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| **Grade 3, Topic 1: Understand Multiplication and Division of Whole Numbers** |

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| **Standards addressed:** | Primary in this topic:3.OA.A.1: Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7*.3.OA.A.2: Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8*.3.OA.A.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.Secondary in this topic:3.OA.B.5: Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* |
| **Aspects of Rigor targeted by the standards:** | Primary in this topic:Conceptual Understanding, ApplicationSecondary in this topic:Procedural Skill and Fluency |
| **Applicable information from the progression documents** | In Equal Groups, the roles of the factors differ. One factor is the number of objects in a group (like any quantity in addition and subtraction situations), and the other is a multiplier that indicates the number of groups. So, for example, 4 groups of 3 objects is arranged differently than 3 groups of 4 objects. Thus there are two kinds of division situations depending on which factor is the unknown (the number of objects in each group or the number of groups). In the Array situations, the roles of the factors do not differ. One factor tells the number of rows in the array, and the other factor tells the number of columns in the situation. But rows and columns depend on the orientation of the array. If an array is rotated 90º, the rows become columns and the columns become rows. This is useful for seeing the commutative property for multiplication in rectangular arrays and areas.Multiplication and division problem representations and solution methods can be considered as falling within three levels... Level 1 is making and counting all of the quantities involved in a multiplication or division. As before, the quantities can be represented by objects or with a diagram, but a diagram affords reflection and sharing when it is drawn on the board and explained by a student. The Grade 2 standards 2.OA.3 and 2.OA.4 are at this level but set the stage for Level 2….Level 2 is repeated counting on by a given number, such as for 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. The count-bys give the running total. The number of 3s said is tracked with fingers or a visual or physical (e.g., head bobs) pattern. For 8 3s, you know the number of 3s and count by 3 until you reach 8 of them. For 24÷3, you count by 3 until you hear 24, then look at your tracking method to see how many 3s you have. Because listening for 24 is easier than monitoring the tracking method for 8 3s to stop at 8, dividing can be easier than multiplying. (See pp. 24-25 in the OA Progressions.) |
| **Essential Question(s)** | How can you represent and find the total number of objects in equal groups in multiplication and division problems?  |



Anchor Tasks

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| **Task** | **Explanation** |
| **1-1 Solve and Share** | This task provides an opportunity for interpreting the product of whole numbers and representing and finding equal groups. |
| **1-3 Solve and Share**  | This task provides an opportunity for interpreting the product of whole numbers and using arrays to represent multiplication problems. |
| [**Grade 3, Module 1, Lesson 2, Concept Development**](https://www.unbounded.org/math/grade-3/module-1/topic-a/lesson-2) | Provides an opportunity to connect skip counting, arrays and equal groups. |
| **1-4 Solve and Share** | This task provides an opportunity for arrays to represent multiplication problems and builds understanding of commutative property. |
| **1-5 Solve and Share** | This task provides an opportunity to interpret whole number quotients and focus on equal shares.  |
| **1-6 Solve and Share**  | This task provides an opportunity for students to identify the number of groups rather than the number of objects in each group.  |
| [**Fish Tanks**](https://www.illustrativemathematics.org/content-standards/3/OA/A/2/tasks/1531) | This task is focused on division and equal groups in order to go deeper on 3.OA.A.2. |
| [**Jan’s Pens**](http://sampleitems.smarterbalanced.org/Item/Details?bankKey=187&itemKey=3625) | This tasks is focused on division and equal groups in order to go deeper on 3.OA.A.2. |

Topic Rules of Thumb

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| **Rule**  | **Why?** |
| Use the definition of multiplication implied by the grade 3 standards (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.) Allow students to use repeated addition as a strategy to find products, but do not use it as a definition of multiplication. | 3.OA.A.1 defines multiplication in terms of equal groups. |
| Use the definition of division implied by the grade 3 standards (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.)Allow students to use repeated subtraction as a strategy to find quotients, but do not use it as a definition of division. | 3.OA.A.2 defines division in terms of equal groups. |
| Do not require students to use repeated addition/subtraction or open number lines to represent multiplication and division.  | 3.OA.A.3 emphasizes situations involving equal groups, arrays, and measurement quantities, but does not require specific strategies for finding products and quotients. |



Assessment Guidance, Topic 1

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| → Topic AssessmentPerformance Assessment  |
| **Item #/Action** | **Why?** |
| 1. Delete
 | 3.OA.A.1 defines multiplication as equal groups, not repeated addition. |
| 1. Modify: Delete text: “Show the problem on a number line. Then write the answer.” Delete the number line.
 | Item requires a specific model or strategy (number line). |
| 1. Delete
 | Does not align to the central concern of 3.OA.A.1. |
| 1. As Is
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| 1. Delete
 | 3.OA.A.1 defines multiplication as equal groups, not repeated addition. |
| 1. As Is
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| 1. Delete
 | Item aligns to 4.OA.A.3 (interpreting remainders). |
| 1. Delete
 | Item requires a specific model or strategy (number line). |
| 9. Modify: Delete diagram. | Item requires a specific model or strategy. |
| 10. Delete | 3.OA.A.2 defines division as equal groups, not repeated subtraction. |
| 11. Modify: Delete Part B. | Item requires a specific model or strategy. |

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| **Grade 3, Topic 2: Multiplication Facts: Use Patterns** |

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| **Standards addressed** | Primary in this topic:3.OA.A.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.3.OA.C.7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.Secondary in this topic:3.OA.B.5: Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*3.OA.A.1: Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7*.3.OA.D.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Conceptual Understanding, ApplicationSecondary in this topic:Procedural Skill and Fluency |
| **Applicable information from the progression documents** | In the Array situations, the roles of the factors do not differ. One factor tells the number of rows in the array, and the other factor tells the number of columns in the situation. But rows and columns depend on the orientation of the array. If an array is rotated 90º, the rows become columns and the columns become rows. This is useful for seeing the commutative property for multiplication in rectangular arrays and areas.Multiplication and division problem representations and solution methods can be considered as falling within three levels related to the levels for addition and subtraction (see Appendix). Level 1 is making and counting all of the quantities involved in a multiplication or division. Level 2 is repeated counting on by a given number, such as for 3 3; 6; 9; 12; 15; 18; 21; 24; 27; 30. The count-bys give the running total. The number of 3s said is tracked with fingers or a visual or physical (e.g., head bobs) pattern. For 8 x 3, you know the number of 3s and count by 3 until you reach 8 of them. For 24$÷$3, you count by 3 until you hear 24, then look at your tracking method to see how many 3s you have. Because listening for 24 is easier than monitoring the tracking method for 8 x 3 is to stop at 8, dividing can be easier than multiplying. The difficulty of saying and remembering the count-by for a given number depends on how closely related it is to 10, the base for our written and spoken numbers. For example, the count-by sequence for 5 is easy, but the count-by sequence for 7 is difficult. Decomposing with respect to a ten can be useful in going over a decade within a count-by. For example, in the count-by for 7, students might use the following mental decompositions of 7 to compose up to and then go over the next decade, e.g., 14 + 7 =14 + 6 + 1= 20 + 1 =21. The count-by sequence can also be said with the factors, such as “one times three is three, two times three is six, three times three is nine, etc.” Seeing as well as hearing the count-bys and the equations for the multiplications or divisions can be helpful. Level 3 methods use the associative property or the distributive property to compose and decompose. These compositions and decompositions may be additive (as for addition and subtraction) or multiplicative. For example, students multiplicatively compose or decompose: 4 x 6 is easier to count by 3 eight times: 4 x 6 = 4 x (2 x 3) = (4 x 2) x 3 = 8 x 3: Students may know a product 1 or 2 ahead of or behind a given product and say: I know 6 x 5 is 30, so 7 x 5 is 30 + 5 more, which is 35. This implicitly uses the distributive property: 7 x 5 = (6 +1) x 5 = 6 x 5 + 1 x 5= 30 + 5 = 35.Patterns make multiplication by some numbers easier to learn than multiplication by others, so approaches may teach multiplications and divisions in various orders depending on what numbers are seen as or are supported to be easiest. All of the understandings of multiplication and division situations, of the levels of representation and solving, and of patterns need to culminate by the end of Grade 3 in fluent multiplying and dividing of all single-digit numbers and 10…Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning. To achieve this by the end of Grade 3, students must begin working toward fluency for the easy numbers as early as possible.(See pp.24-27 in the OA Progressions.) |
| **Essential Question(s)** | How can I use what I know about equal groups to help me quickly multiply numbers? (Focus on 0,1,2,5, and 10.) |



Anchor Tasks

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| **Task** | **Explanation** |
| **2-5 Solve and Share and Visual Learning Bridge** (problem only)  | Beginning with this lesson will allow students to make connections to skip counting work in grade 2 and give teachers information about how students are developing fluency with multiplication facts. |
| **2.1 Solve and Share** (problem only) | This problem highlights multiplication by 2s.  |
| **2-1 Intervention Activity** | Provides opportunity to connect skip counting by 2s and 5s to array work in Topic 1. |
| **2-4 Solve and Share** | This problem highlights students using patterns to connect skip counting by 10s to multiplication by 10s.  |
| **2.3 Visual Learning Bridge and Solve and Share**  | Beginning with the Visual Learning Bridge provides students with an opportunity to start think about equal groups, so they can extend that thinking as they work with zero as a factor.  |
| **5-1 Intervention Activity** plus question of “How can you explain this?” for each pattern identified  | This activity is moved up from Topic 5 so that students can explore patterns in the multiplication table as they are identifying multiplication patterns. This will allow students to understand the patterns on a multiplication table and will allow for the table to be a resource for students. Each time students find a pattern ask, “how can you explain this?” to ensure students are connecting the identification of the pattern to what they know about equal groups. |
| [**Patterns in the Multiplication Table**](https://www.illustrativemathematics.org/content-standards/3/OA/D/9/tasks/956) | This task builds on student understanding developed in the 5-1 Intervention Activity and builds deeper understanding of why patterns exist in the multiplication table. |

Topic Rules of Thumb

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| **Rule**  | **Why?** |
| Use the definition of multiplication implied by the grade 3 standards (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.) Allow students to use repeated addition as a strategy to find products, but do not use it as a definition of multiplication. | 3.OA.A.1 defines multiplication in terms of equal groups. |
| Use the definition of division implied by the grade 3 standards (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.)Allow students to use repeated subtraction as a strategy to find quotients, but do not use it as a definition of division. | 3.OA.A.2 defines division in terms of equal groups. |
| Connect the work for looking for patterns in multiplication facts to grade 2 work with skip counting.  | Level 2 is repeated counting on by a given number, such as for 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. The count-bys give the running total. The number of 3s said is tracked with fingers or a visual or physical (e.g., head bobs) pattern. For 8 x 3, you know the number of 3s and count by 3 until you reach 8 of them. (CC/OA Progression) |
| Do not introduce multiplying by 9s in this topic. Move lesson 2-2 into Topic 3. This should be problem only.  | Multiplying by 9s, for many students, requires using easier facts; moving the lesson later will allow them to have experience with more multiplication facts and make connections. Patterns make multiplication by some numbers easier to learn than multiplication by others, so approaches may teach multiplications and divisions in various orders depending on what numbers are seen as or are supported to be easiest. (CC/OA Progression) |
| Connect all strategies to conceptual understanding. Do not teach any tricks.  | As implied in the foregoing, this isn’t a matter of instilling facts divorced from their meanings, but rather the outcome of a carefully designed learning process that heavily involves the interplay of practice and reasoning. All of the work on how different numbers fit with the base-ten numbers culminates in these “just know” products and is necessary for learning products. (OA Progression) |
| As students encounter the anchor tasks and other features of the program, make sure students are accessing a range of factors in multiplication and division. | By the end of grade 3, students should be in fluent multiplying and dividing of all single-digit numbers and 10. |



Assessment Guidance, Topic 2

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| → Topic AssessmentPerformance Assessment  |
| **Item #/Action** | **Why?** |
| 1. Delete
 | Work with multiples of 9 was moved to Topic 3, per Topic Rules of Thumb above.  |
| 1. Modify: Delete Part A and Part B.
 | Item requires a specific model or strategy. |
| 1. As Is
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| 1. Delete
 | Work with multiples of 9 was moved to Topic 3, per Topic Rules of Thumb above. |
| 1. As Is
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| 1. As Is
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| 1. As Is
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| 1. As Is
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| 9. As Is |  |
| 10. As Is |  |
| 11. As Is |  |
| 12. As Is |  |
| 13. Delete | Item aligns to 4.OA.A.3 (interpreting remainders). |

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| **Grade 3, Topic 3: Apply Properties: Multiplication Facts for 3, 4, 6, 7, 8 (and 9)** |

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| **Standards addressed** | Primary in this topic:3.OA.B.5: Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*3.OA.C.7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.Secondary in this topic:3.OA.D.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*.3.OA.A.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Conceptual UnderstandingSecondary in this topic:Procedural Skill and Fluency, Application |
| **Applicable information from the progression documents** | Multiplication and division problem representations and solution methods can be considered as falling within three levels related to the levels for addition and subtraction (see Appendix). Level 1 is making and counting all of the quantities involved in a multiplication or division. Level 2 is repeated counting on by a given number, such as for 3 3; 6; 9; 12; 15; 18; 21; 24; 27; 30. The count-bys give the running total. The number of 3s said is tracked with fingers or a visual or physical (e.g., head bobs) pattern. For 8 x 3, you know the number of 3s and count by 3 until you reach 8 of them. For 24$÷$3, you count by 3 until you hear 24, then look at your tracking method to see how many 3’s you have. Because listening for 24 is easier than monitoring the tracking method for 8 x 3s to stop at 8, dividing can be easier than multiplying. The difficulty of saying and remembering the count-by for a given number depends on how closely related it is to 10, the base for our written and spoken numbers. For example, the count-by sequence for 5 is easy, but the count-by sequence for 7 is difficult. Decomposing with respect to a ten can be useful in going over a decade within a count-by. For example, in the count-by for 7, students might use the following mental decompositions of 7 to compose up to and then go over the next decade, e.g., 14 + 7 =14 + 6 + 1= 20 + 1 =21. The count-by sequence can also be said with the factors, such as “one times three is three, two times three is six, three times three is nine, etc.” Seeing as well as hearing the count-bys and the equations for the multiplications or divisions can be helpful. Level 3 methods use the associative property or the distributive property to compose and decompose. These compositions and decompositions may be additive (as for addition and subtraction) or multiplicative. For example, students multiplicatively compose or decompose: 4x 6 is easier to count by 3 eight times: 4 x 6 = 4 x (2 x 3) = (4 x 2) x 3 = 8 x 3: Students may know a product 1 or 2 ahead of or behind a given product and say: I know 6 x 5 is 30, so 7 x 5 is 30 + 5 more, which is 35. This implicitly uses the distributive property: 7 x 5 = (6 + 1) x 5 = 6 x 5 +1 x 5= 30 + 5 = 35.Patterns make multiplication by some numbers easier to learn than multiplication by others, so approaches may teach multiplications and divisions in various orders depending on what numbers are seen as or are supported to be easiest.All of the understandings of multiplication and division situations, of the levels of representation and solving, and of patterns need to culminate by the end of Grade 3 in fluent multiplying and dividing of all single-digit numbers and 10.Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning. To achieve this by the end of Grade 3, students must begin working toward fluency for the easy numbers as early as possible.(See pp. 24-27 in the OA Progressions.) |
| **Essential Question(s)** | How can unknown multiplication facts be found using known facts? |



Anchor Tasks

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| **Task** | **Explanation** |
| **3-1 Solve and Share** | This task provides a good opportunity for use of the distributive property, which is part of 3.OA.B.5. |
| **3-2 Solve and Share** | This task provides a good opportunity for use of the distributive property, which is part of 3.OA.B.5. |
| **3-3 Solve and Share** | This task provides a good opportunity for use of the distributive property, which is part of 3.OA.B.5. |
| **3-4 Solve and Share** | This task provides a good opportunity for use of the distributive property, which is part of 3.OA.B.5. |
| **3-5 Solve and Share** | This task provides a good opportunity for use of the distributive property, which is part of 3.OA.B.5. |
| **2-2 Solve and Share**(problem only) | For many students, multiplying by 9s requires using easier facts. Moving the lesson to this topic will allow them to have experience with more multiplication facts and make connections.  |
| [**Valid Equalities? (Part 2)**](https://www.illustrativemathematics.org/content-standards/3/OA/B/5/tasks/1821) | These problems offer students the opportunity to think about multiplication equations without context, to apply the facts they have learned over Topics 1 and 2, and to use the properties of operations to solve multiplication problems.  |
| **3-7 Solve and Share** | This problem offers an opportunity for students to use the associative property, which is part of 3.OA.B.5. |

Topic Rules of Thumb

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| **Rule**  | **Why?** |
| Highlight student thinking based on the application of properties of operations. Include a variety of strategies so that students can see how to determine unknown facts from facts that they personally know. | To make sure 3.OA.B.5 is fully realized and students understand how applying properties helps lead toward fluency. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property) |
| Do not require students to use formal notation to represent the distributive property.  | Parenthetical notation is reserved for grade 5 (5.OA.A.1). |
| As students encounter the anchor tasks and other features of the program, make sure students are accessing a range of factors in multiplication and division. | By the end of grade 3, students should be in fluent multiplying and dividing of all single-digit numbers and 10. |



Assessment Guidance, Topic 3

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| → Topic AssessmentPerformance Assessment  |
| **Item #/Action** | **Why?** |
| 1. Delete  | Item aligns to 5.OA.A.1 (using parentheses). |
| 2. As Is |  |
| 3. Delete | Does not align to the central concern of 3.OA.B.5. |
| 4. As Is |  |
| 5. As Is |  |
| 6. As Is |  |
| 7. Delete | Item aligns to 5.OA.A.1 (using parentheses). |
| 8. As Is |  |
| 9. Delete | Does not align to the central concern of 3.OA.A.3 or 3.OA.B.5. |
| 10. As Is |  |
| 11. Delete | Does not align to the central concern of 3.OA.A.3 or 3.OA.B.5. |
| 12. As Is |  |

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| **Grade 3, Topic 4: Use Multiplication To Divide: Division Facts and****Topic 5: Fluently Multiply and Divide Within 100** |

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| **Standards addressed** | Primary in this topic:3.OA.B.6: Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8*.3.OA.D.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*.3.OA.A.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = \_ ÷ 3, 6 × 6 = ?*3:OA.C.7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.Secondary in this topic:3.OA.A.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.3.OA.D.8: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Conceptual Understanding, Procedural Skill and FluencySecondary in this topic:Application |
| **Applicable information from the progression documents** | In the Array situations, the roles of the factors do not differ. One factor tells the number of rows in the array, and the other factor tells the number of columns in the situation. But rows and columns depend on the orientation of the array. If an array is rotated 90º, the rows become columns and the columns become rows. This is useful for seeing the commutative property for multiplication in rectangular arrays and areas.Relating Equal Group situations to Arrays, and indicating rows or columns within arrays, can help students see that a corner object in an array (or a corner square in an area model) is not double counted: at a given time, it is counted as part of a row or as a part of a column but not both.Problems in terms of “rows” and “columns,” e.g., “The apples in the grocery window are in 3 rows and 6 columns,” are difficult because of the distinction between the number of things in a row and the number of rows. There are 3 rows but the number of columns (6) tells how many are in each row. There are 6 columns but the number of rows (3) tells how many are in each column. Students do need to be able to use and understand these words, but this understanding can grow over time while students also learn and use the language in the other multiplication and division situations.Multiplication and division problem representations and solution methods can be considered as falling within three levels related to the levels for addition and subtraction (see Appendix). Level 1 is making and counting all of the quantities involved in a multiplication or division. Level 2 is repeated counting on by a given number, such as for 3: 3; 6; 9; 12; 15; 18; 21; 24; 27; 30. The count-bys give the running total. The number of 3s said is tracked with fingers or a visual or physical (e.g., head bobs) pattern. For 8 \* 3, you know the number of 3s and count by 3 until you reach 8 of them. For 24$÷$3, you count by 3 until you hear 24, then look at your tracking method to see how many 3s you have. Because listening for 24 is easier than monitoring the tracking method for 8 x 3s to stop at 8, dividing can be easier than multiplying. The difficulty of saying and remembering the count-by for a given number depends on how closely related it is to 10, the base for our written and spoken numbers. For example, the count-by sequence for 5 is easy, but the count-by sequence for 7 is difficult. Decomposing with respect to a ten can be useful in going over a decade within a count-by. For example, in the count-by for 7, students might use the following mental decompositions of 7 to compose up to and then go over the next decade, e.g., 14+7 =14+6+1= 20+1 =21. The count-by sequence can also be said with the factors, such as “one times three is three, two times three is six, three times three is nine, etc.” Seeing as well as hearing the count-bys and the equations for the multiplications or divisions can be helpful. Level 3 methods use the associative property or the distributive property to compose and decompose. These compositions and decompositions may be additive (as for addition and subtraction) or multiplicative. For example, students multiplicatively compose or decompose: 4 x 6 is easier to count by 3 eight times: 4 x 6 = 4 x (2 x 3) = (4 x 2) x 3 = 8 x 3: Students may know a product 1 or 2 ahead of or behind a given product and say: “I know 6 x 5 is 30, so 7 x 5 is 30 + 5 more, which is 35. This implicitly uses the distributive property: 7 x 5 = (6 +1) x 5 = 6 x 5 + 1 x 5 = 30 + 5 = 35.Multiplications and divisions can be learned at the same time and can reinforce each other. Level 2 methods can be particularly easy for division, as discussed above. Level 3 methods may be more difficult for division than for multiplication. Throughout multiplication and division learning, students gain fluency and begin to know certain products and unknown factors.Use of two-step problems involving easy or middle difficulty adding and subtracting within 1,000 or one such adding or subtracting with one step of multiplication or division can help to maintain fluency with addition and subtraction while giving the needed time to the major Grade 3 multiplication and division standards.All of the understandings of multiplication and division situations, of the levels of representation and solving, and of patterns need to culminate by the end of Grade 3 in fluent multiplying and dividing of all single-digit numbers and 10.Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning. To achieve this by the end of Grade 3, students must begin working toward fluency for the easy numbers as early as possible.Patterns make multiplication by some numbers easier to learn than multiplication by others, so approaches may teach multiplications and divisions in various orders depending on what numbers are seen as or are supported to be easiest. (See pp.24-27 in the OA Progressions.) |
| **Essential Question(s)** | How are multiplication and division related? How can unknown division facts be found using known multiplication facts? How do you use strategies that are accurate and efficient to multiply and divide? |



Anchor Tasks

|  |  |
| --- | --- |
| **Task** | **Explanation** |
| **5-6 Solve and Share and 5-7 Solve and Share** | Combining these tasks on writing multiplication and division story problems provides an opportunity for students to connect the multiplication and division work they have completed in previous topics to do the numerical work of connecting multiplication and division.  |
| **4-1 Solve and Share** | This task provides an opportunity to make connections between multiplication and division and supports understanding required by 3.OA.B.6. |
| **4-2 Solve and Share and 4-4 Solve and Share** | Together, these tasks offer an opportunity to solve division problems targeting unknown number of groups and unknown number of objects in each group. These tasks reinforce the connections between multiplication and division. |
| [**Finding the Unknown in a Division Equation**](https://www.illustrativemathematics.org/content-standards/3/OA/A/4/tasks/1814) | This task provides an opportunity to reinforce the connections between multiplication and division and identify student misconceptions. |
| **4-6 Visual Learning Bridge and Solve and Share** | These tasks offer an opportunity to extend understandings of division to the factors of 0 and 1. Starting with the Visual Learning Bridge will help students think about what the factors 0 and 1 represent in division problems.  |
| **4-7 Solve and Share** | This task provides an opportunity to solve for unknown factor as required by 3.OA.B.6. |
| **5-2 Solve and Share and 5-3 Solve and Share** | These tasks support student understanding of the relationship between multiplication and division, as they will find the solutions to multiplication and division problems in the multiplication table. The tasks also provide students with an opportunity to practice and develop fluency. |
| [**Isabelle’s Garden**](http://schools.nyc.gov/NR/rdonlyres/067DB512-9685-43CD-9ACE-246CBD0B2A89/0/NYCDOE_G3_Math_IsabellasGarden_FINAL.pdf) | These tasks support student understanding of the relationship between multiplication and division, as they will find the solutions to multiplication and division problems in the multiplication table. The tasks also provide students with an opportunity to practice and develop fluency. |

Topic Rules of Thumb

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| --- | --- |
| **Rule**  | **Why?** |
| Highlight student thinking based on the relationship between multiplication and division and application of properties of operations.  | To make sure 3.OA.B.5 is fully realized and students understand how applying properties helps lead toward fluency. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property) |
| Include a variety of strategies so that students can see how to determine unknown facts build from facts that they personally know. | 3.OA.C.7 suggests strategies such as the relationship between multiplication and division and the properties of operations. Strategies that use the relationship between multiplication and division help students to learn unknown facts from known facts, but not all students will have the same “known” facts.  |
| As student encounter the Anchor Tasks and other features of the program, make sure students are accessing a range of factors in multiplication and division. | By the end of grade 3, students should be in fluent multiplying and dividing of all single-digit numbers and 10. |
| Teachers do not need to expect students to have mastered multiplication and division fluency by the end of this topic; rather, fluency practice should continue throughout the year.  | 3.OA.C.7 requires that, by the end of grade 3, students should be fluent multiplying and dividing of all single-digit numbers and 10. |

Assessment Guidance, Topics 4 and 5

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| → Topic Assessment (Topic 4 only) Performance Assessment  |
| To assess the fluency requirements of 3.OA.C.7, use the [Multiplication and Division within 100 Mini-Assessment.](https://achievethecore.org/page/861/multiplication-and-division-within-100-mini-assessment) |
| **Item #/Action** | **Why?** |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. Delete
 | Item aligns to 4.OA.A.3 (interpreting remainders). |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. Delete
 | Item aligns to 4.OA.A.3 (interpreting remainders). |
| 9. Delete | Item requires a specific model or strategy. |
| 10. Delete | Item aligns to 4.OA.A.3 (interpreting remainders). |
| 11. As Is |  |
| 12. As Is |  |
| 13. Modify: Delete the bar diagram. | Item requires a specific model or strategy. |
| 14. Delete | Item requires a specific model or strategy. |
| 15. Delete | Item aligns to 2.OA.C.3 (even and odd numbers). |
| 16. As Is |  |
| 17. Delete | Repeats content from previous questions |
| 18. Modify: Delete Part A and Part B. | Does not align to the central concern of 3.OA.A.3 |

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| **Grade 3, Topic 6: Connect Area To Multiplication and Addition** |

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| --- | --- |
| **Standards addressed** | Primary in this topic:3.MD.C.5a: A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.3.MD.C.6: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).3.MD.C.7a: Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.3.MD.C.7c: Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.3.MD.C.7d: Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.Secondary in this topic:3.MD.C.5b: A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.3.MD.C.6: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).3.MD.C.7.b: Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Conceptual UnderstandingSecondary in this topic:Procedural Skill and Fluency, Application |
| **Applicable information from the progression documents** | Students need to learn to conceptualize area as the amount of two-dimensional space in a bounded region and to measure it by choosing a unit of area, often a square. A two-dimensional geometric figure that is covered by a certain number of squares without gaps or overlaps can be said to have an area of that number of square units.…students can be taught to multiply length measurements to find the area of a rectangular region. But, in order that they make sense of these quantities, they first learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows.They also learn to understand and explain that the area of a rectangular region of, for example, 12 length-units by 5 length-units can be found either by multiplying 12 x 5 or by adding two products, e.g., 10 x 5 and 2 x 5, illustrating the distributive property.(See pp. 16-18 in the MD Progressions.) |
| **Essential Question(s)** | How can you find the area of a figure?How does area connect to multiplication and addition?  |



Anchor Tasks

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| --- | --- |
| **Task** | **Explanation** |
| [**Understand area as an attribute of plane figures**](https://www.unbounded.org/downloads/4924/preview?slug_id=37374) | Establishes concept of area. |
| **6-1 Solve and Share and Visual Learning Bridge** (problem only) | These tasks develop students’ understanding of a unit square as required by 3.MD.C.5a and 3.MD.C.6.  |
| [**Finding the Area of Polygons**](https://www.illustrativemathematics.org/content-standards/3/MD/C/6/tasks/1515) | This task builds on students’ understanding of a unit square as required by 3.MD.C.5a and 3.MD.C.6.  |
| **6-2 Solve and Share** | This task provides an opportunity for students to connect their emerging understanding of a unit square to measuring and making sense of area.  |
| **6-3 Solve and Share** | This task provides students with an opportunity to deepen their conceptual understanding of a square unit and area.  |
| **6-4 Solve and Share**  | These tasks connect students’ emerging understanding of area to their understanding of multiplication and arrays. |
| **6-5 Solve and Share**  | These tasks provide students with an opportunity to develop conceptual understanding of area and connect their understanding of the distributive property of multiplication to finding area.  |
| **6-6 Solve and Share**  | These tasks allow students to extend their understanding of area to apply to irregular shapes and connects to students’ previous work in breaking apart arrays. |
| **6-7 Solve and Share**  | This task builds on students’ understanding of area and targets recognizing area as additive (3.MD.C.7d). |

Topic Rules of Thumb

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| **Rule**  | **Why?** |
| As students employ the distributive property to find area, do not hold students responsible for writing equations using parentheses. | Grouping symbols are not required until 5.OA.A.1. |
| To avoid confusion in later grades, use precise language when talking about definition of area. Do not define area as length times width; instead, refer to the definition provided in the Measurement and Data Progression Document.  | Students need to learn to conceptualize area as the amount of two-dimensional space in a bounded region and to measure it by choosing a unit of area, often a square. (Progression Document Measurement and Data) |
| Connect students’ work in Topics 1–5 using arrays to the area model to reinforce the connection between the two models. | Strengthens coherence between 3.OA standards and 3.MD.C.7. |



Assessment Guidance, Topic 6

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| → Topic Assessment Performance Assessment  |
| **Item #/Action** | **Why?** |
| 1. As Is
 |  |
| 1. Delete
 | Item aligns to 5.OA.A.1 (using parentheses). |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 9. As Is |  |
| 10. As Is |  |
| 11. As Is |  |
| 12. As Is |  |
| 13. As Is |  |
| 14. As Is |  |
| 15. As Is |  |
| 16. Delete | Repeats content from question #8. |
| 17. Delete | Does not align to central concern of 3.MD.C.6 or 3.MD.C.7a. |
| 18.Modify: Delete Part A. | Item aligns to 4.OA.A.3. |

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| **Grade 3, Topic 7: Represent and Interpret Data** |

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| **Standards addressed** | Primary in this topic:3.MD.B.3: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets*.Secondary in this topic:3.OA.A.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.3.OA.D.8: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:ApplicationSecondary in this topic:Procedural Skill and Fluency, Conceptual Understanding |
| **Applicable information from the progression documents** | In Grade 3, the most important development in data representation for categorical data is that students now draw picture graphs in which each picture represents more than one object, and they draw bar graphs in which the height of a given bar in tick marks must be multiplied by the scale factor in order to yield the number of objects in the given category. These developments connect with the emphasis on multiplication in this grade. At the end of Grade 3, students can draw a scaled picture graph or a scaled bar graph to represent a data set with several categories (six or fewer categories)…Students can gather categorical data in authentic contexts, including contexts arising in their study of science, history, health, and so on. Of course, students do not have to generate the data every time they work on making bar graphs and picture graphs. That would be too time-consuming. After some experiences in generating the data, most work in producing bar graphs and picture graphs can be done by providing students with data sets. The Standards in Grades 1–3 do not require students to gather categorical data.(See p. 7 in the MD Progressions.)Use of two-step problems involving easy or middle difficulty adding and subtracting within 1,000 or one such adding or subtracting with one step of multiplication or division can help to maintain fluency with addition and subtraction while giving the needed time to the major Grade 3 multiplication and division standards.(See p. 28 of the OA Progressions.) |
| **Essential Question(s)** | N/A |



Assessment Guidance, Topic 7

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| → Topic Assessment Performance Assessment  |
| **Item #/Action** | **Why?** |
| 1. Modify: Delete Part A.
 | Part A has multiple correct answers. |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 9. Delete | Item has multiple correct answers. |
| 10. As Is |  |
| 11. As Is |  |
| 12. Delete | Repeats content from other questions. |
| 13. Delete | Repeats content from other questions. |
| 14. Delete | Repeats content from other questions. |
| 15. Delete | Repeats content from other questions. |
| 16. As Is |  |
| 17. As Is |  |
| 18. Delete | Does not align to the central concern of 3.MD.B.3, |

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| **Grade 3, Topic 8: Use Strategies and Properties To Add and Subtract****Note: This topic is not aligned to grade 3 expectations. The recommendation is to incorporate some of the rounding work into Topic 9 but to skip this topic.** |

|  |  |
| --- | --- |
| **Standards addressed** | Primary in this topic:3.NBT.A.2: Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.Secondary in this topic:3.OA.D.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*.3.NBT.A.1: Use place value understanding to round whole numbers to the nearest 10 or 100. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Procedural Skill and FluencySecondary in this topic:Conceptual Understanding |
| **Applicable information from the progression documents** | At Grade 3, the major focus is multiplication, so students' work with addition and subtraction is limited to maintenance of fluency within 1000 for some students and building fluency to within 1000 for others...They focus on methods that generalize readily to larger numbers so that these methods can be extended to 1,000,000 in Grade 4 and fluency can be reached with such larger numbers. Fluency within 1000 implies that students use written methods without concrete models or drawings, though concrete models or drawings can be used with explanations to overcome errors and to continue to build understanding as needed.[Students] need to understand that when moving to the right across the places in a number (e.g., 456), the digits represent smaller units. When rounding to the nearest 10 or 100, the goal is to approximate the number by the closest number with no ones or no tens and ones (e.g., so 456 to the nearest ten is 460; and to the nearest hundred is 500). Rounding to the unit represented by the leftmost place is typically the sort of estimate that is easiest for students and often is sufficient for practical purposes. Rounding to the unit represented by a place in the middle of a number may be more difficult for students (the surrounding digits are sometimes distracting). Rounding two numbers before computing can take as long as just computing their sum or difference.(See p. 12 in the NBT Progressions.) |
| **Essential Question(s)** | N/A |

Assessment Guidance, Topic 8

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| Topic 8 assessments do not align to 3.NBT.A.2 |

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| **Grade 3, Topic 9: Fluently Add and Subtract Within 1,000** |

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| --- | --- |
| **Standards addressed** | Primary in this topic:3.NBT.A.2: Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Procedural Skill and Fluency |
| **Applicable information from the progression documents** | At Grade 3, the major focus is multiplication, so students' work with addition and subtraction is limited to maintenance of fluency within 1000 for some students and building fluency to within 1000 for others...They focus on methods that generalize readily to larger numbers so that these methods can be extended to 1,000,000 in Grade 4 and fluency can be reached with such larger numbers. Fluency within 1000 implies that students use written methods without concrete models or drawings, though concrete models or drawings can be used with explanations to overcome errors and to continue to build understanding as needed.(See p. 12 in the NBT Progressions.) |
| **Essential Question(s)** | N/A  |



Assessment Guidance, Topic 9

|  |  |
| --- | --- |
| **Item #/Action** | **Why?** |
| Use an assessment for this topic that more closely resembles the fluency expectations for this standard, such as examples found here:https://achievethecore.org/content/upload/SBAC\_3.NBT.A.2.pdf | The *enVisionmath 2.0* assessment does not adequately assess the procedural skill and fluency expectations of 3.NBT.A.2. |

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| **Grade 3, Topic 10: Multiply by Multiples of 10** |

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| --- | --- |
| **Standards addressed** | Primary in this topic:3.NBT.A.3: Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Conceptual UnderstandingSecondary in this topic:Procedural Skill and Fluency |
| **Applicable information from the progression documents** | [T]he product 3 x 50 can be represented as 3 groups of 5 tens, which is 15 tens, which is 150. This reasoning relies on the associative property of multiplication: 3 x 50 = 3 x (5 x 10) = (3 x 5) x 10 = 15 x 10 = 150. It is an example of how to explain an instance of a calculation pattern for these products: calculate the product of the non-zero digits, then shift the product one place to the left to make the result ten times as large.(See p. 12 in the NBT Progressions.) |
| **Essential Question(s)** | N/A |



Assessment Guidance, Topic 10

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| → Topic Assessment Performance Assessment  |
| **Item #/Action** | **Why?** |
| 1. Modify: Delete last sentence of the question and the model.
 | Item requires a specific strategy or model not based on place value (open number line).  |
| 1. Delete
 | Item aligns to 5.OA.A.1(using parentheses). |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. Delete
 | Item aligns to 5.OA.A.1(using parentheses). |
| 1. As Is
 |  |
| 1. Delete
 | Item aligns to 5.OA.A.1(using parentheses). |
| 1. Modify: Delete last sentence of the question and the model.
 | Item requires a specific strategy or model not based on place value (open number line).  |
| 9. As Is |  |
| 10.Delete | Item aligns to 5.OA.A.1(using parentheses). |
| 11. Modify: Delete Part A and Part B. | Item requires a specific strategy or model not based on place value (open number line).  |

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| **Grade 3, Topic 11: Use Operations With Whole Numbers To Solve Problems** |

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| **Standards addressed** | Primary in this topic:3.OA.D.8: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:ApplicationSecondary in this topic:Procedural Skill and Fluency, Conceptual Understanding |
| **Applicable information from the progression documents** | Use of two-step problems involving easy or middle difficulty adding and subtracting within 1,000 or one such adding or subtracting with one step of multiplication or division can help to maintain fluency with addition and subtraction while giving the needed time to the major Grade 3 multiplication and division standards.(See p. 28 in the OA Progressions.) |
| **Essential Questions** | How can you use the four operations to solve problems and check for reasonableness in your answers? |



Anchor Tasks

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| --- | --- |
| **Task** | **Explanation** |
| **11-1 Solve and Share** (problem only) | This task provides students with an opportunity to make sense of the problem and use their understanding of the four operations to solve.  |
| **11-2 Solve and Share and Visual Learning Bridge** (problem only) | These tasks provide students with an opportunity to make sense of the problem and use their understanding of the four operations to solve. |
| **11-3 Solve and Share and Visual Learning Bridge** (problem only) | These tasks provide students with an opportunity to make sense of the problem and use their understanding of the four operations to solve. |

Topic Rules of Thumb

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| --- | --- |
| **Rule**  | **Why?** |
| Prompt students to assess the reasonableness of their answers and classmates’ answers.  | Targets the third sentence of 3.OA.D.8. (“Assess the reasonableness of answers using mental computation and estimation strategies including rounding.”) |
| As much as possible, present word problems to students and encourage them to use the models and strategies that are most helpful to them. Avoid requiring a specific model, strategy, or “first step” for solving.  | MP1 requires students to make sense of problems and persevere in solving them. |
| Record student work with equations using variables and ask students to do the same. | Targets second sentence of 3.OA.D.8. |



Assessment Guidance, Topic 11

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| → Topic Assessment Performance Assessment  |
| **Item #/Action** | **Why?** |
| 1. Modify: Keep first paragraph of the question. Then add the directions “Use equations to represent the problem.”
 | Does not align to the central concern of 3.OA.D.8. |
| 1. Delete
 | 3.OA.D.8 requires students to use “a letter standing for the unknown.” Problems that structure this using two equations with two different unknowns are above the standard. |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. Modify: Delete Part B.
 | Item does not align to the central concern of 3.OA.D.8 (interpreting equations). |
| 1. As Is
 |  |
| 1. Modify: Change last sentence to “How much did he spend on lunches?”
 | Item requires a specific strategy or model.  |
| 9. Delete | Does not align to the central concern of 3.OA.D.8. |
| 10. Delete | Does not align to the central concern of 3.OA.D.8. |

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| **Grade 3, Topic 12: Understand Fractions As Numbers** |

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| **Standards addressed** | Primary in this topic:3.NF.A.1: Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.3.NF.A.3c: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.3.NF.A.2a: Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.3.NF.A.2b: Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.3.MD.B.4: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.Secondary in this topic:3.G.A.2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape*. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Conceptual UnderstandingSecondary in this topic:Application |
| **Applicable information from the progression documents** | Grade 3 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and taking one part, e.g., if a whole is partitioned into 4 equal parts then each part is $\frac{1}{4}$ of the whole, and 4 copies of that part make the whole. Next, students build fractions from unit fractions, seeing the numerator 3 of $\frac{3}{4}$ as saying that $\frac{3}{4}$ is the quantity you get by putting 3 of the $\frac{1}{4}$ 's together. They read any fraction this way, and in particular there is no need to introduce "proper fractions" and "improper fractions" initially; $\frac{5}{3}$ is the quantity you get by combining 5 parts together when the whole is divided into 3 equal parts.As students experiment on number line diagrams they discover that many fractions label the same point on the number line, and are therefore equal; that is, they are equivalent fractions. For example, the fraction $\frac{1}{2}$ is equal to $\frac{2}{4}$ and also to $\frac{3}{6}$. Students can also use fractions strips to see fraction equivalence.Previously, in Grade 2, students compared lengths using a standard measurement unit. In Grade 3, they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions. For example, [a] segment from 0 to $\frac{3}{4}$ is shorter than the segment from 0 to $\frac{5}{4}$ because it measures 3 units of $\frac{1}{4}$ as opposed to 5 units of $\frac{1}{4}$. Therefore $\frac{3}{4}$ < $\frac{5}{4}$.To construct a unit fraction on a number line diagram, e.g., $\frac{1}{3}$, students partition the unit interval into 3 intervals of equal length and recognize that each has length $\frac{1}{3}$. They locate the number $\frac{1}{3}$ on the number line by marking off this length from 0, and locate other fractions with denominator 3 by marking off the number of lengths indicated by the numerator.The number line reinforces the analogy between fractions and whole numbers. Just as 5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so $\frac{5}{3}$ is the point obtained in the same way using a different interval as the basic unit of length, namely the interval from 0 to $\frac{1}{3}$.Students also develop competence in the composition and decomposition of rectangular regions, that is spatially structuring rectangular arrays. They learn to partition a rectangle into identical squares by anticipating the final structure and thus forming the array by drawing rows and columns.In Grade 3, students are beginning to learn fraction concepts (3.NF). They understand fraction equivalence in simple cases, and they use visual fraction models to represent and order fractions. Grade 3 students also measure lengths using rulers marked with halves and fourths of an inch. They use their developing knowledge of fractions and number lines to extend their work from the previous grade by working with measurement data involving fractional measurement values.For example, every student in the class might measure the height of a bamboo shoot growing in the classroom, leading to the data set shown in the table. (Again, this illustration shows a larger data set than students would normally work with in elementary grades.) To make a line plot from the data in the table, the student can ascertain the greatest and least values in the data: 13$\frac{1}{2}$ inches and 14$\frac{3}{4}$ inches. The student can draw a segment of a number line diagram that includes these extremes, with tick marks indicating specific values on the measurement scale. This is just like part of the scale on a ruler. Having drawn the number line diagram, the student can proceed through the data set recording each observation by drawing a symbol, such as a dot, above the proper tick mark. As with Grade 2 line plots, if a particular data value appears many times in the data set, dots will “pile up” above that value. There is no need to sort the observations, or to do any counting of them, before producing the line plot. Students can pose questions about data presented in line plots, such as how many students obtained measurements larger than 14$\frac{1}{4}$ inches.(See p. 3-4, in the NF Progressions. See p. 13 in the Geometry Progressions. See p. 10-11 in the MD Progressions.) |
| **Essential Question(s)** | What is a fraction?  |



Anchor Tasks

|  |  |
| --- | --- |
| **Task** | **Explanation** |
| [**Halves, Thirds and Sixths**](https://www.illustrativemathematics.org/content-standards/3/NF/A/1/tasks/1502) | Begins unit by relating area model of fractions to work in Topic 6. |
| **12-2 Solve and Share** | This task targets the concept of equal parts and builds on students’ work of partitioning from earlier grades. |
| **12-3 Visual Learning Bridge problem only** | Opportunity for students to consider unit fractions in relation to the whole on a number line. |
| **12-4 Solve and Share**  | This task provides an opportunity for students to understand fractions as numbers on the number line. |
| [**Fractions on a Number Line with Endpoints 0 and 1**](https://www.unbounded.org/math/grade-3/module-5/topic-d/lesson-14) | This task provides an opportunity for students to continue developing their understanding of fractions as numbers on a number line. |
| **12-5 Solve and Share and Visual Learning Bridge**  | These tasks provide an opportunity for students to continue developing their understanding of fractions as numbers on a number line. |
| [**Placing Various Fractions on the Number Line**](https://www.unbounded.org/math/grade-3/module-5/topic-d/lesson-17) | This task provides an opportunity for students to continue developing their understanding of fractions as numbers on a number line and that fractions can be numbers less than and greater than one.  |
| **12-6 Solve and Share** | This task provides an opportunity for students to continue developing their understanding of fractions as numbers on a number line and connects work with representing and interpreting data to understanding fractions as numbers. |
| **12-7 Visual Learning Bridge** | This task provides an opportunity for students to continue developing their understanding of fractions as numbers on a number line and connects work with representing and interpreting data to understanding fractions as numbers. |

Topic Rules of Thumb

|  |  |
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| **Rule**  | **Why?** |
| In all lessons, emphasis should be that fractions are numbers.  | This matches the language of the 3.NF cluster. |
| When referencing the numerator and denominator, use terminology focused on equal parts (for example, ¼ is 1 part when the whole is divided into 4 equal parts).  | This matches the language used in 3.NF.A.1.  |
| Use Topics 12-6 and 12-7 to reinforce student understanding of fractions on the number line. | Connects work in 3.NF and 3.MD.  |

Assessment Guidance, Topic 12

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| --- |
| → Topic Assessment Performance Assessment  |
| **Item #/Action** | **Why?** |
| 1. As Is
 |  |
| 1. Delete
 | Item requires Application. |
| 1. As Is
 |  |
| 1. Modify: Delete label of ⅛ from the number line.
 | Does not align to central concern of 3.NF.A.2. |
| 1. As Is
 |  |
| 1. Delete
 | Item requires Application. |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 9. As Is |  |
| 10. Delete | Item requires Application. |
| 11. As Is |  |
| 12. Delete | Item aligns to 4.NF.B.3 (adding fractions). |
| 13. Delete | Item requires Application. |
| 14. Modify: Change directions to “Which point represents 6/4 on the number line?” | Clarifies direction to better assess 3.NF.A.2. |
| 15. As Is |  |
| 16. As Is |  |
| 17. As Is |  |
| 18. As Is |  |
| 19. As Is |  |
| 20. As Is |  |

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| **Grade 3, Topic 13: Fraction Equivalence and Comparison** |

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| **Standards addressed** | Primary in this topic:3.NF.A.3a: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.3.NF.A.3d: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.3.NF.A.3c: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.3.NF.A.3b: Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Conceptual Understanding |
| **Applicable information from the progression documents** | As students experiment on number line diagrams they discover that many fractions label the same point on the number line, and are therefore equal; that is, they are equivalent fractions. For example, the fraction $\frac{1}{2}$ is equal to $\frac{2}{4}$ and also to $\frac{3}{6}$. Students can also use fractions strips to see fraction equivalence.Previously, in Grade 2, students compared lengths using a standard measurement unit. In Grade 3, they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions. For example, [a] segment from 0 to $\frac{3}{4}$ is shorter than the segment from 0 to $\frac{5}{4}$ because it measures 3 units of $\frac{1}{4}$ as opposed to 5 units of $\frac{1}{4}$. Therefore $\frac{3}{4}$ < $\frac{5}{4}$.(See p. 4 in the NF Progressions.) |
| **Essential Question(s)** | How can different fractions name the same part of a whole?What are different ways to compare fractions? |



Anchor Tasks

|  |  |
| --- | --- |
| **Task** | **Explanation** |
| **13-1 Solve and Share** (problem only) | This task allows students to generate equivalent fractions and explain why the fractions are equivalent. |
| **13-2 Solve and Share**  | This task builds on students understanding of fractions as numbers on the number line and allows students to visually represent equivalent fractions.  |
| [**Generate Equivalent Fractions Using Visual Models and the Number Line**](https://www.unbounded.org/math/grade-3/module-5/topic-e/lesson-23) | This task builds on students understanding of fractions as numbers on the number line and allows students to visually represent equivalent fractions.  |
| **13-3 Solve and Share** (problem only) | This task allows students to compare fractions and reason about their size.  |
| **13-4 Solve and Share**  | This task allows students to compare fractions and reason about their size.  |
| **13-5 Solve and Share** (problem only) | This task allows students to compare fractions and reason about their size.  |
| **13-6 Solve and Share**  | This task allows students to compare fractions and reason about their size.  |
| [**Compare Fractions with the Same Numerator Using <, >, =**](https://www.unbounded.org/math/grade-3/module-5/topic-f/lesson-29) | These tasks allow students to compare fractions with the same numerator, to reason about their size, and to compare fractions using the symbols <, >, and =.  |
| **13-7 Visual Learning Bridge**  | This task builds on students’ understanding of fractions as numbers on the number line and equivalent fractions to explore whole numbers as fractions. |

Topic Rules of Thumb

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| **Rule**  | **Why?** |
| In all lessons, emphasis should be that fractions are numbers.  | This matches the language of the 3.NF cluster. |
| Teachers should make sure that students are comfortable with the models and strategies used throughout the topic. Underscore misunderstands of equal parts and the whole if students are not able to replicate a model or strategy on their own. | Any model or strategy should connect back to the expectations of the standards in 3.NF. |



Assessment Guidance, Topic 13

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| → Topic Assessment Performance Assessment  |
| **Item #/Action** | **Why?** |
| 1. Delete
 | Item aligns to 4.NF.B.3 (adding fractions). |
| 1. As Is
 |  |
| 1. Modify: Delete number lines.
 | Item requires a specific model or strategy. |
| 1. Delete
 | Item aligns to 4.NF.B.3 (adding fractions). |
| 1. Modify: Delete fraction models.
 | Item requires a specific model or strategy. |
| 1. Modify: Rewrite question to remove context, e.g. “Which fraction is closer to 0 than to 1: 2/6 or ⅚?”
 | Item requires Application. |
| 1. Modify: Delete fraction models.
 | Item requires a specific model or strategy. |
| 1. As Is
 |  |
| 9. Modify: Delete fraction models. | Item requires a specific model or strategy. |
| 10. Delete | Item requires Application. Repeats content from previous questions. |
| 11. Modify: Remove number line model. Change last sentence of question to “Draw a model or explain how you know that the fractions are equivalent.” | Item requires a specific model or strategy. |
| 12. As Is |  |
| 13. Delete | Item aligns to 6.RP.A (rates). |
| 14. Delete | Doesn’t align to the central concern of 3.NF.A.3b. |
| 15. delete | Item requires Application. Repeats content from previous questions. |
| 16. As Is |  |
| 17. As Is |  |
| 18. As Is |  |
| 19. Delete | Item aligns to 4.NF.A.2 (comparing fractions with unlike numerator and denominators).  |
| 20. As Is |  |

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| **Grade 3, Topic 14: Solve Time, Capacity, and Mass Problems** |

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| --- | --- |
| **Standards addressed** | Primary in this topic:3.MD.A.1: Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.3.MD.A.2: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).1 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:ApplicationSecondary in this topic:Procedural Skill and Fluency, Conceptual Understanding |
| **Applicable information from the progression documents** | Students in Grade 3 learn to solve a variety of problems involving measurement and such attributes as length and area, liquid volume, mass, and time.3.MD.1, 3.MD.2 Many such problems support the Grade 3 emphasis on multiplication (see Table 1) and the mathematical practices of making sense of problems (MP1) and representing them with equations, drawings, or diagrams (MP4). Such work will involve units of mass such as the kilogram.A few words on volume are relevant. Compared to the work in area, volume introduces more complexity, not only in adding a third dimension and thus presenting a significant challenge to students’ spatial structuring, but also in the materials whose volumes are measured. These materials may be solid or fluid, so their volumes are generally measured with one of two methods, e.g., “packing” a right rectangular prism with cubic units or “filling” a shape such as a right circular cylinder. Liquid measurement, for many third graders, may be limited to a one-dimensional unit structure (i.e., simple iterative counting of height that is not processed as three-dimensional). Thus, third graders can learn to measure with liquid volume and to solve problems requiring the use of the four arithmetic operations, when liquid volumes are given in the same units throughout each problem. Because liquid measurement can be represented with one-dimensional scales, problems may be presented with drawings or diagrams, such as measurements on a beaker with a measurement scale in milliliters.(See pp. 18-19 in the MD Progressions.) |
| **Essential Question(s)** | How can time, capacity, and mass be measured and found? |



Anchor Tasks

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| --- | --- |
| **Task** | **Explanation** |
| **14-1 Solve and Share**  | This task targets students’ ability to tell and write time to the nearest minute.  |
| **14-2 Solve and Share and Visual Learning Bridge** | These tasks provide an opportunity for students to solve word problems involving addition and subtraction of time intervals. |
| **14-9 Solve and Share and Visual Learning Bridge** | These tasks provide an opportunity for students to solve word problems involving addition and subtraction of time intervals. |
| **14-4 Solve and Share** | This task provides students with an opportunity to estimate and measure liquid volume.  |
| **14-5 Solve and Share**  | This task provides students with an opportunity to measure liquid volume.  |
| **14-6 Visual Learning Bridge**  | This task provides students with an opportunity to estimate and measure mass. |
| **14-7 Solve and Share**  | This task provides students with an opportunity to estimate and measure mass. |
| **14-8 Solve and Share**  | This task provides students with an opportunity to apply their understanding of multiplication to word problems involving mass.  |

Topic Rules of Thumb

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| **Rule**  | **Why?** |
| Ensure students are encountering numbers of the appropriate magnitude for third grade. Students should be adding and subtracting within 1,000 and multiplying and dividing within 100.  | The content limits for grade 3 are defined in 3.NBT.A.2 and 3.OA.C. |



Assessment Guidance, Topic 14

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| → Topic Assessment Performance Assessment  |
| **Item #/Action** | **Why?** |
| 1. As Is
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| 1. As Is
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| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. Modify: Delete Part A and Part B. Just give the story problem.
 | Does not align to the central concern of 3.MD.A.1.  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 9. As Is |  |
| 10. As Is |  |
| 11. As Is |  |
| 12. Delete | Repeats content from previous questions. |
| 13. Delete | Repeats content from previous questions. |
| 14. Delete | Repeats content from previous questions. |
| 15. As Is |  |
| 16. As Is |  |
| 17. As Is |  |
| 18. Delete | Repeats content from previous questions. |
| 19. As Is |  |

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| **Grade 3, Topic 15: Attributes of Two-Dimensional Shapes** |

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| **Standards addressed** | Primary in this topic:3.G.A.1: Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.Secondary in this topic:3.MD.C.5b: A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:Conceptual Understanding |
| **Applicable information from the progression documents** | Students analyze, compare, and classify two-dimensional shapes by their properties (3.G.1). They explicitly relate and combine these classifications. Because they have built a firm foundation of several shape categories, these categories can be the raw material for thinking about the relationships between classes. For example, students can form larger, superordinate, categories, such as the class of all shapes with four sides, or quadrilaterals, and recognize that it includes other categories, such as squares, rectangles, rhombuses, parallelograms, and trapezoids. They also recognize that there are quadrilaterals that are not in any of those subcategories…The Standards do not require that such representations be constructed by Grade 3 students, but they should be able to draw examples of quadrilaterals that are not in the subcategories.Similarly, students learn to draw shapes with prespecified attributes, without making a priori assumptions regarding their classification.MP1 For example, they could solve the problem of making a shape with two long sides of the same length and two short sides of the same length that is not a rectangle.Students investigate, describe, and reason about decomposing and composing polygons to make other polygons. Problems such as finding all the possible different compositions of a set of shapes involve geometric problem solving and notions of congruence and symmetry (MP7). They also involve the practices of making and testing conjectures (MP1), and convincing others that conjectures are correct (or not) (MP3). Such problems can be posed even for sets of simple shapes such as tetrominoes, four squares arranged to form a shape so that every square shares at least one side and sides coincide or share only a vertex.More advanced paper-folding (origami) tasks afford the same mathematical practices of seeing and using structure, conjecturing, and justifying conjectures. Paper folding can also illustrate many geometric concepts. For example, folding a piece of paper creates a line segment. Folding a square of paper twice, horizontal edge to horizontal edge, then vertical edge to vertical edge, creates a right angle, which can be unfolded to show four right angles. Students can be challenged to find ways to fold paper into rectangles or squares and to explain why the shapes belong in those categories. Students also develop more competence in the composition and decomposition of rectangular regions, that is, spatially structuring rectangular arrays. They learn to partition a rectangle into identical squares by anticipating the final structure and thus forming the array by drawing rows and columns (see the bottom right example on p. 11; some students may still need work building or drawing squares inside the rectangle first). They count by the number of columns or rows, or use multiplication to determine the number of squares in the array. They also learn to rotate these arrays physically and mentally to view them as composed of smaller arrays, allowing illustrations of properties of multiplication (e.g., the commutative property and the distributive property).Students need to learn to conceptualize area as the amount of two-dimensional space in a bounded region and to measure it by choosing a unit of area, often a square. A two-dimensional geometric figure that is covered by a certain number of squares without gaps or overlaps can be said to have an area of that number of square units.(See pp. 13-14 in the Geometry Progressions. See p. 16 in the MD Progressions.) |
| **Essential Question(s)** | N/A |



Assessment Guidance, Topic 15

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| → Topic Assessment Performance Assessment  |
| **Item #/Action** | **Why?** |
| 1. As Is
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| 1. As Is
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| 1. As Is
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| 1. As Is
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| 1. As Is
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| 1. As Is
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| 1. As Is
 |  |
| 1. As Is
 |  |
| 9. As Is |  |
| 10. As Is |  |
| 11. As Is |  |

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| **Grade 3, Topic 16: Solve Perimeter Problems** |

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| **Standards addressed** | Primary in this topic:3.MD.D.8: Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.Secondary in this topic:3.MD.C.7b: Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. |
| **Aspects of Rigor targeted by the standards** | Primary in this topic:ApplicationSecondary in this topic:Procedural Skill and Fluency, Conceptual Understanding |
| **Applicable information from the progression documents** | Third graders focus on solving real-world and mathematical problems involving perimeters of polygons (3.MD.8). A perimeter is the boundary of a two-dimensional shape. For a polygon, the length of the perimeter is the sum of the lengths of the sides. Initially, it is useful to have sides marked with unit length marks, allowing students to count the unit lengths. Later, the lengths of the sides can be labeled with numerals. As with all length tasks, students need to count the length-units and not the end-points. Next, students learn to mark off unit lengths with a ruler and label the length of each side of the polygon. For rectangles, parallelograms, and regular polygons, students can discuss and justify faster ways to find the perimeter length than just adding all of the lengths (MP3). Rectangles and parallelograms have opposite sides of equal length, so students can double the lengths of adjacent sides and add those numbers or add lengths of two adjacent sides and double that number. A regular polygon has all sides of equal length, so its perimeter length is the product of one side length and the number of sides. Perimeter problems for rectangles and parallelograms often give only the lengths of two adjacent sides or only show numbers for these sides in a drawing of the shape. The common error is to add just those two numbers. Having students first label the lengths of the other two sides as a reminder is helpful. Students then find unknown side lengths in more difficult “missing measurements” problems and other types of perimeter problems.Students can be taught to multiply length measurements to find the area of a rectangular region. But, in order that they make sense of these quantities (MP2), they first learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows.They also learn to understand and explain that the area of a rectangular region of, for example, 12 length-units by 5 length-units can be found either by multiplying 12 x 5 or by adding two products, e.g., 10 x 5 and 2 x 5, illustrating the distributive property.(See pp. 16-18 in the MD Progressions.) |
| **Essential Question(s)** | N/A |



Assessment Guidance, Topic 16

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| → Topic Assessment Performance Assessment  |
| **Item #/Action** | **Why?** |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
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| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 1. As Is
 |  |
| 9.As Is |  |
| 10.As Is |  |
| 11. Delete | Magnitude of numbers is beyond grade 3 expectations (multiplying anddividing within 100). |