Task

a. Locate 1 on the number line. Label the point. Be as exact as possible.

b. Locate 1 on the number line. Label the point. Be as exact as possible.

Task:	
The Math	*What student work would demonstrate understanding of the task?*
Addressing	*What fraction understanding is this task addressing specifically?*
Building On	*What fraction understanding(s) is this task building on?*
Building Toward	*What fraction understanding(s) is this task building toward?*

Grade 3

The meaning of fractions In Grades 1 and 2, students use fraction language to describe partitions of shapes into equal shares.[2.G.3] In Grade 3 they start to develop the idea of a fraction more formally, building on the idea of partitioning a whole into equal parts. The whole can be a shape such as a circle or rectangle, a line segment, or any one finite entity susceptible to subdivision and measurement. In Grade 4, this is extended to include wholes that are collections of objects.

[2.G.3] Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Grade 3 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and taking one part, e.g., if a whole is partitioned into 4 equal parts then each part is $\frac{1}{4}$ of the whole, and 4 copies of that part make the whole. Next, students build fractions from unit fractions, seeing the numerator 3 of $\frac{3}{4}$ as saying that $\frac{3}{4}$ is the quantity you get by putting 3 of the $\frac{1}{4}$'s together.[3.NF.1] They read any fraction this way, and in particular there is no need to introduce "proper fractions" and "improper fractions" initially; $\frac{5}{3}$ is the quantity you get by combining 5 parts together when the whole is divided into 3 equal parts.

[3.NF.1] Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

The importance of specifying the whole

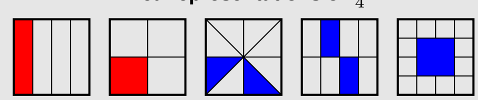
Without specifying the whole it is not reasonable to ask what fraction is represented by the shaded area. If the left square is the whole, the shaded area represents the fraction $\frac{3}{2}$; if the entire rectangle is the whole, the shaded area represents $\frac{3}{4}$.

Two important aspects of fractions provide opportunities for the mathematical practice of attending to precision (MP6):

- Specifying the whole.
- Explaining what is meant by "equal parts."

Initially, students can use an intuitive notion of congruence ("same size and same shape") to explain why the parts are equal, e.g., when they divide a square into four equal squares or four equal rectangles.

Area representations of $\frac{1}{4}$



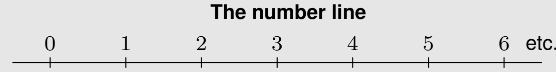
In each representation the square is the whole. The two squares on the left are divided into four parts that have the same size and shape, and so the same area. In the three squares on the right, the shaded area is $\frac{1}{4}$ of the whole area, even though it is not easily seen as one part in a division of the square into four parts of the same shape and size.

Students come to understand a more precise meaning for "equal parts" as "parts with equal measurements." For example, when a ruler is partitioned into halves or quarters of an inch, they see that each subdivision has the same length. In area models they reason about the area of a shaded region to decide what fraction of the whole it represents (MP3).

The goal is for students to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of the whole numbers; just as every whole number is obtained by combining a sufficient number of 1s, every fraction is obtained by combining a sufficient number of unit fractions.

The number line and number line diagrams On the number line, the whole is the *unit interval*, that is, the interval from 0 to 1, measured by length. Iterating this whole to the right marks off the whole numbers, so that the intervals between consecutive whole numbers, from 0 to 1, 1 to 2, 2 to 3, etc., are all of the same length, as shown. Students might think of the number line as an infinite ruler.

The number line



To construct a unit fraction on a number line diagram, e.g. $\frac{1}{3}$, students partition the unit interval into 3 intervals of equal length and recognize that each has length $\frac{1}{3}$. They locate the number $\frac{1}{3}$ on the number line by marking off this length from 0, and locate other fractions with denominator 3 by marking off the number of lengths indicated by the numerator.[3.NF.2]

Students sometimes have difficulty perceiving the unit on a number line diagram. When locating a fraction on a number line diagram, they might use as the unit the entire portion of the number line that is shown on the diagram, for example indicating the number 3 when asked to show $\frac{3}{4}$ on a number line diagram marked from 0 to 4. Although number line diagrams are important representations for students as they develop an understanding of a fraction as a number, in the early stages of the NF Progression they use other representations such as area models, tape diagrams, and strips of paper. These, like number line diagrams, can be subdivided, representing an important aspect of fractions.

The number line reinforces the analogy between fractions and whole numbers. Just as 5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so $\frac{5}{3}$ is the point obtained in the same way using a different interval as the basic unit of length, namely the interval from 0 to $\frac{1}{3}$.

Equivalent fractions Grade 3 students do some preliminary reasoning about equivalent fractions, in preparation for work in Grade 4. As students experiment on number line diagrams they discover that many fractions label the same point on the number line, and are therefore equal; that is, they are *equivalent fractions*. For example, the fraction $\frac{1}{2}$ is equal to $\frac{2}{4}$ and also to $\frac{3}{6}$. Students can also use fraction strips to see fraction equivalence.[3.NF.3ab]

In particular, students in Grade 3 see whole numbers as fractions, recognizing, for example, that the point on the number line designated by 2 is now also designated by $\frac{2}{1}$, $\frac{4}{2}$, $\frac{6}{3}$, $\frac{8}{4}$, etc. so that[3.NF.3c]

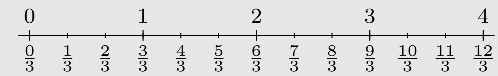
$$2 = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \cdots .$$

Of particular importance are the ways of writing 1 as a fraction:

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \cdots .$$

Comparing fractions Previously, in Grade 2, students compared lengths using a standard measurement unit.[2.MD.3] In Grade 3 they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions. For example, segment from 0 to $\frac{3}{4}$ is shorter than the segment from 0 to $\frac{5}{4}$ because it measures 3 units of $\frac{1}{4}$ as opposed to 5 units of $\frac{1}{4}$. Therefore $\frac{3}{4} < \frac{5}{4}$.[3.NF.3d]

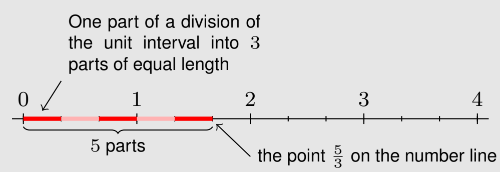
The number line marked off in thirds



3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

- a Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
- b Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

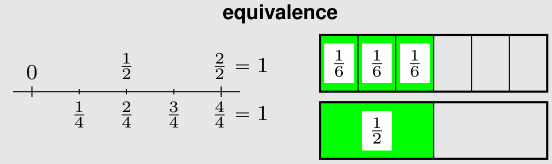
Number line representation of $\frac{5}{3}$



3.NF.3abc Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- a Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- b Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- c Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Using the number line and fraction strips to see fraction equivalence



2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

3.NF.3d Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

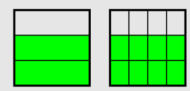
Grade 4

Grade 4 students learn a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction. This property forms the basis for much of their other work in Grade 4, including the comparison, addition, and subtraction of fractions and the introduction of finite decimals.

Equivalent fractions Students can use area models and number line diagrams to reason about equivalence.4.NF.1 They see that the numerical process of multiplying the numerator and denominator of a fraction by the same number, *n*, corresponds physically to partitioning each unit fraction piece into *n* smaller equal pieces. The whole is then partitioned into *n* times as many pieces, and there are *n* times as many smaller unit fraction pieces as in the original fraction.

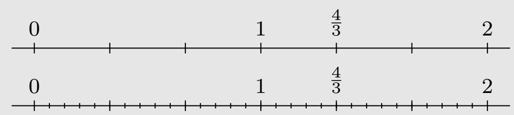
4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Using an area model to show that $\frac{2}{3} = \frac{4\times 2}{4\times 3}$



The whole is the square, measured by area. On the left it is divided horizontally into 3 rectangles of equal area, and the shaded region is 2 of these and so represents $\frac{2}{3}$. On the right it is divided into 4×3 small rectangles of equal area, and the shaded area comprises 4×2 of these, and so it represents $\frac{4\times 2}{4\times 3}$.

Using the number line to show that $\frac{4}{3} = \frac{5\times 4}{5\times 3}$



$\frac{4}{3}$ is 4 parts when each part is $\frac{1}{3}$, and we want to see that this is also 5×4 parts when each part is $\frac{1}{5\times 3}$. Divide each of the intervals of length $\frac{1}{3}$ into 5 parts of equal length. There are 5×3 parts of equal length in the unit interval, and $\frac{4}{3}$ is 5×4 of these. Therefore $\frac{4}{3} = \frac{5\times 4}{5\times 3} = \frac{20}{15}$.

This argument, once understood for a range of examples, can be seen as a general argument, working directly from the Grade 3 understanding of a fraction as a point on the number line.

The fundamental property can be presented in terms of division, as in, e.g.

$$\frac{28}{36} = \frac{28 \div 4}{36 \div 4} = \frac{7}{9}.$$

Because the equations $28 \div 4 = 7$ and $36 \div 4 = 9$ tell us that $28 = 4 \times 7$ and $36 = 4 \times 9$, this is the fundamental fact in disguise:

$$\frac{4 \times 7}{4 \times 9} = \frac{7}{9}.$$

It is possible to over-emphasize the importance of simplifying fractions in this way. There is no mathematical reason why fractions must be written in simplified form, although it may be convenient to do so in some cases.

Grade 4 students use their understanding of equivalent fractions to compare fractions with different numerators and different denominators.4.NF.2 For example, to compare $\frac{5}{8}$ and $\frac{7}{12}$ they rewrite both fractions as

$$\frac{60}{96}\left(= \frac{12 \times 5}{12 \times 8}\right) \quad \text{and} \quad \frac{56}{96}\left(= \frac{7 \times 8}{12 \times 8}\right)$$

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Because $\frac{60}{96}$ and $\frac{56}{96}$ have the same denominator, students can compare them using Grade 3 methods and see that $\frac{56}{96}$ is smaller, so

$$\frac{7}{12} < \frac{5}{8}.$$

Students also reason using benchmarks such as $\frac{1}{2}$ and 1. For example, they see that $\frac{7}{8} < \frac{13}{12}$ because $\frac{7}{8}$ is less than 1 (and is therefore to the left of 1) but $\frac{13}{12}$ is greater than 1 (and is therefore to the right of 1).

Grade 5 students who have learned about fraction multiplication can see equivalence as "multiplying by 1":

$$\frac{7}{9} = \frac{7}{9} \times 1 = \frac{7}{9} \times \frac{4}{4} = \frac{28}{36}$$

However, although a useful mnemonic device, this does not constitute a valid argument at this grade, since students have not yet learned fraction multiplication.

Adding and subtracting fractions The meaning of addition is the same for both fractions and whole numbers, even though algorithms for calculating their sums can be different. Just as the sum of 4 and 7 can be seen as the length of the segment obtained by joining together two segments of lengths 4 and 7, so the sum of $\frac{2}{3}$ and $\frac{8}{5}$ can be seen as the length of the segment obtained joining together two segments of length $\frac{2}{3}$ and $\frac{8}{5}$. It is not necessary to know how much $\frac{2}{3} + \frac{8}{5}$ is exactly in order to know what the sum means. This is analogous to understanding 51×78 as 51 groups of 78, without necessarily knowing its exact value.

This simple understanding of addition as putting together allows students to see in a new light the way fractions are built up from unit fractions. The same representation that students used in Grade 4 to see a fraction as a point on the number line now allows them to see a fraction as a sum of unit fractions: just as $5 = 1+1+1+1+1$, so

$$\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

because $\frac{5}{3}$ is the total length of 5 copies of $\frac{1}{3}$.[4.NF.3]

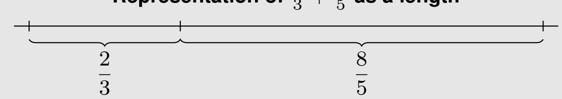
Armed with this insight, students decompose and compose fractions with the same denominator. They add fractions with the same denominator:[4.NF.3c]

$$\begin{aligned}
\frac{7}{5} + \frac{4}{5} &= \overbrace{\frac{1}{5} + \cdots \frac{1}{5}}^{7} + \overbrace{\frac{1}{5} + \cdots \frac{1}{5}}^{4} \\
&= \frac{\overbrace{1 + 1 + \cdots + 1}^{7+4}}{5} \\
&= \frac{7+4}{5}.
\end{aligned}$$

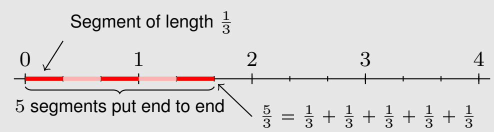
Using the understanding gained from work with whole numbers of the relationship between addition and subtraction, they also subtract fractions with the same denominator. For example, to subtract $\frac{5}{6}$ from $\frac{17}{6}$, they decompose

$$\frac{17}{6} = \frac{12}{6} + \frac{5}{6}, \quad \text{so} \quad \frac{17}{6} - \frac{5}{6} = \frac{17-5}{6} = \frac{12}{6} = 2.$$

Representation of $\frac{2}{3} + \frac{8}{5}$ as a length



Using the number line to see that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$



4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Grade 5

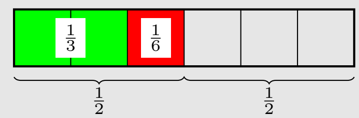
Adding and subtracting fractions In Grade 4, students have some experience calculating sums of fractions with different denominators in their work with decimals, where they add fractions with denominators 10 and 100, such as

$$\frac{2}{10} + \frac{7}{100} = \frac{20}{100} + \frac{7}{100} = \frac{27}{100}.$$

Note that this is a situation where one denominator is a divisor of the other, so that only one fraction has to be changed. They might have encountered other similar situations, for example using a fraction strip to reason that

$$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}.$$

Using a fraction strip to show that $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$



They understand the process as expressing both summands in terms of the same unit fraction so that they can be added. Grade 5 students extend this reasoning to situations where it is necessary to re-express both fractions in terms of a new denominator.[5.NF.1] For example, in calculating $\frac{2}{3} + \frac{5}{4}$ they reason that if each third in $\frac{2}{3}$ is subdivided into fourths, and if each fourth in $\frac{5}{4}$ is subdivided into thirds, then each fraction will be a sum of unit fractions with denominator $3 \times 4 = 4 \times 3 = 12$:

$$\frac{2}{3} + \frac{5}{4} = \frac{2 \times 4}{3 \times 4} + \frac{5 \times 3}{4 \times 3} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}.$$

In general two fractions can be added by subdividing the unit fractions in one using the denominator of the other:

$$\frac{a}{b} + \frac{c}{d} = \frac{a \times d}{b \times d} + \frac{c \times b}{d \times b} = \frac{ad + bc}{bd}.$$

It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding algorithms for adding fractions.

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

Students make sense of fractional quantities when solving word problems, estimating answers mentally to see if they make sense.[5.NF.2] For example in the problem

> Ludmilla and Lazarus each have a lemon. They need a cup of lemon juice to make hummus for a party. Ludmilla squeezes $\frac{1}{2}$ a cup from hers and Lazarus squeezes $\frac{2}{5}$ of a cup from his. How much lemon juice to they have? Is it enough?

students estimate that there is almost but not quite one cup of lemon juice, because $\frac{2}{5} < \frac{1}{2}$. They calculate $\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$, and see this as $\frac{1}{10}$ less than 1, which is probably a small enough shortfall that it will not ruin the recipe. They detect an incorrect result such as $\frac{1}{2} + \frac{2}{5} = \frac{3}{7}$ by noticing that $\frac{3}{7} < \frac{1}{2}$.

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Multiplying and dividing fractions In Grade 4 students connected fractions with addition and multiplication, understanding that

$$\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 5 \times \frac{1}{3}.$$

In Grade 5, they connect fractions with division, understanding that

$$5 \div 3 = \frac{5}{3},$$

or, more generally, $\frac{a}{b} = a \div b$ for whole numbers a and b, with b not equal to zero.[5.NF.3] They can explain this by working with their understanding of division as equal sharing (see figure in margin). They also create story contexts to represent problems involving division of whole numbers. For example, they see that

> If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get?

can be solved in two ways. First, they might partition each pound among the 9 people, so that each person gets $50 \times \frac{1}{9} = \frac{50}{9}$ pounds. Second, they might use the equation $9 \times 5 = 45$ to see that each person can be given 5 pounds, with 5 pounds remaining. Partitioning the remainder gives $5\frac{5}{9}$ pounds for each person.

Students have, since Grade 1, been using language such as "third of" to describe one part when a whole is partitioned into three parts. With their new understanding of the connection between fractions and division, students now see that $\frac{5}{3}$ is one third of 5, which leads to the meaning of multiplication by a unit fraction:

$$\frac{1}{3} \times 5 = \frac{5}{3}.$$

This in turn extends to multiplication of any quantity by a fraction.[5.NF.4a] Just as

$$\frac{1}{3} \times 5 \text{ is one part when } 5 \text{ is partitioned into } 3 \text{ parts,}$$

so

$$\frac{4}{3} \times 5 \text{ is } 4 \text{ parts when } 5 \text{ is partitioned into } 3 \text{ parts.}$$

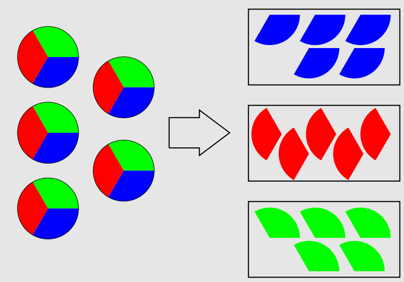
Using this understanding of multiplication by a fraction, students develop the general formula for the product of two fractions,

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd},$$

for whole numbers a, b, c, d, with b, d not zero. Grade 5 students need not express the formula in this general algebraic form, but rather reason out many examples using fraction strips and number line diagrams.

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

How to share 5 objects equally among 3 shares:
$5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$

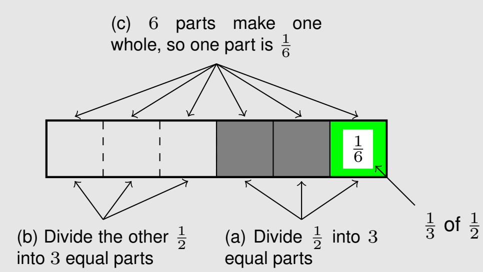


If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute $\frac{1}{3}$ of itself to each share. Thus each share consists of 5 pieces, each of which is $\frac{1}{3}$ of an object, and so each share is $5 \times \frac{1}{3} = \frac{5}{3}$ of an object.

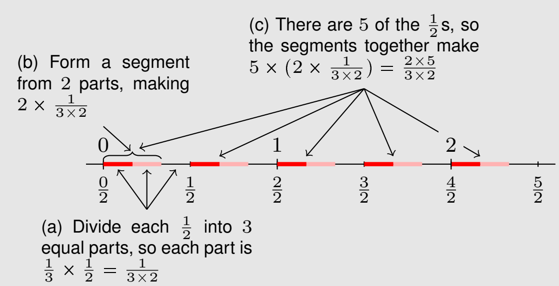
5.NF.4a Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

Using a fraction strip to show that $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$



Using a number line to show that $\frac{2}{3} \times \frac{5}{2} = \frac{2 \times 5}{3 \times 2}$



Draft, 19 September 2013, comment at commoncoretools.wordpress.com.

Task

Jon and Charlie plan to run together. They are arguing about how far to run. Charlie says,

> ***I run $\frac{3}{6}$ of a mile each day.***

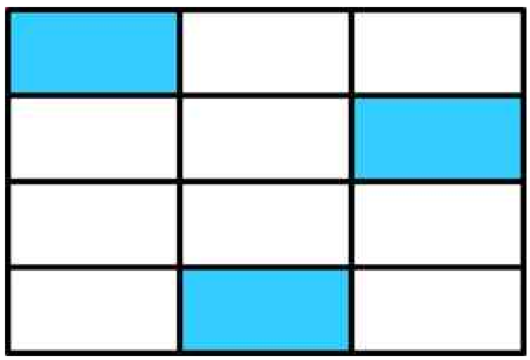
Jon says,

> ***I can only run $\frac{1}{2}$ of a mile.***

If Charlie runs $\frac{3}{6}$ of a mile and Jon runs $\frac{1}{2}$ of a mile, explain why it is silly for them to argue. Draw a picture or a number line to support your reasoning.

Task

a.
What fraction of the rectangle below is shaded?



b. Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct? Explain why or why not by using the picture.

Task

a. To add fractions, we usually first find a common denominator.

i. Find two different common denominators for $\frac{1}{5}$ and $\frac{1}{15}$.

ii. Use each common denominator to find the value of $\frac{1}{5} + \frac{1}{15}$. Draw a picture that shows your solution.

b. Find $\frac{3}{4} + \frac{1}{5}$. Draw a picture that shows your solution.

c. Find $\frac{14}{8} + \frac{15}{12}$.

3.NF Jon and Charlie's Run

Alignments to Content Standards: 3.NF.A.3.a

Task

Jon and Charlie plan to run together. They are arguing about how far to run. Charlie says,

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If Charlie runs $\frac{3}{6}$ of a mile and Jon runs $\frac{1}{2}$ of a mile, explain why it is silly for them to argue. Draw a picture or a number line to support your reasoning.

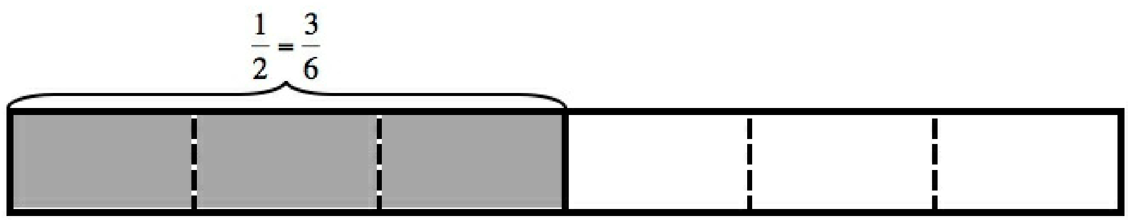
IM Commentary

The purpose of this task is to present students with a context where they need to explain why two simple fractions are equivalent and is most appropriate for instruction. Students can illustrate the fact that the two fractions are equivalent by showing they belong at the same point on the number line or are represented by the same area in an appropriate diagram. Students will benefit from discussing their reasoning in small groups or as a class.

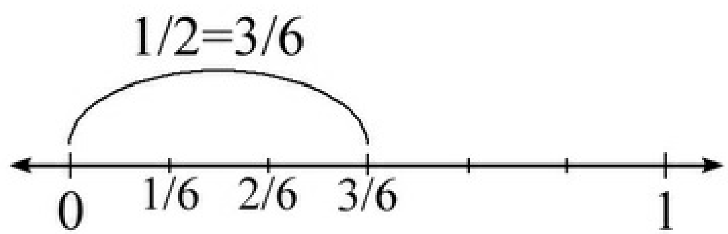
Edit this solution

Solution

It is silly for Jon and Charlie to argue about the length of their run because the two lengths that each boy will run are equal. Consider the rectangle below that has a length of 1 which represents 1 mile. It is divided into 6 equal pieces and 3 are shaded. So the 3 shaded pieces represent $\frac{3}{6}$ of a mile.



Since all 6 pieces are the same length, the three shaded pieces together have the same length as the three unshaded pieces, so we know that the shaded area also represents $\frac{1}{2}$ mile. We can also see this equivalence when we plot $\frac{1}{2}$ and $\frac{3}{6}$ on a number line. If we place 5 tick marks at even intervals between 0 and 1, we will divide the segment between 0 and 1 into 6 equal length pieces. $\frac{3}{6}$ will be plotted at the third tick mark to the right of 0.



This tick mark also marks the half-way point of the segment between 0 and 1, so we will also plot $\frac{1}{2}$ there. This means that these two fractions are equivalent.

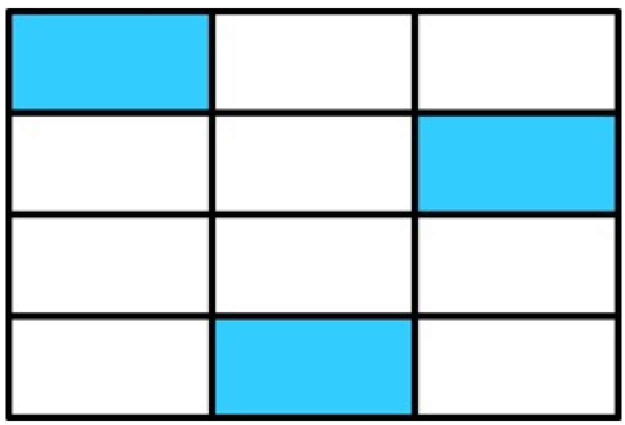
So Jon and Charlie will both run the same distance and can run together the whole way.

4.NF Fractions and Rectangles

Alignments to Content Standards: 4.NF.A.1

Task

a. What fraction of the rectangle below is shaded?



b. Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct? Explain why or why not by using the picture.

IM Commentary

The primary goal of this task is for students to use pictures to explain the equivalence between $\frac{3}{12}$ and $\frac{1}{4}$. This is a step toward students understanding the general idea that

$$\frac{a \times n}{b \times n} = \frac{a}{b}$$

Students usually first see fraction equivalence by subdividing the pieces into smaller, equal sized pieces and realizing that you haven't changed the amount that represents the fraction. This explains why if you multiply the numerator and denominator by the same whole number, you get an equivalent fraction:

$$\frac{a}{b} = \frac{a \times n}{b \times n}$$

This task helps set students on the path to understanding that if you divide the numerator and denominator by the same whole number, you get an equivalent fraction. It is as if we are "putting smaller pieces together to form bigger pieces," as opposed to subdividing the pieces as seen in, e.g. 4.NF Explaining Fraction Equivalence with Pictures.

Students don't necessarily generate representations like the one shown in the task stem where the pieces are "scattered around," so it is good for them to see and think about such representations.

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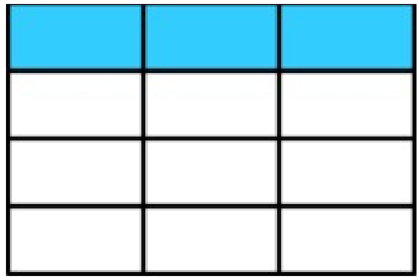
Solution

a. In this picture the entire rectangle has been divided into 12 equal-sized pieces. So we can say that each piece is $\frac{1}{12}$ of the entire rectangle. Since 3 out of the total 12 pieces have been shaded, the shaded portion represents

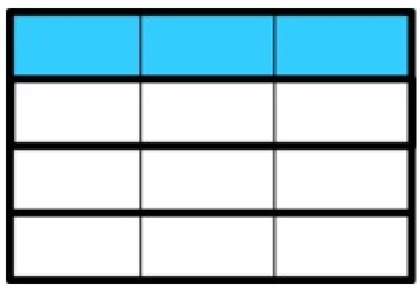
$$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} = 3 \times \frac{1}{12} = \frac{3}{12}$$

of the total rectangle. We can see the number in the denominator of the fraction represents how many pieces the entire rectangle is broken up into and the numerator represents how many of those pieces are shaded.

b. Laura is correct. We can see this by simply re-arranging the shaded parts of the rectangle to form a single row (See picture below.)



To make it clear that this row represents $\frac{1}{4}$ of the rectangle we can merge all of the pieces in a row together.



Since the rectangle is now made up of 4 equal sized pieces, the shaded region (which is just one of the four pieces) represents $\frac{1}{4}$ of the entire rectangle.

5.NF Finding Common Denominators to Add

Alignments to Content Standards: 5.NF.A.1

Task

a. To add fractions, we usually first find a common denominator.

i. Find two different common denominators for $\frac{1}{5}$ and $\frac{1}{15}$.

ii. Use each common denominator to find the value of $\frac{1}{5} + \frac{1}{15}$. Draw a picture that shows your solution.

b. Find $\frac{3}{4} + \frac{1}{5}$. Draw a picture that shows your solution.

c. Find $\frac{14}{8} + \frac{15}{12}$.

IM Commentary

Part (a) of this task asks students to find and use two different common denominators to add the given fractions. The purpose of this question is to help students realize that they can use any common denominator to find a solution, not just the least common denominator. Part (b) does not ask students to solve the given addition problem in more than one way. Instead, the purpose of this question is to give students an opportunity to choose a denominator and possibly to compare their solution method with another student who chose a different denominator. The two most likely student choices are the least common denominator and the product of the two denominators.

Parts (a) and (b) ask students to draw pictures to help them to see why finding a common denominator is an important part of solving the given addition problems. Part (c) does not ask students to draw a picture. Instead, the purpose of part (c) is to give students who are ready to work symbolically a chance to work more efficiently.

The numbers were chosen in each part for a specific reason. Part (a) asks students to add unit fractions, helping to emphasize the need for a common denominator. Additionally, one denominator is a multiple of the other, so students should recognize it as a common denominator for both fractions. Part (b) asks students to add a unit fraction and a non-unit fraction. Part (c) asks students to add fractions where one of the addends is greater than 1. It also gives students a chance to write one of the fractions with a smaller denominator before adding, making the arithmetic that follows easier.

Solutions

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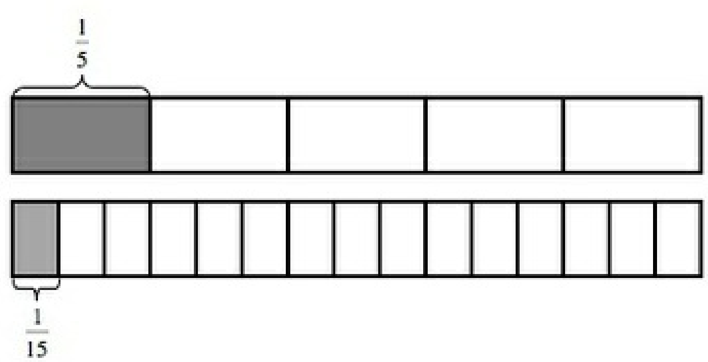
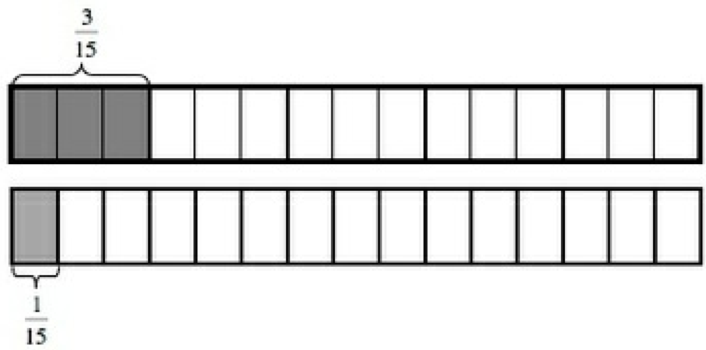
Solution: Adding fractions

a. i. The denominators of the fractions in the first problem are 5 and 15. 5 divides evenly into 15, meaning that 15 is a multiple of 5, so 15 itself is a common denominator for the fractions $\frac{1}{5}$ and $\frac{1}{15}$. The picture below shows this:

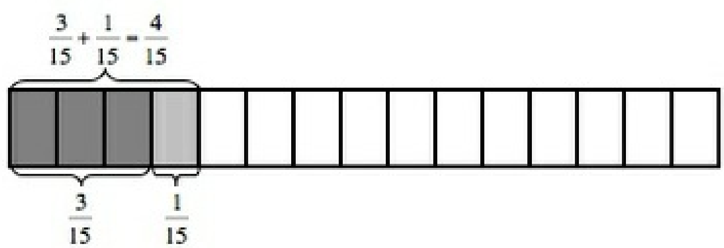


Here is a picture showing the fractions when they are both written in terms of fifteenths.



Any multiple of 15 is also a common multiple of 5 and 15. This means that $2 \times 15 = 30$ is a common multiple and is a common denominator for the fractions $\frac{1}{5}$ and $\frac{1}{15}$. Thus, 15 and 30 are two different common denominators for the fractions $\frac{1}{5}$ and $\frac{1}{15}$.

ii. The first common denominator that we identified in part (i) was 15. Here is a picture that represents $\frac{1}{5} + \frac{1}{15}$:

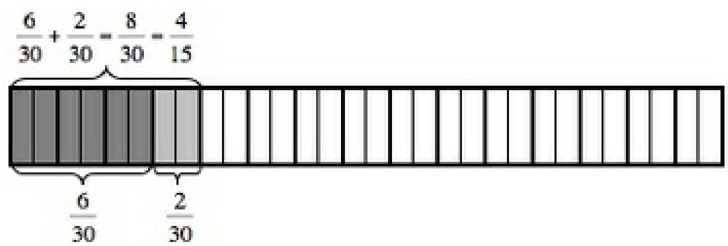


This is how we can write the process of finding this common denominator and adding using symbols:

$$\begin{aligned}\frac{1}{5} + \frac{1}{15} &= \frac{1 \times 3}{5 \times 3} + \frac{1}{15} \\ &= \frac{3}{15} + \frac{1}{15} \\ &= \frac{3+1}{15} \\ &= \frac{4}{15}\end{aligned}$$

The picture shows that after we convert $\frac{1}{5}$ to $\frac{3}{15}$ we can add $\frac{1}{15}$ and we are left with $\frac{4}{15}$, as we found symbolically.

The second common denominator that we identified in part (i) was 30. Here is a picture that represents $\frac{1}{5} + \frac{1}{15}$:

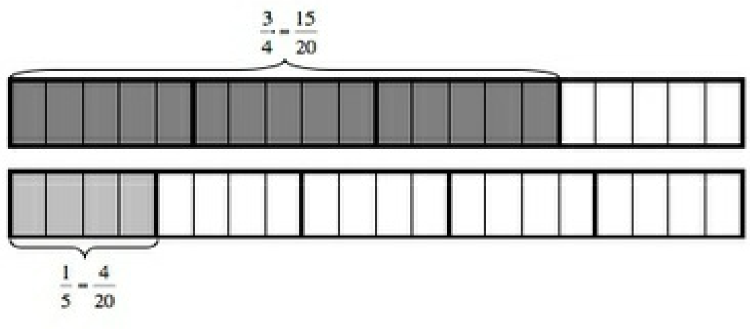


This is how we can write the process of finding this common denominator and adding using symbols:

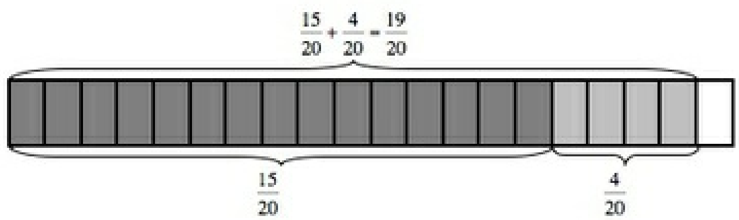
$$\begin{aligned}\frac{1}{5} + \frac{1}{15} &= \frac{1 \times 6}{5 \times 6} + \frac{1 \times 2}{15 \times 2} \\ &= \frac{6}{30} + \frac{2}{30} \\ &= \frac{6+2}{30} \\ &= \frac{8}{30} \\ &= \frac{4 \times 2}{15 \times 2} \\ &= \frac{4}{15}\end{aligned}$$

The picture shows that after we convert $\frac{1}{5}$ to $\frac{6}{30}$ and $\frac{1}{15}$ to $\frac{2}{30}$ we can add the thirtieths and are left with $\frac{8}{30}$. This solution is equivalent to $\frac{4\times2}{15\times2}\,\frac{4}{15}$, as we found above. Thus, we get the same answer using the two different common denominators that we identified in part (i), as we would expect.

iii. In order to find the solution to the addition problem $\frac{3}{4} + \frac{1}{5}$, we can first find a common denominator. 20 is a multiple of 4 and 5 because $4 \times 5 = 20$, meaning that 20 is a common denominator for the fractions $\frac{3}{4}$ and $\frac{1}{5}$. Here is a picture that shows these fractions when they are both written in terms of twentieths:



Here is a picture that represents $\frac{3}{4} + \frac{1}{5}$:



This is how we can write the process of finding a common denominator and adding using symbols:

$$\begin{aligned}\frac{3}{4} + \frac{1}{5} &= \frac{3 \times 5}{4 \times 5} + \frac{1 \times 4}{5 \times 4} \\ &= \frac{15}{20} + \frac{4}{20} \\ &= \frac{15 + 4}{20} \\ &= \frac{19}{20}\end{aligned}$$

The picture shows that when we convert $\frac{3}{4}$ to $\frac{15}{20}$ and $\frac{1}{5}$ to $\frac{4}{20}$ and add twentieths, we have $\frac{19}{20}$, as we found symbolically.

iv. In order to solve this addition problem we again find a common denominator for the fractions $\frac{14}{8}$ and $\frac{15}{12}$. 24 is a common multiple for the denominators 8 and 12 because $8 \times 3 = 24$ and $12 \times 2 = 24$. This means that 24 is a common denominator for the fractions $\frac{14}{8}$ and $\frac{15}{12}$. This is how we can write the process of finding a common denominator and adding using symbols:

$$\begin{aligned}\frac{14}{8}+\frac{5}{12} &= \frac{14\times 3}{8\times 3}+\frac{5\times 2}{12\times 2}\\ &= \frac{42}{24}+\frac{10}{24}\\ &= \frac{52}{24}\end{aligned}$$

If we pause for a moment, we will see that $\frac{52}{24}$ is equivalent to $\frac{13}{6}$, although either representation of the sum is correct.

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Solution: Alternative approach to part (c)

A student with a robust understanding of fraction equivalence might note that $\frac{14}{8}$ is equivalent to $\frac{7\times2}{4\times2}=\frac{7}{4}$. So:

$$\frac{14}{8}+\frac{5}{12}=\frac{7}{4}+\frac{5}{12}$$

12 is a common denominator for these two fractions, so:

$$\begin{aligned}\frac{14}{8}+\frac{5}{12} &= \frac{7}{4}+\frac{5}{12}\\ &= \frac{7\times 3}{4\times 3}+\frac{5}{12}\\ &= \frac{21}{12}+\frac{5}{12}\\ &= \frac{26}{12}\end{aligned}$$

Note that $\frac{26}{12}$ is equivalent to $\frac{52}{24}$ as well as $\frac{13}{6}$; any of these answers are acceptable.

Task:	
The Math	*What student work would demonstrate understanding of the task?*
Addressing	*What fraction understanding is this task addressing specifically?*
Building On or Building Toward	*How did this task either build on from the first task, or build toward the first task (Locate 1)?*
Progressions Connections	*What connections do you see in this task to the Progressions document pages you read?*