 domain of Equations and Expressions. • At ~1:04, a student reads the lesson's goal, "By the end of this lesson, you will be able to identify and create equivalent expressions using numbers properly." Standard(s) addressed in this lesson: • It isn't clear if the actual target of the lesson was 6.EE.A.3 and/or 6.EE.A.4. To determine whether expressions were equivalent, some students applied the properties of operations to generate equivalent expressions (6.EE.A.3) and others substituted values in for the expressions to see if they named the same number (6.EE.A.4).
B. The enacted lesson appropriately relates new content to math content within or across grades.	• The teacher describes how the lesson builds upon students' prior skills in the introductory statements she makes (~ 0:27) and with the small group discussion about using the properties they learned recently (~18:04). • The lesson brings in content and skills from other standards in the EE domain including evaluating expressions with variables (6.EE.A.2) and using mathematical terms to identify parts of an expression (6.EE.A.2b).
C. The enacted lesson intentionally targets the aspects of Rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.	Aspect(s) of Rigor targeted in the standard(s) addressed in this lesson: • Standard: 6.EE.A.4 is primarily conceptual while 6.EE.A.3 is more procedural Aspect(s) of Rigor targeted in this lesson: • Lesson: procedural skill - 6.EE.A.4: It was unclear whether students accessed the conceptual understanding aspects of this standard. While some students substituted for n to determine if expressions were equivalent, it was not explicitly discussed (or evident that students understood) that to prove expressions are equivalent, the expressions must produce the same quantity

Indicator	Evidence
	for all values of the variable. At ~25:50, a student shares how one expression is equivalent to the starting expression by substituting two values; the teacher discusses one value but doesn't address the other or emphasize the definition of equivalence from 6.EE.A.4. At ~27:54, the teacher redirects a student who chooses substitution as a strategy which implies her goal in the lesson is more aligned to 6.EE.A.3. - 6.EE.A.3: Some students applied properties and combined like terms to determine if expressions were equivalent, which are procedural skills.
Core Action 2: Employ Instructional practices that allow all students to learn the content of the lesson.	
A. The teacher makes the mathematics of the lesson explicit through the use of explanations, representations, tasks, and/or examples.	• The mathematics of the lesson is not explicit. • The opening activity which uses the toothpick model does not directly relate to the lesson's learning target of identifying and creating equivalent expressions. Not all the expressions on the worksheet could be represented by the physical model. If it had been possible to use toothpicks to model more of the expressions from the lesson, the teacher would have been able to clearly connect equivalent expressions to the physical models. • At ~15:15, the goal of the main problem set is to determine whether the expressions are equivalent (yes/no) and use mathematical evidence to support your answer. However, there are missed opportunities for the teacher and students to talk in-depth about the mathematical explanations about why the expressions were equivalent. • At ~24:36, students are struggling to combine like terms with n-n. The teacher gives several examples of the expression with different values for n. However, she doesn't give students time to articulate the generalization she is trying to support them in understanding.
B. The teacher strengthens all students understanding of the content by strategically sharing students' representations and/or solution methods.	• The teacher calls on a small number of students to share their thinking with the whole class. • At ~10:50, a student shows the whole class how the expression 2s + 2 relates to the table ("the shape number, as seen in the table, we have shape number 1, 1 times 2 equals 2 and add 2 you get four which is the number of toothpicks. So that is how the

Indicator	Evidence
	algebraic expression relates to the table." The student continues to show how the expression 2s + 2 also works when you substitute for s =2. No other student was asked to share their expression or approach in this part of the lesson and the teacher did not discuss why it was important for the student to make sure the expression worked for all values. It is not clear if the sharing strengthened the understanding of the content for all students. • At ~17:30, the teacher is working with a student who is struggling with the work. When the student is not able to explain his work, the teacher asks the girl sitting next to him to explain her work again.
C. The teacher deliberately checks for understanding throughout the lesson to surface misconceptions and opportunities for growth, and adapts the lesson according to student understanding.	• Throughout the lesson, the teacher often circulates through the room questioning or observing student work (e.g., at ~16:12: "Is that equal to our target expression. You say no. I wonder why. You (another student) says yes. I don't know which is right," at ~7:22 "Tell me about this"; ~20: "Good keep going"). • At ~25:10, the teacher says, "Let's do some checking" and puts the correct answers on the SMARTBoard. All students seem to be checking and correcting their answers. The teacher asks students to use their self-check to identify a problem they want to talk about. Because this occurs toward the end of the class period, it is not clear whether the teacher is able to adapt the closing of the lesson based on what she saw from students. • At ~30:12, the teacher is circulating to look at student work and listen in on the turn and talk conversations.
D. The teacher facilitates the summary of the mathematics with references to student work and discussion in order to reinforce the purpose of the lesson.	• The teacher begins summarizing the lesson at 25:42, which leaves about 6 minutes for the lesson summary. • At ~29:29, the summary of the lesson begins. The summary gives students an opportunity to discuss the strategies they used to determine if expressions were equivalent. (The focus question on SMARTBoard was slightly different: "How do we know whether expressions are equivalent?") At ~ 30:40, a student shares that "once we answered the expression..., after we simplified everything, ... we looked back at Kailee's answer and we checked to see if it was the same...kind of the same thing"; the teacher responds by discussing simplifying, having students name the properties and the phrase "collecting like terms", rather than on the meaning of equivalence. Specific student work is not

Indicator	Evidence
	referenced during the summary. • The teacher's use of turn and talk without a whole group discussion of the key idea, risks that not all students walk away with the same understanding of the mathematics of the lesson.

Core Action 3: Provide all students opportunities to exhibit mathematical practices while engaging with the content of the lesson.	
A. The teacher provides opportunities for all students to work with and practice grade-level problems and exercises. Students work with and practice grade-level problems and exercises.	• All students are completing grade-level exercises/problems. • The students are given opportunities to write algebraic expressions consistent with the expectations of 6.EE.A.2. They also work on generating equivalent expressions using the properties of operations (6.EE.A.3) and some students exhibit partial understanding of the skills articulated in 6.EE.A.4 showing that substituting a particular value into different forms of an expression yields the same value if the expressions are equivalent.
B. The teacher cultivates reasoning and problem solving by allowing students to productively struggle. Students persevere in solving problems in the face of difficulty.	• In an interaction with a student at ~9:14, the teacher encourages the student to "make an expression, just put it on the paper and check it" which encourages risk taking and perseverance. • At ~19:36, a student seems to be struggling. The teacher's response helped the student and her table get the correct answer rather than encouraging perseverance. • At ~28:15, the teacher calls on a specific student to "explain what I would do to simplify." The student responds, "It's hard because it's, like, variables." The teacher then moves on to call on a stronger student to answer the question.
C. The teacher poses questions and problems that prompt students to explain their thinking about the content of the lesson. Students share their thinking about the content of the lesson beyond just stating answers.	• In the first part of the lesson, the toothpick problem was presented which encouraged the students to share their developing thinking. At ~3:27, the teacher says, "Please, talk to each other." • At ~16:40, teacher asks, "How do you know?" and "Give us an example of how [2 times n and n times n] are different?" At ~25:50, a student explains how to use two different values for the variable in order to show that two expressions are equivalent. • In small groups, the teacher encourages many students explain their thinking (e.g., ~17:28 "Does that makes sense? Can you explain it back to me? ... Let's have Amber explain it to [you] one more time." ~ 17:44: "Let's talk to the group.")

Indicator	Evidence
	• Throughout the rest of the lesson the teacher asks a lot of yes/no questions like: "Does this make sense? Can you see this? Do you have the right expression?" These did not invite students to share their developing thinking and there was little evidence of students responding by sharing their thinking about mathematical ideas.
D. The teacher creates the conditions for student conversations where students are encouraged to talk about each other's thinking. Students talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.	• At ~ 5:27, the teacher directs students to have conversations about their work as they create their descriptions and tables. Students seem to be asking questions of each other at their tables. • At ~7:45, the teacher says, "You started at 5, and, Denise, it looks like you were maybe starting at 1. Is that OK? Why is it OK?" • "At ~16:13, the teacher asks a table of students whether #2 is equal to the target expression. She points out that students at the table have different answers and says, "Which is right? I don't know." She walks away from the table and students use an example of substitution to justify their reasoning. • At ~19:47, the teacher asks a student whether a given expression is equivalent to the target expression. When the student says no, the teacher follows up by asking, "Why not?" Two students discuss how they can use collecting like terms to prove the equivalence, with some support from the teacher. • At ~ 21:50, there are two students excitedly discussing their solution methods. In contrast, when individual students are sharing their answers with the whole group, there isn't a lot of interaction with the student presenting. • At ~ 11:45, a student poses a question to clarify his understanding of the expression another student wrote 2s + 2 ("Don't you have to use the number of toothpicks in it?"). The student presenting her work responds but doesn't seem to understand her question, so the teacher jumps in to clarify.
E. The teacher connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas. Students use increasingly precise mathematical language and ideas.	• The teacher makes an effort to use precise language to refer to the properties and algebraic terms (e.g., ~6:09, ~13:08). • When the teacher describes the task at ~4:42, she says, "I'm going to ask you to create an algebraic expression, complete the table and write a description in words of the pattern." In this direction, she is not precise about the fact that all of these representations show the number of toothpicks in a given shape. This lack of precision in defining variables and the meaning of

Indicator	Evidence
	expressions is reflected in the way that students talk about their work. • At ~12:58, a student uses the precise language for expressions/equations when she suggests that what is written in the header should be "algebraic expressions or equations." • At ~18:20, the teacher connects the distributive property to the game Angry Birds, "So, remember if we distribute this Angry Bird to this pig, that's Angry Bird times pig... So, this Angry Bird to this Angry Bird, Angry Bird times Angry Bird."

If any uncorrected mathematical errors are made during the context of the lesson (instruction, materials, or classroom displays), note them here:

• There are issues of mathematical correctness and precision. At ~13:58, the teacher says "s is for shape" instead of precisely stating the meaning of the variable (e.g., "s represents the stage number").

Lesson Plan Analysis

Lesson: Grade 6 - Equivalent Expressions

Use this document to record information/evidence from the sample lesson plan. Evidence should consider the Core Actions. Evidence recorded will be integrated into the Feedback Summary worksheet.

Core Action 1:
Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by the college- and career-ready standards in mathematics.
Discussion Questions

- **Which standard(s) and/or cluster(s) are targeted in this lesson? Does the lesson address a part of the standard(s) or all aspects of the standard(s)? Are they grade-level standards? Are they part of the Major Work of the grade?**
- **If the standard(s) targeted are Supporting Work of the grade, how will connections be made to engage students in the Major Work of the grade?**
- **What is the mathematical learning goal for students in this lesson?**
- **Which aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, and application) do the targeted standards require? What features of the lesson support the aspect(s) of Rigor present in the targeted standards?**
- **How does the teacher plan to make explicit connections to build on students' prior skills and understandings? What will the teacher say to students or show students to make this connection clear?**

The lesson plan lists 6.EE.A.4 – identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them) – as the targeted standard for the lesson, which is on grade-level. The learning goal is for students to be able to identify and create equivalent expressions using number properties. The learning goal signals that the lesson will be more focused on 6.EE.A.3 rather than the targeted standard, 6.EE.A.4. The Expressions and Equations domain is part of the Major Work of the grade for grade 6.

The plan states that "students will revisit the concept of describing linear algebraic patterns that lead to 2 or more equivalent expressions to provide opportunities for deeper understanding." However, there is not clear evidence as to what the teacher will show or tell students to make the connection to past learning.

Identifying equivalence requires conceptual understanding. During the "Summarize 2" section, the question, "How did you determine if the expressions were equivalent?" may allow students to describe their understanding of what equivalence means. The exit slip rubric calls for "well-developed, clearly written and accurate explanation(s)." However, there is no mention in the plan of the accurate definition of equivalence (when the two expressions name the same number regardless of which value is substituted into them).

Discussion Questions

- **How does the teacher plan to use explanations, representations, tasks, and/or examples that will make the mathematics of this lesson clear to students?**
- **What will students produce? Are they expected to produce only answers?**
- **What ideas/concepts will be the focus of discussions?**
- **How will students share/present their mathematical work to support all students' understanding of the topic?**
- **When in the lesson does the teacher plan to check for understanding?**
- **How does the teacher plan to summarize the mathematics of the lesson? Will the summary include student work and discussion to reinforce the mathematical learning goal of the lesson?**

There is not clear evidence that the teacher plans to use explanations, representations, or tasks to help students answer the Focus Question of the lesson. In Launch 1/Explore 1 time is planned for students to extend a pattern and then create a table, expression, model, and description of that pattern. This work is aligned to 6.EE.C.9. In Launch 2 the teacher plans to have students determine if expressions are equivalent to the target expression: "You can use your model, your words or your table to determine if your expression is equivalent to these expressions and are these expressions equivalent to each other." In Explore 2 the teacher asks students to prove if expressions are equivalent using properties. Both align to 6.EE.A.3 or 6.EE.A.4.

In Explore 1, the teacher plans to circulate the room to identify two students who correctly connected at least two representations in different ways. A list of eight "advancing and assessing" questions are listed within the plan. A formal check for understanding is planned with an exit slip.

Summary 2 includes the following questions to reinforce the mathematics of the lesson: "How did you determine if the expressions were equivalent? What are the strategies we used to determine if expressions were equivalent? What properties did we use to prove this?"

Core Action 3:
Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

Discussion Questions

- **What mathematical language will be used in this lesson? How will the teacher support students' use of increasingly precise language, including for English language learners if applicable?**
- **Are mathematical models, mathematical representations, mathematical arguments, and mathematical counter-arguments expected from students, as required by the standards? What problem(s) and question(s) will allow students to share their thinking and/or justify their conclusions?**
- **When will students be doing grade-level problems and exercises? Will all students have this opportunity?**

The mathematical language named in the lesson plan include: "expressions," "equivalent," and "properties." How the teacher will support students' use of this language is not described. The problem set in Explore 2 asks students to use mathematical evidence to support their answers.

Student Work Analysis: Model Response

Lesson: Grade 6 - Equivalent Expressions

*Use this document to record information/evidence from the sample student work. Evidence should consider the Core Actions. Evidence recorded will be integrated into the Feedback Summary worksheet. **Before analyzing student work, be sure to have first completed the student assignment.***

General notes and observations about the task:

1. **Which standard(s) and/or cluster(s) are targeted in this assignment? Are they grade-level standards?**
   In order for students to show mathematical evidence that expressions are equivalent they have to be able to apply the properties of operations to generate equivalent expressions (6.EE.A.3) and substitute values in for n for evidence of a non-example (6.EE.A.4).

2. **What is the mathematical purpose of the assignment?**
   Students need to determine whether expressions are equivalent to a target expression and share mathematical evidence to support their answer.

3. **What aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, and application) does the assignment address? Explain.**
   Conceptual understanding (knowing what equivalence means and what constitutes mathematical evidence that two expressions are equivalent) and procedural skill (the ability to evaluate expressions at specific values of their variables (6.EE.A.2c) and/or apply the properties of operations).

Analyzing individual student samples (worksheet on next page):

1. **What does the student's work demonstrate about his/her understanding of the expectations of the assignment?**

2. **What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?**

(See worksheet)

Student Work Analysis Worksheet

Student Work Sample	What does the student's work demonstrate about his/her understanding of the expectations of the assignment?	What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?
Student A	Student A understands that he/she is supposed to indicate whether each expression is equivalent to the target expression. For #3, the erased work shows that the student started to use substitution to look for equivalence but didn't complete the process.	Since no mathematical evidence is shared, it is hard to determine whether Student A applied properties, evaluated expressions mentally, or just guessed about the equivalence of expressions.
Student B	Student B seems to understand the task of identifying equivalent expressions and names the distributive property when they use it. The student provides mathematical evidence for questions #2 and #4 but not for the other problems. This may be due to not quite fully understanding how to provide evidence in all cases (especially to support an answer of no) or he/she might just need some prompting.	Although Student B is able to distribute correctly (#4, #6 and #9) and collect like terms on occasion (#2), he/she cannot consistently combine like terms (see errors in combining like terms in #6 and lack of combining like terms in #7 and #9). It is possible that the student is not able to apply the associative property, as all of those problems require rearranging terms. He/she is not able to consistently identify equivalent expressions. The student does not seem to know how to show mathematical evidence for when expressions are not equivalent.
Student C	Student C understands that he/she is supposed to indicate whether each expression is equivalent to the target expression. For some of the expressions, the student has shown some work that was used to determine if the expressions are equivalent, but it doesn't seem that he/she knows how to provide mathematical evidence to justify their answer.	Student C shows correct evidence in #5 and #8 for why the expressions are not equivalent to the target expression. Self-grading shows all problems were correctly identified with yes or no with the exception of #6, #7, and #9. There is some work next to several questions that shows he/she is correctly combining like terms (#2), applying the associative property (#8) and applying the distributive property (#4, #5, #6). Work for #7 shows a possible misconception on the conditions needed to use the distributive property.
Student D	Student D understands that he/she is supposed to indicate whether each expression is equivalent to the target expression. The student provides mathematical evidence for questions #2, #4, #5 & #6 but not for the other problems. This may be due to not quite fully understanding how to provide evidence in all cases (especially to support an answer of no) or the student might just need some minimal prompting.	Self-grading shows all problems except #7 and #9 were correctly identified with yes or no. Student D was accurately able to identify equivalent expressions. The student shows that he/she is consistently able to apply the distributive property (#4, #5, #6, #9) and are sometimes able to combine like terms (#2) but sometimes not (#9.)

Student Work Sample	What does the student's work demonstrate about his/her understanding of the expectations of the assignment?	What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?
Student E	Student E seems to understand that he/she is supposed to determine whether or not each expression is equivalent to the given expression. The student does not, however, write 'yes/no' next to each expression but his/her self-grading indicates they understand how their work corresponds to a 'yes' or 'no' on the posted answers.	Student E shows an understanding that expressions are equivalent if they have the same value when the value of the variable is substituted by the same number. The student used the fact that when n=1, 2n + 1 = 4 to determine if the expressions were equivalent to 2n + 1. He/she seems to stop short of substituting a second value, so it is possible that the student will get some false positives. It is likely that this student does not understand that this way of determining equivalence only works if the expressions are equivalent with any value of n.
Student F	Student F understands that he/she is supposed to indicate whether each expression is equivalent to the target expression and provide mathematical evidence to support his/her answer (#1-#5). It's not clear why the student changed the process for #6-#9.	Student F shows an understanding that expressions are equivalent if they have the same value when the value of the variable is substituted by the same number, but stops short of substituting a second value, so it is possible that the student will get some false positives (#6, #7). The student also demonstrates an understanding of using properties of operations to see that a given expression is equivalent to the target expressions (#1, 2, 4).

Note: For a collection of more than six samples of student work, print this page multiple times.

1. **How did the directions and/or prompts for the assignment allow students to demonstrate the requirements of the targeted standard(s)?**
   The prompt to "use mathematical evidence" was not clear enough to allow most students to support their answers successfully; many students simply wrote yes or no and the suggestion of using properties was only brought into the prompt as a bonus. Students needed more explicit directions (and probably more space for their answer) to be able to demonstrate the requirements of 6.EE.A.3. and 6.EE.A.4.

2. **How did the mathematical content of the assignment allow students to demonstrate the requirements of the targeted standard(s)?**
   Substituting for n=1 (first value in student's table) was successful for all expressions that were equivalent to 2n + 1. Because so many students used 1 as the value for n, it would have been useful to have a problem that was a false positive for n=1, (e.g., 2n2 + 2) in order to illuminate errors in student thinking that substituting one value was proof of equivalence.

3. **What patterns do you notice in the student work?**
   - **What did students do consistently well?**
     Most students understood that they were supposed to determine if the expressions were equivalent to a target expression. Overall, most answers correctly identified whether the expressions were equivalent to the targeted expression. Some students applied properties of operations to the expressions to see if they could be represented by 2n + 2 when put into a different form. Some students substituted the value n = 1 to see if the expression would be equivalent to 4 since 2n + 2 = 4 for n= 1. Students were generally successful in applying the distributive property.

   - **Were there any common errors?**
     Students were not consistently successful in combining like terms, which resulted in #6, #7, and #9 being the most challenging for the students. For students who substituted for n, none showed an understanding that to be equivalent, the value of the expression must be the same as the targeted for all values of n. Few students were able to show mathematical evidence for all nine problems on their sheet.

Feedback Summary

Lesson: <u>Grade 6 - Equivalent Expressions</u>

Using the completed Instructional Practice Guide, the Lesson Plan Analysis, and Student Work Analysis, consider the aggregate strengths and considerations for the lesson. Choose relevant Beyond the Lesson questions to guide longer-term reflection.

Note for Facilitator: *The italicized statements can be used for group discussions, as a basis for developing questions for a coaching conversation with the teacher, or for participants to take a deeper dive into adapting the lesson and deepening their understanding of mathematics and the Shifts required by college- and career-ready standards.*

Evidence of the Shifts and standards-aligned practice	Areas where alignment to the Shifts and standards can improve
Core Action 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by the college- and career-ready standards in mathematics.	
- The mathematics planned for and executed in the lesson is on grade-level. - Teacher intentionally planned for the lesson to build students' conceptual understanding of identifying when two expressions are equivalent. (e.g., there was an expectation set in the main problem set and in exit slip rubric for students to share mathematical evidence for why expressions were or were not equivalent, the focus question was "How can we show whether expressions are equivalent?") - Students were asked to apply the properties they had previously learned to the learning goal for this lesson.	- The lack of clarity around the content of the lesson and whether or not the focus was on 6.EE.A.3 and/or 6.EE.A.4 was evident throughout the lesson, the lesson plan, and in the student work samples. The lesson plan references using the model/words/table to determine equivalence (Launch 2) and using properties (Explore 2). How these two methods of determining equivalency relate was not discussed however, nor was the focus question "How can we show whether expressions are equivalent?" fully developed. *Consider alterations to the lesson to align activities, questions, and student assignments to the mathematical ideas if the lesson was targeting both 6.EE.A.3 and 6.EE.A.4.* - The mathematical connection between the "toothpick activity" and the learning goal was unclear. *Consider ways to make the connection between the "toothpick activity" and the focus of the lesson more evident.* - Students struggled with using properties, particularly combining like terms. *Consider ways to incorporate unfinished learning into the context of this lesson.*
Core Action 2: Employ instructional practices that allow all students to learn the content of the lesson.	
- Eight "advancing and assessing" questions are noted in the lesson plan to support checking for understanding. - The teacher planned for a lesson summary that reinforces the focus question.	There was no evidence of the teacher monitoring all students' proficiency with the target of the lesson or evidence that the lesson was adapted based on what was being observed. *Consider adaptations that could be made in the lesson based on seeing the student work samples or consider ways the lesson could be altered*

Evidence of the Shifts and standards-aligned practice	Areas where alignment to the Shifts and standards can improve
• The set-up of the classroom and the structure of the lesson allowed opportunities for students to share their solution methods.	*that would have allowed for the exit slip to be administered (along with any needed edits to the exit slip prompts).* • The summary of mathematics was directed toward naming properties and the phrase "combining like terms" rather than reinforcing the purpose of the lesson. *Consider ways to summarize the lesson to reinforce the idea of identifying when two expressions are equivalent using substitution or properties of operations with references to student work and discussions that were observed.*
Core Action 3: Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.	
• Students are given opportunities to work with grade-level problems and exercises. • A problem was presented at the beginning of the lesson that encouraged students to share their developing thinking. • There is evidence of the teacher and the students using precise mathematical language. • The teacher has created classroom conditions that allow students to talk to each other, share their thinking, and ask each other questions.	• At times the questions the teacher asks of students and the students ask of each other do not support students' thinking of the mathematical ideas. Consider additional questions that could be asked of students that would invite them to share their thinking of the mathematical ideas.
Beyond the Lesson *Choose relevant Beyond the Lesson questions to guide longer-term reflection.*	

- Summarize how this lesson fits within the unit. Describe how the other lessons and tasks in this unit are intentionally sequenced to help students develop increasingly sophisticated understanding, skills, and practices.

- In what ways have your students made progress towards mastering the grade/course-level content standards? How are you monitoring and tracking their achievement of the Standards? What work still needs to be done to ensure all students achieve mastery of each standard by the end of the year?

- How will you meet all students' needs while working on grade/course-level content in this unit? (e.g., How will you provide scaffolding for students below grade/course level so they can reach the grade/course-level expectations? How will you create opportunities for students who are advanced to go deeper into the grade/course-level content?)

- In what ways have you seen your students increase their independence in applying the Standards for Mathematical Practice in learning content this year? Which practice standards do students still need to develop and how can you support them in doing so?

Implications and Next Steps

Note for facilitator: Participants could use this space to reflect on questions 1 & 2, the role-specific questions, or one or more of the italicized statements from above.

1. Based on your role in the learning community, how did examining all aspects of this lesson impact your work?
2. Based on your role in the learning community, what resources and strategies could be used to encourage and support aligned instructional practice in the classroom?

Role-Specific Reflection Questions:

- **Superintendent/District Leader** – How can I direct resources to improve standards-aligned instruction in classrooms?
- **School Leader** – What building conditions must exist to support standards-aligned instruction in classrooms?
- **Coach** – How can content-based feedback help prioritize professional learning and coaching activities to support teachers with standards-aligned instruction?
- **Teacher** – Which aspects of your instructional practice provide all students with access to grade-level standards-based content and tasks? Which aspects do not?
- **Parent** – Where do you see evidence of standards-aligned instruction in your child's classroom?
- **Partner organization** – How does our organization's theory of action and activities with districts and partners support standards aligned instruction in classrooms?