Reviewing Using the IMET: Mathematics

Module 103: Standards for Mathematical Practice and Access for All Learners (Alignment Criteria 2 and 3)
Participant Materials

Essential Questions:

- How does the **Instructional Materials Evaluation Tool (IMET)** reflect the major features of the Standards and the Shifts?
- What understandings support high-quality, accurate application of the IMET metrics?

Goals:

- ✓ Understand how aligned materials embody the shifts inherent in the Common Core State Standards
- ✓ Understand the precise meaning of each metric of the IMET
- ✓ Recognize examples and non-examples related to each metric of Alignment Criteria 2 and 3 of the IMET

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

AC Metric 2B: Materials attend to the full meaning of each practice standard.

- Over the course of any given year of instruction, is each mathematical practice Standard meaningfully present in the form of assignments, activities, or problems that stimulate students to develop the habits of mind described in the practice Standard?

Standards: MP.1 Make sense of problems and persevere in solving them.
4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

COMMON CORE PROFESSIONAL DEVELOPMENT

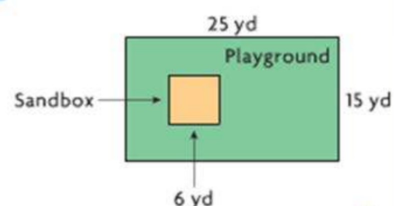
Building Mathematical Practices

CC.K–12.MP.1 Make sense of problems and persevere in solving them.

In this lesson, students are given the dimensions of a large rectangle and a small rectangle within it. Then students are asked to find the area inside the large rectangle that does not include the area of the small rectangle. Students must analyze the information and see the relationship between the areas of the two rectangles: the small area is part of the large area.

UNLOCK the Problem REAL WORLD

A landscaper is laying turf for a rectangular playground. The turf will cover the whole playground except for a square sandbox. The diagram shows the playground and sandbox. How many square yards of turf will the landscaper use?



Use the graphic organizer below to solve the problem.

Read the Problem	Solve the Problem
What do I need to find? I need to find how many square yards of turf the landscaper will use.	First, find the area of the playground. $A = b \times h$ $= 25 \times 15$ $= 375$ square yards
What information do I need to use? The turf will cover the playground. The turf will not cover the sandbox. The length and width of the playground are 25 yards and 15 yards. The side length of the square sandbox is 6 yards.	Next, find the area of the sandbox. $A = s \times s$ $= 6 \times 6$ $= 36$ square yards Last, subtract the area of the sandbox from the area of the playground. 375 − 36 339 square yards
How will I use the information? I can solve simpler problems. Find the area of the playground. Find the area of the sandbox.	So, the landscaper will use 339 square yards of turf to cover the playground.

AC Metric 2B: Materials attend to the full meaning of each practice standard.

- Are there teacher directed materials that explain the role of the practice Standards in the classroom and in students' mathematical development? Are alignments to practice Standards accurate?

Standards: MP.7 Look for and make use of structure.
4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

MATHEMATICAL PRACTICES

MP.7 Use Structure When students apply compensation to multiply using mental math they are using structure to solve a mathematical problem.

Pose the Problem ***Suppose you ride your bicycle 27 miles each week. What multiplication sentence can you write to represent the number of miles you ride in 3 weeks? How can you multiply mentally to find the product?*** Give students time to find the answer mentally any way they choose. Since this is a new skill for students, allow them to record their thinking on paper. Then have them share their methods and answers.

Model and Demonstrate On the board, write 3×27. ***What number can we substitute for 27 to make the mental multiplication easier?*** [30; 27 is close to 30] ***What is 3×30?*** [90] ***Multiplying three times 30 is easy to do mentally, but now we need to adjust our answer. How do you get from 27 to 30?*** [Add 3] ***To adjust, use inverse relationships and take away, or subtract, 3 groups of 3 from 90 to find the answer. What is 90 minus 9?*** [81]

AC Metric 2B: Materials attend to the full meaning of each practice standard.

- Are there teacher directed materials that explain the role of the practice Standards in the classroom and in students' mathematical development? Are alignments to practice

6 Attend to precision.

Precision refers not only to the correctness of arithmetic calculations, algebraic manipulations, and geometric reasoning but also to the proper use of mathematical language, symbols, and units to communicate mathematical ideas. Throughout [redacted] you will demonstrate your skills in these areas when you are asked to calculate, describe, show, explain, prove, and predict.

[redacted]

1 Make sense of problems and persevere in solving them.

In [redacted] you will work through Explores and Examples that present a solution pathway for you to follow. You are asked questions along the way so that you gain an understanding of the solution process, and then you will apply what you've learned in the Try This and Practice for the lesson.

[redacted]

<u>AC Metric 2B:</u> Materials attend to the full meaning of each practice standard.
Do the materials treat the practice standards as developing across grades or grade bands? Are the practice standards in early grades appropriately simple? Do they display an arc of growing sophistication across the grades?

<u>Standards:</u>

K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings2, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
1.OA.A.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

a) Ms. Munson has 42 pencils. She gives an equal number of pencils to 7 students. Which equation has an unknown value equal to the number of pencils each student gets?

A. 42 - ? =7

B. 7 + ? = 42

C. 42 - 7 = ?

D. 7 x ? = 42

b) Rachel collected bottles 3 days this week.

Monday	4 bottles
Tuesday	7 bottles
Wednesday	6 bottles

Write an equation to show the total number of bottles Rachel collected.

c) Darcy is putting 13 basketballs on a rack. Each row of the rack holds 5 basketballs. She fills 2 rows and puts the rest of the basketballs in the top row. How many balls are in the top row?

Select **all** the equations that correctly model this problem.

- ❏ 5 + 2 + 13 = ?
- ❏ 13 = 5 + 5 + ?
- ❏ 5 x 2 x ? = 13
- ❏ ? + 5 x 2 = 13

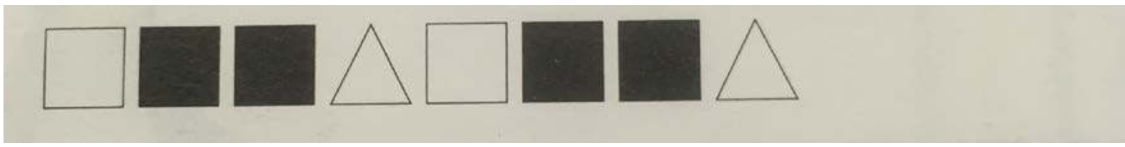
d) *An excerpt of a story read to students:*
Just when Sam was about to give up on the problem, his cousin, Tulip the two-toed sloth, came over to the sandbox from across the playground.

Sam explained that he was having trouble adding 3+4 because he only had three toes on each hand. Tulip thought she could help.

Think of a way that Tulip and Sam can work together using their sloth toes to figure out 3 + 4. (Remember that Tulip is a two-toed sloth.)

AC Metric 2A: Materials address the practice Standards in such a way as to enrich the Major Work of the grade; practices strengthen the focus on Major Work instead of detracting from it, in both teacher and student materials
Standard: MP.7 Look for and make use of structure.

Extend Patterns



Assume the pattern above is an ABBC repeating pattern.

1. **What will be the 10th shape in the pattern? ____________**
2. **What will be the 40th shape in the pattern? ____________**
3. **What will be the 50th shape in the pattern? ____________**

MP.7 Look for and make use of structure.
Discuss with students how figuring out the 10th shape is different than finding the 40th or 50th shape in the pattern. Encourage students to use division and their understanding of the meaning of remainders to reason about future shapes in the pattern without having to draw them all out. Be sure discussion is structured to bring out the following points:

- To determine the 40th shape in the pattern, you can divide 40 by 4 since there are four repeating terms in the pattern. The quotient is 10. So, after 10 full repeats of the pattern, the 40th shape will be a triangle since that is the last shape in the pattern.
- To determine the 50th shape in the pattern, you can divide 50 by 4 to get the quotient 12 R2. This tells us that after 12 full repeats, the 48th shape will be a triangle since that is the last shape in the pattern. So the 50th shape is the same as the second shape in the pattern, which is a black square.

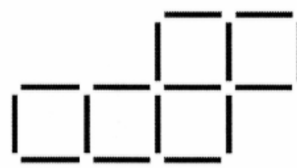
Throughout the conversation, make sure that students are connecting the pattern work to their understanding of the meaning of division and remainders.

AC Metric 2A: Materials address the practice Standards in such a way as to enrich the Major Work of the grade; practices strengthen the focus on Major Work instead of detracting from it, in both teacher and student materials
Standard: MP.7 Look for and make use of structure.

TOOTHPICK CHALLENGES

This challenge will require you to visualize new shapes that can be made from the toothpick designs below. Keep in mind that for no puzzle should you have toothpicks that overlap or "cross" each other. To help you solve the challenges below, you may want to get toothpicks and use them to build models.

a. The design at right is made with 16 toothpicks.



i. Move only 2 toothpicks so that the result has only 4 congruent (identical) squares.

ii. Starting with the original design, move 2 toothpicks so that the design has a total of 6 squares. Note: The squares do not need to have the same area.

iii. Starting with the original design, which 2 toothpicks could you move so that 5 squares of the same size remain? Is there more than one way to do this?

AC Metric 2C: Materials support the Standards' emphasis on mathematical reasoning**.**

- Do the materials support students in constructing viable arguments and critiquing the arguments of others concerning grade-level mathematics that is detailed in the content Standards?

Standard: MP.3 Construct viable arguments and critique the reasoning of others.

What's the Error?

Dear Math Students,

My friends and I are helping build flower boxes for a community garden. We are going to build 42 flower boxes. The building plans say each box needs 13 nails. I rounded to estimate how many nails we'll need. Since 40 × 10 = 400. I bought a box of 400 nails.

My friends say we won't have enough nails. Did I make a mistake? Can you help me estimate how many nails we need?

1. Write a response to the letter. Make sure to answer both questions.

Alignment Criteria 2: Materials must authentically connect content standards and practice standards
Standard/Cluster: 4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Lesson 1.2.3 What can a rectangular array show?

Characteristics of Numbers

Lesson Objective: Students will represent whole numbers with rectangular arrays and categorize numbers as prime, composite, odd, and/or even.

Mathematical Practices: Today students will continue building on the **structures** of the past two lessons, adding in the concepts of even, odd, composite, and prime. As students discuss these ideas, encourage them to also **attend to precision** in their use of vocabulary.

Length of Activity: One day (approximately 45 minutes)

Core Problems: Problems 1-62 through 1-65 (at least one part)

Materials: Lesson 1.2.3 Resource Page, one copy for class display

Pennies, at least 40 per team

Suggested Lesson Activity: Ask for a volunteer to read the lesson introduction and focus questions. If necessary, clarify the idea of a rectangular array. Then distribute pennies and direct teams to start on problem 1-62.

This might be a good time for a Participation Quiz where your focus is on Team Roles. As you circulate, encourage teams to use pennies as they think about the different arrangements. If you notice that students are recording their answers by drawing each dot in the rectangular array, ask them if they can think of an easier way to represent the size of the array without drawing each dot. Some students might suggest drawing a rectangle and labeling its length and width to represent the number of rows and columns in the array. Another option would be to describe the array's dimensions (such as "2 by 18") or by writing a numerical representation (such as 2(18)) instead of drawing a diagram.

Problem 1-63 introduces the terms "composite" and "prime." As you circulate, you can ask teams, *"Are all even numbers composite?"* or *"Is 0 prime or composite?"* to encourage further conversation. Ask teams to justify their ideas.

Some students will be challenged by problem 1-65 and some teams may not be able to finish. If this is the case, then the remainder of the problem can be skipped. This lesson is a good opportunity to promote the mathematical practice of making sense of problems and persevering in solving them and to support students as they begin to develop good communication and teamwork skills. Encourage students to use expressions and diagrams to help clarify their thinking and refer to the Team Support tab of this Teacher Edition for ideas about study team strategies that may be helpful. If students do not know how to start, for example, you might call a Huddle and provide the tip that students could create 4 equal rows (of 2 pennies, 3 pennies, etc.) with 3 left over, and check to see if any of those totals fit the other clues. Ask clarifying questions such as "Is the number even? How do you know?" (It can't be even because it has a remainder when divided into 2 rows). Continue to encourage students to test their thinking by forming pennies into rectangular arrays.

Universal Access:

Academic Literacy and Language Support: Consider using the strategy of paraphrasing for problems 1-62 through 1-65. For more information about the strategy of paraphrasing, refer to the Universal Access section of this Teacher Edition.

The mathematical terms odd and even can be especially challenging for English learners due to their multiple meanings in common English (**odd** – strange or unusual, **even** – tied at the end of a race, smooth surface, to divide evenly with no remainder, as in "...divide **evenly** into...", or "**even** if you could..."). Make sure that students have a grasp of the mathematical definitions of odd and even in addition to these other meanings.

Scaffolding: Problem 1-70 in the "Review & Preview" section might be particularly challenging for students who have a hard time visualizing quantities. Refer to the "Homework" section of these lesson notes for suggestions for supporting these students.

Additional Challenge: Problem 1-67 is provided as an additional challenge for students or teams who are interested or who have time. Students may recognize that what they are looking for in finding dimensions of possible prisms are unique factor triples, or unique sets of three factors that multiply to the given number.

Team Strategies:

While teams are working together, it is important to limit your interactions with individual students, as they can impair team conversation. Students bring up issues frequently that can draw you into an individual conversation at the expense of a conversation with the team. In addition, if students recognize that they can approach you for a direct answer, they will be less likely to turn to their team with questions. In general, team questions should come from the Resource Manager.

Mathematical Background:

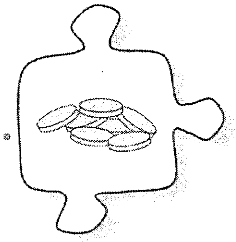
This section reminds students about some of the most basic descriptors of whole numbers. **Prime** numbers are those that have exactly two factor: themselves and 1. The five smallest prime numbers are 2, 3, 5, 7, and 11.

If a whole number is not prime, then it is **composite** and has other divisors between 1 and itself. The five smallest composite numbers are 4, 6, 8, 9, and 10.

The numbers 0 and 1 are not considered to be prime or composite. Negative integers are not covered by these definitions.

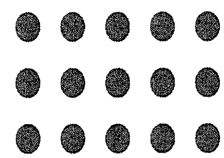
Even numbers are those that are evenly divisible by 2. **Odd** numbers are those that leave a remainder of 1 when divided by 2. So 0, 2, 4, and 106 are even; 1, 5, and 11 are odd. The definition of even and odd also applies to negative numbers.

1.2.3 What can a rectangular array show?



Characteristics of Numbers

In Lesson 1.2.1, you worked with your team to find different ways of showing different numbers of pennies. One arrangement that can be used to represent any whole number is a **rectangular array**. An example is shown at right. The horizontal lines of pennies are called *rows*, while the vertical lines of pennies are called *columns*.



Rectangular array for the number 15 with 3 rows and 5 columns

In this lesson, you will use rectangular arrays to investigate properties of numbers. Use the following questions to help focus your team's discussion today.

Can all numbers be represented the same way?

What can we learn about a number from its representations?

1-62. HOW MANY PENNIES? Part One

Jenny, Ann, and Gigi have different numbers of pennies. Each girl has between 10 and 40 pennies. Work with your team to figure out all the possible numbers of pennies that each girl could have. Use the clues given below. Be ready to explain your thinking to the class.

a. Jenny can arrange all of her pennies into a rectangular array that looks like a square. Looking like a square means it has the same number of rows as columns.
[Jenny has 16, 25, or 36 pennies.]

b. Ann can arrange all of her pennies into five different rectangular arrays.
[She could have 36. 36: 1 by 36, 2 by 18, 3 by 12, 4 by 9, 6 by 6]

c. Whenever Gigi arranges her pennies into a rectangular array with more than one row or column, she has a **remainder** (in this case leftover pennies). **[11, 13, 17, 19, 23, 29, 31, or 37]**

1-63. What can you learn about a number from its rectangular arrays? Consider this question as you complete parts (a) and (b) below.

a. A number that can be arranged into more than one rectangular array, such as Ann's in part (b) of problem 1-62, is called a **composite number**. List all composite numbers less than 15. **[4, 6, 8, 9, 10, 12, 14]**

b. Consider the number 17, which could be Gigi's number. Seventeen pennies can be arranged into only one rectangular array: 1 penny by 17 pennies. Any number, like 17, that can form only one rectangular array is called a **prime number**. Work with your team to find all prime numbers less than 25. **[2, 3, 5, 7, 11, 13, 17, 19, and 23]**

1-64. Jenny, Ann, and Gigi were thinking about **odd** and **even** numbers. (When **even** numbers are divided by two, there is no remainder. When **odd** numbers are divided by two, there is a remainder of one.) Jenny said, *"Odd numbers cannot be formed into a rectangle with two rows. Does that mean they are prime?"*

Consider Jenny's question with your team. Are all odd numbers prime? If so, explain how you know. If not, find a **counterexample**. A counterexample is an example that can be used to show a statement is false (in this case, finding a number that is odd but not prime). **[No, not all odd numbers are prime. Composite odd numbers less than 25: 9, 15, 21]**

1-65. HOW MANY PENNIES? Part Two

Work with your team to figure out how many pennies (between 10 and 40) each person could have. Use the clues given below. You may want to use diagrams or expressions to help you determine your answers. Can you find more than one possible answer?

a. When Xander arranges his pennies into a rectangle with more than one row, he always has some leftover pennies. When he uses two equal rows or three equal rows, he has one leftover penny. When he arranges them into a rectangle with four equal rows, he has three leftover pennies. **[One possible solution: 19 (2 by 9 R1, 3 by 6 R1, 4 by 4 R3)]**

b. When Jorge arranges his pennies into a rectangle with two equal rows, three equal rows, or five equal rows, he has one leftover penny. When he arranges his pennies into a rectangle with four equal rows, he has three leftover pennies. How many pennies could Jorge have? **[One possible solution: 31 (2 by 15 R1, 3 by 10 R1, 5 by 6 R1, 4 by 7 R3)]**

c. When Louisa arranges her pennies into a rectangle with two equal rows, three equal rows, four equal rows, or six equal rows, she has one leftover penny. When she arranges her pennies into a rectangle with five equal rows, she has two leftover pennies. How many pennies could Louisa have? **[One possible solution: 37 (2 by 18 R1, 3 by 12 R1, 4 by 9 R1, 6 by 6 R1, 5 by 7 R2)]**

Guidelines for Design of Mathematics Instruction and Materials for ELLs

Keep tasks focused on high cognitive demand, conceptual understanding, and correspondences among representations. Mathematics instruction for ELLs should follow the general recommendations for high-quality mathematics instruction: a) Focus on mathematical concepts and connections among those concepts; and b) Use and maintain high cognitive-demand mathematical tasks, for example by encouraging students to explain their problem solving and reasoning.

Explanations and justification need not always include words. Instruction should support students in learning to develop oral and written explanations, but students can also show conceptual understanding by using diagrams and other representations. For example, students might use an area model to show that two fractions are equivalent of how multiplication by a positive fraction smaller than one makes the result smaller.

Understanding Language. (2012). Guidelines for Design of Mathematics Instruction and Materials for ELLs. Available online http://ell.stanford.edu/

<u>AC Metric 3A:</u> Support for English Language Learners and other special populations is thoughtful and helps those students meet the same Standards as all other students. The language in which problems are posed is carefully considered.

T: (Write $\frac{1}{2} = \frac{ \times }{ \times } = \frac{2}{}$. Point to $\frac{1}{2}$.) Say the unit fraction.

S: 1 half.

T: On your personal white boards, fill in the unknown numbers to make an equivalent fraction.

S: (Write $\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$.)

Continue with the following possible sequence: $\frac{1}{2} = \frac{4}{8}$, $\frac{1}{3} = \frac{2}{6}$, $\frac{1}{3} = \frac{3}{9}$, $\frac{1}{4} = \frac{4}{16}$, $\frac{1}{5} = \frac{3}{15}$.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Fluency drills are fun, fast-paced math games. Be careful not to leave English language learners behind. Make sure to clarify that *common unit, common denominator, like unit,* and *like denominator* are terms that refer to the same thing and are often used in math class interchangeably.

<u>AC Metric 3B:</u> Materials provide appropriate level and type of scaffolding, differentiation, intervention, and support for a broad range of learners with gradual removal of supports, when needed, to allow students to demonstrate their mathematical understanding independently.

5. Compare each pair of fractions using >, <, or =. Draw a model if you choose to.

a. $\frac{3}{4}$ ________ $\frac{3}{7}$

b. $\frac{4}{5}$ ________ $\frac{8}{12}$

c. $\frac{7}{10}$ ________ $\frac{3}{5}$

l. $\frac{2}{3}$ ________ $\frac{11}{15}$

e. $\frac{3}{4}$ ________ $\frac{11}{12}$

$\frac{7}{3}$ ________ $\frac{7}{4}$

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Support English language learners as they explain their reasoning for Problems 5(a), 5(d), and 5(f) on the Problem Set. Provide a word bank with corresponding pictures.

Possible words for the word bank are listed below:

fourth seventh third fifteenth
whole ninth one closer
greater than less than almost
tape diagram

AC Metric 3C: Design of lessons recommends and facilitates a mix of instructional approaches for a variety of learners (e.g., using multiple representations, asking a range of questions, checking for understanding, flexible grouping, pair-share, deconstructing/reconstructing the language of problems).

EMBEDDED INSTRUCTIONAL STRATEGIES

A wide variety of instructional strategies are implemented throughout the Math Series textbooks. These strategies are designed to support student attainment of the Standards for Mathematical Practice. In each example shown, one or two connections to the Mathematical Practice Standards are made explicit. However, each strategy supports multiple standards.

Problem Type: Real-World Connections

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that the quantitative relationships seen in the real world are no different than the quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.

Problem Type: Worked Examples

Research shows students learn best when they are actively engaged with a task. Often students only focus or mentally engage with a problem when they're required to produce a "product" or "answer." We offer a different approach to worked examples to help students better benefit from this mode of instruction. Many students need a model to know how to engage effectively with worked examples. Students need to be able to question their understanding, make connections with steps, and ultimately self-explain (the progression of the steps and the final outcome).

Problem Type: Analyzing Student Methods

Pre-written student methods provide a framework that allows students the opportunity to analyze viable methods and problem-solving strategies. Questions are presented along with the student methods to help students think more deeply about the various strategies. This problem type is designed to foster flexibility and a student's internal dialogue about the mathematics and strategies used to solve problems.

Problem Type: Analyzing Correct and Incorrect Responses

This problem type shifts the focus to an analysis of correct and incorrect responses. One goal of these problems is to help students make inferences about correct responses. Research shows that only providing positive examples does not eliminate some of the things students may think; it is also important to show negative examples. From the incorrect responses, students will learn to determine where the error is, why it is an error, and how to correct it. These types of problems will help students analyze their own work for errors and correctness.

Problem Type: Who's Correct?

"Who's Correct?" problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not told who is correct. Students have to think more deeply about what the strategies really mean and whether the solutions make sense. Students will determine what is correct and what is incorrect, and then explain their reasoning. These types of problems will help students analyze their own work for errors and correctness.

Problem Type: Manipulatives

Manipulatives are used throughout the curricula to foster a conceptual understanding of mathematical concepts. These activities provide students with opportunities to develop strategies and reasoning that will serve as the foundation for learning more abstract mathematics. Our goal is for students to ultimately perform operations and exhibit their understanding without using manipulatives. To foster the transfer of student understanding from concrete manipulatives to the abstract procedures, we use a variety of instructional prompts.

Problem Type: Matching, Sorting, and Exploring

Students will experience various hands-on activities that match or sort verbal descriptions, tables, and graphs. These activities help develop skills of recognizing and categorizing patterns in mathematics.

Problem Type: Using Technology

Step-by-step instructions provide students with opportunities to understand how to use graphing calculators.

Problem Type: Graphic Organizers

Students will use graphic organizers to create their own references of key mathematical concepts.

Problem Type: Talk the Talk

In Talk the Talk, open-ended questions require students to summarize and generalize their mathematical understandings and key concepts. An authored review of the major mathematical concept(s) or rule(s) from the lesson is stated so that students have a concise, accurate reference for review.

Alignment Criteria 3: Materials must provide supports for English Language Learners and other special populations.
Standard: 4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

UNIT 7 LESSON 2

Fractions on the Number Line

LESSON FOCUS

- Use the number-line model for fractions.

CA CC
Mathematical Practice
MP.1, MP.2, MP.3, MP.4, MP.5, MP.6, MP.7
Mathematical Content
4.NF.2

The Day at a Glance

USE MATH TALK TODAY!

Teaching the Lesson

MATH BACKGROUND for this lesson is included on pp. 597BB–597EE.

ACTIVITY FOCUS

Activity 1 Locate fractions and mixed numbers on number lines.
Activity 2 Practice with number lines.
Activity 3 Use benchmarks to compare fractions.

MATERIALS

- Student Activity Book pp. 251–254 or Student Hardcover Book pp. 251–254 and Activity Workbook p. 95
- Whiteboard

Differentiated Instruction • Universal Access

MATERIALS

- Activity Cards 7-2 • Measuring Lengths (TRB M19) • Inch ruler • Math Journals
- Personal Math Trainer • MegaMath • Destination Math®
- RtI Tier 1 • Tier 2 • Tier 3

Homework and Spiral Review

MATERIALS

- Homework and Remembering pp. 151–152

QUICK PRACTICE 5 MINUTES

Goal Practice Comparing Fractions
Write the fraction pairs $\frac{1}{3}$ and $\frac{1}{5}$, $\frac{4}{12}$ and $\frac{7}{12}$, and $\frac{3}{8}$ and $\frac{3}{10}$ on the board, leaving space between the fractions in each pair for a comparison symbol. Invite a **Student Leader** to come to the board.

Student Leader (pointing to the smaller denominator of the first pair) "Fewer parts means..."

Class "Larger parts, so greater fractions."

The Student Leader writes > between the fractions.

Continue with two more Student Leaders for the fraction pairs $\frac{4}{12} < \frac{7}{12}$ and $\frac{3}{8} > \frac{3}{10}$.

Teaching the Lesson | Differentiated Instruction | Homework and Spiral Review

CA CC

Mathematical Practice
MP.2, MP.3, MP.6

Mathematical Content
4.NF.2

20 MINUTES

FOCUS Locate fractions and mixed numbers on number lines.

MATERIALS Student Activity Book p. 251 or Student Hardcover Book p. 251, transparency of Student Activity Book p. 251 (optional)

GO DIGITAL

Whiteboard Lesson

Teaching Note

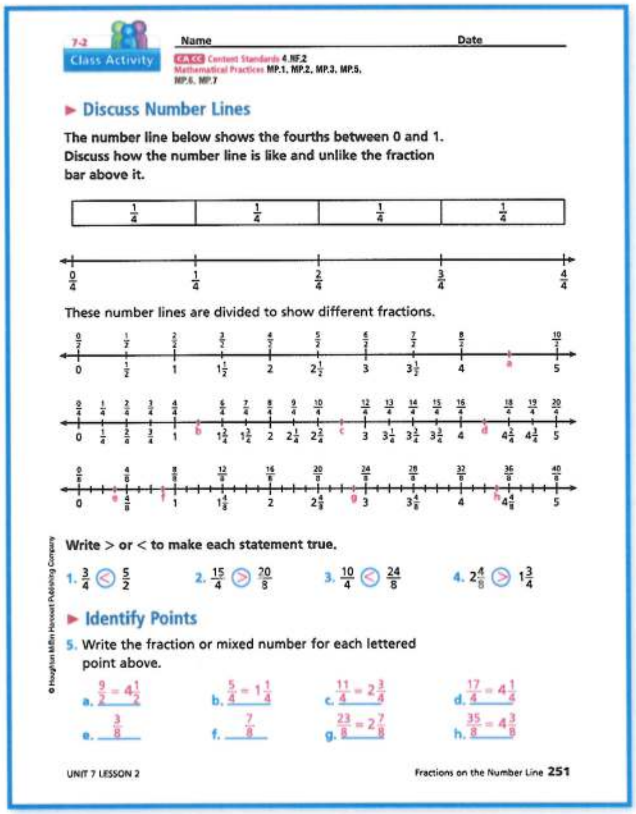
What to Expect from Students
Number lines are more difficult than fraction bars for students to understand. Number lines, like fractions bars, are length models. However, the labeling at the end of a fractional length instead of in the middle is confusing for some students. They don't realize that each fraction length is the total of the unit fractions from the beginning of the number line. Looking at the fraction bar above the number line can help them see this conceptual difference.

The Learning Community—Best Practices

MATH TALK Encourage students to respond before you do. Allow time for students to make comments or ask questions about each other's work before you begin to speak. If you tend to speak first, the students will not take ownership of their role as crucial participants in the discourse; they will look to you instead.

ACTIVITY 1

Number Lines



7-2 Class Activity

Name Date

CA CC Content Standards 4.NF.2
Mathematical Practices MP.1, MP.2, MP.3, MP.5, MP.6, MP.7

▶ Discuss Number Lines

The number line below shows the fourths between 0 and 1. Discuss how the number line is like and unlike the fraction bar above it.

These number lines are divided to show different fractions.

Write > or < to make each statement true.

1. $\frac{3}{4} < \frac{5}{2}$ 2. $\frac{15}{4} > \frac{20}{8}$ 3. $\frac{10}{4} < \frac{24}{8}$ 4. $2\frac{4}{8} > 1\frac{3}{4}$

▶ Identify Points

5. Write the fraction or mixed number for each lettered point above.

a. $\frac{9}{2} = 4\frac{1}{2}$ b. $\frac{5}{4} = 1\frac{1}{4}$ c. $\frac{11}{4} = 2\frac{3}{4}$ d. $\frac{17}{4} = 4\frac{1}{4}$

e. $\frac{3}{8}$ f. $\frac{7}{8}$ g. $\frac{23}{8} = 2\frac{7}{8}$ h. $\frac{35}{8} = 4\frac{3}{8}$

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UNIT 7 LESSON 2 Fractions on the Number Line 251

Student Activity Book page 251

▶ Discuss Number Lines WHOLE CLASS MATH TALK

MP.6 Attend to Precision Explain a Representation Have students look at the number line and fraction bar at the top of the Student Book page 251. Discuss how the number line is like the fraction bar and how it is different. Be sure the following points are discussed:

- In the fraction bar, each unit fraction is labeled separately, with the label $\frac{1}{4}$ inside each section. The fraction $\frac{1}{4}$ is represented by 1 section, the fraction $\frac{2}{4}$ is represented by two sections, and so on.
- In a number line, the labeling is cumulative from the 0. The label appears at the end of a fraction division. For example, the label $\frac{1}{4}$ is at the point $\frac{1}{4}$ of the distance from 0 to 1. The label $\frac{2}{4}$ is at the point $\frac{2}{4}$ of the distance from 0 to 1, and so on.

Different Fractional Divisions on Number Lines Discuss the number lines in the middle of the student page. You can use an overhead transparency of the student page to help illustrate the main points of the discussion.

Help students see that the number of equal divisions between 0 and 1 determine the unit fraction. For example, on the first number line, the interval between 0 and 1 is divided into two equal parts, so each mark indicates $\frac{1}{2}$. Emphasize that the labels—$\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, and so on—show the total lengths from 0 to the mark. Point out that the labels above the number lines show numbers greater than 1 as improper fractions. The labels below the line show the numbers as mixed numbers.

Explain that, because the number lines are aligned, we can use them to compare fractions even if there are neither like numerators nor like denominators. Have students set a straightedge on the page to verify $\frac{3}{4}$ is less than $\frac{5}{2}$. Have them record this result in Exercise 1. Have students use this method to complete Exercises 2–4, and then discuss the results.

▶ Identify Points PAIRS

MP.2 Reason Abstractly and Quantitatively Connect Symbols and Models Have students work in **Student Pairs** to complete Exercise 5, which asks them to identify the lettered points on the number line.

ACTIVITY 2

Practice With Number Lines

▶ Number Lines for Thirds and Sixths WHOLE CLASS

Use **Solve and Discuss** for Exercises 6–8 on Student Book page 252. You may wish to display an overhead transparency of the page.

The number line in Exercise 6 shows thirds, but some students may see fourths because they will count the division marks instead of the units between the marks.

In the number line for Exercise 7, each $\frac{1}{3}$ has been 2-fractured to make sixths. Each unit length is $\frac{1}{6}$. The thirds are still visible because the thirds marks are a little longer.

Activity continued ▶

English Learners

Draw a 0 to 1 number line showing points that are intervals of fourths. Have students read the labels. Write ***interval***.

EMERGING

Say: **From one point to the next point is an interval. Each interval equals $\frac{1}{4}$.** Have students repeat.

EXPANDING

Ask: **Is this a number line or a fraction bar?** number line Say: **Each interval equals __.** $\frac{1}{4}$

BRIDGING

Have students describe the intervals on the number line.

CA CC
Mathematical Practice
MP.5, MP.7
Mathematical Content
4.NF.2

20 MINUTES

FOCUS Practice with number lines.

MATERIALS Student Activity Book p. 252 or Student Hardcover Book p. 252 and Activity Workbook p. 95, transparency of Student Activity Book p. 252 (optional)

GO DIGITAL

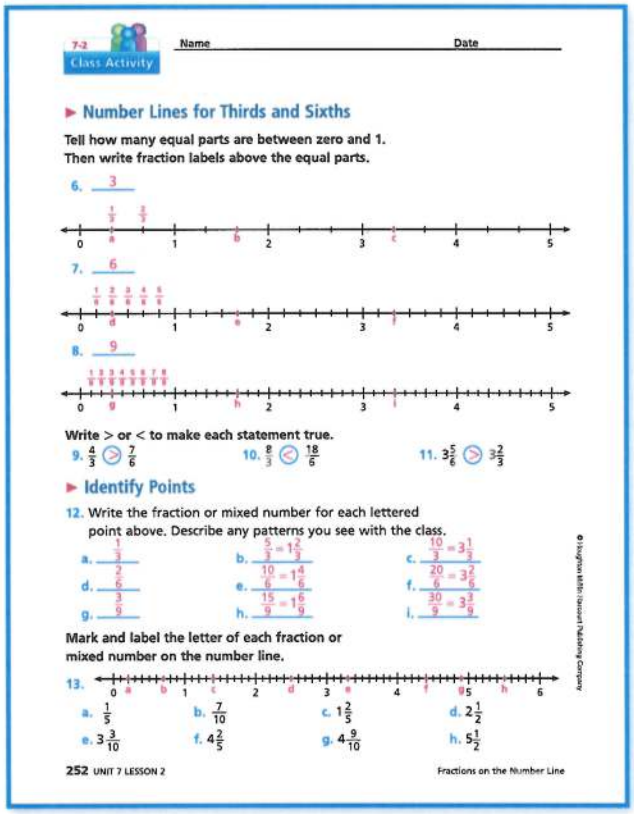
Whiteboard Lesson

The Learning Community—Best Practices

Helping Community Create a classroom where students are not competing, but desire to collaborate and help one another. Communicate often that your goal as a class is that everyone understands the math you are studying. Tell students that this will require everyone working together to help each other.

Teaching Note

What to Expect from Students Some students may notice by observing the number lines that they show different names for the same fraction, such as $\frac{2}{3}$ and $\frac{6}{9}$. Point out that fractions that represent the same part of a whole are called *equivalent fractions*, and that students will learn about equivalent fractions in Lesson 4 of this unit.


7-2 Class Activity

Name ____ Date ____

▶ Number Lines for Thirds and Sixths

Tell how many equal parts are between zero and 1. Then write fraction labels above the equal parts.

6. 3

7. 6

8. 9

Write > or < to make each statement true.

9. $\frac{4}{3} > \frac{7}{6}$ 10. $\frac{8}{3} < \frac{18}{6}$ 11. $3\frac{5}{6} > 3\frac{2}{3}$

▶ Identify Points

12. Write the fraction or mixed number for each lettered point above. Describe any patterns you see with the class.

a. $\frac{1}{3}$ b. $\frac{5}{3} = 1\frac{2}{3}$ c. $\frac{10}{3} = 3\frac{1}{3}$

d. $\frac{2}{6}$ e. $\frac{10}{6} = 1\frac{4}{6}$ f. $\frac{20}{6} = 3\frac{2}{6}$

g. $\frac{3}{9}$ h. $\frac{15}{9} = 1\frac{6}{9}$ i. $\frac{30}{9} = 3\frac{3}{9}$

Mark and label the letter of each fraction or mixed number on the number line.

13.

a. $\frac{1}{5}$ b. $\frac{7}{10}$ c. $1\frac{2}{5}$ d. $2\frac{1}{2}$

e. $3\frac{3}{10}$ f. $4\frac{2}{5}$ g. $4\frac{9}{10}$ h. $5\frac{1}{2}$

252 UNIT 7 LESSON 2 Fractions on the Number Line

Student Activity Book page 252

MP.5 Use Appropriate Tools Draw a Diagram Students may want to draw a number line or use the diagrams from Exercises 6–8 to help them complete Exercises 9–11. Students should circle lengths on the number lines to compare the fractions.

▶ Identify Points PAIRS

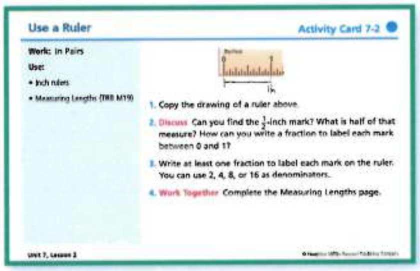
MP.7 Look for Structure Identify Relationships Have students work in **Student Pairs** to complete Exercises 12 and 13. Again, emphasize that the numbers for the points mean the total lengths from 0 to that point.

In Exercise 12, students should notice that the fractions or mixed numbers in each column name the same point on the number line. Some students may also notice that for each fraction in the second row, the numerators and denominators are twice those of the fraction above. Some may even recognize that they could use this pattern to create additional fractions that name the same number.

Intervention

for students having difficulty

RtI Tier 2



Use a Ruler — Activity Card 7-2

Work: In Pairs

Use:
- Inch rulers
- Measuring Lengths (TRB M19)

1. Copy the drawing of a ruler above.
2. Discuss Can you find the $\frac{1}{2}$-inch mark? What is half of that measure? How can you write a fraction to label each mark between 0 and 1?
3. Write at least one fraction to label each mark on the ruler. You can use 2, 4, 8, or 16 as denominators.
4. Work Together Complete the Measuring Lengths page.

Unit 7, Lesson 2

Activity Notes This activity will introduce students to another model for representing fractions. Point out that the marks are different lengths. The marks for halves are the longest and the marks for sixteenths are the shortest.

Encourage students to discuss their results with other students. Students may notice that there are different fraction names for the same fraction, such as $\frac{8}{16}$ and $\frac{1}{2}$. This may help to get them ready to learn about equivalent fractions later in this unit.

Math Writing Prompt

Describe the Process Describe how you would draw a number line showing fourths between 0 and 3. What other fractions (besides fourths) could you show on that number line?

Personal Math Trainer

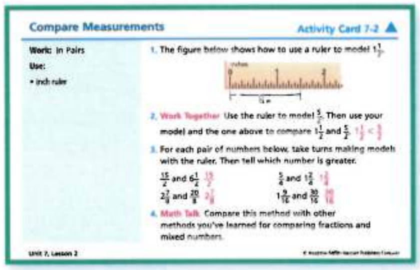
Digital Support

4.NF.2

On Level

for students having success

RtI Tier 1



Compare Measurements — Activity Card 7-2

Work: In Pairs

Use:
- Inch ruler

1. The figure below shows how to use a ruler to model $1\frac{1}{2}$.
2. Work Together Use the ruler to model $\frac{5}{2}$. Then use your model and the one above to compare $1\frac{1}{2}$ and $\frac{5}{2}$. $1\frac{1}{2} < \frac{5}{2}$
3. For each pair of numbers below, take turns making models with the ruler. Then tell which number is greater.

   $\frac{15}{2}$ and $6\frac{1}{2}$ $\frac{15}{2}$ — $\frac{5}{4}$ and $1\frac{2}{4}$ $1\frac{2}{4}$

   $2\frac{7}{8}$ and $\frac{20}{8}$ $2\frac{7}{8}$ — $1\frac{9}{16}$ and $\frac{30}{16}$ $\frac{30}{16}$
4. Math Talk Compare this method with other methods you've learned for comparing fractions and mixed numbers.

Unit 7, Lesson 2

Activity Notes In this activity, students will use rulers as models for comparing fractions and mixed numbers with like denominators.

When students compare this method with others they've learned so far in this unit, they should recognize that this method for comparing fractions and mixed numbers is limited to denominators of 2, 4, 8, and 16. Some students may also notice that this method can be used to compare some fractions that have neither like denominators nor like numerators.

Math Writing Prompt

Compare Models How is showing a fraction on a number line the same as showing the fraction with fractions bars? How is it different?

MegaMath

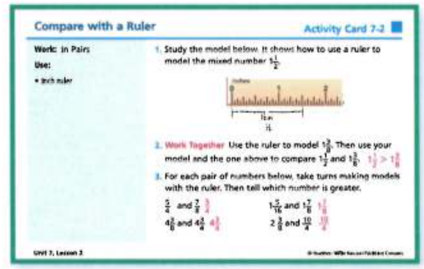
Digital Support

Fraction Action

Number Line Mine, Level G

Challenge

for students seeking a challenge



Compare with a Ruler — Activity Card 7-2

Work: In Pairs

Use:
- Inch ruler

1. Study the model below. It shows how to use a ruler to model the mixed number $1\frac{1}{2}$.
2. Work Together Use the ruler to model $1\frac{3}{8}$. Then use your model and the one above to compare $1\frac{1}{2}$ and $1\frac{3}{8}$. $1\frac{1}{2} > 1\frac{3}{8}$
3. For each pair of numbers below, take turns making models with the ruler. Then tell which number is greater.

   $\frac{5}{4}$ and $\frac{7}{8}$ $\frac{5}{4}$ — $1\frac{5}{16}$ and $1\frac{7}{8}$ $1\frac{7}{8}$

   $4\frac{5}{8}$ and $4\frac{3}{4}$ $4\frac{3}{4}$ — $2\frac{3}{8}$ and $\frac{10}{4}$ $\frac{10}{4}$

Unit 7, Lesson 2

Activity Notes Students will use rulers as models for comparing fractions (including fractions greater than 1) and mixed numbers that have neither like denominators nor like numerators. Later in this unit, students will learn multiple methods for comparing such fractions, and this activity gives them a preview.

If time allows, encourage students to repeat the last part of the activity for fractions or mixed numbers of their choice. They should be aware that the denominators of the fractions should be 2, 4, 8, or 16.

Math Writing Prompt

Models for Equivalent Fractions To compare two equivalent fractions using fraction strips, you show that the strips are the same length. How can you compare two equivalent fractions using a number line?

Destination Math®

Digital Support

Course III: Module 3: Unit 1: Session 4: Ordering and Rounding Fractions